

CLASSIC 2 FLAVOUR COLOR SUPERCONDUCTIVITY AND ORDINARY NUCLEAR MATTER-A NEW PARADIGM STATEMENT

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Abstract- A system of ordinary nuclear matter, the resultant of classic 2-flavor color superconductivity is investigated. It is shown that the time independence of the contributions one system to another without the transitional phase portrays another system by itself and constitutes the equilibrium solution of the original time independent system. Methodology is accentuated with the explanations, we write the governing equations with the nomenclature for the systems in the foregoing. Further papers extensively draw inferences upon such concatenation process, ipsofacto.

Index Terms- CCFSC, ORDINARY NUCLEAR MATTER, QCD, QGP

I. INTRODUCTION

Frank Wilczek expatiated on a first cut at applying the lessons learned from color-flavor locking and quark-hadron continuity to real QCD, which is complicated by splitting between strange and light quarks. Both classic 2-flavor color superconductivity (with the strange quark passive) and color-flavor locking are valid ground states in different parameter regimes at high density. An extremely intriguing possibility, matter or QCD matter refers to any of a number of theorized phases of matter whose degrees of freedom include quarks and gluons. These theoretical phases would occur at extremely high temperatures and densities, billions of times higher than can be produced in equilibrium in laboratories. Under such extreme conditions, the familiar structure of matter, where the basic constituents are nuclei (consisting of nucleons which are bound states of quarks) and electrons, is disrupted. In quark matter it is more appropriate to treat the quarks themselves as the basic degrees of freedom.

In the standard model of particle physics, the strong force is described by the theory of quantum chromo dynamics (QCD). At ordinary temperatures or densities this force just confines the quarks into composite particles (hadrons) of size around $10-15$ m = 1 femtometer = 1 fm (corresponding to the QCD energy scale $\Lambda_{\text{QCD}} \approx 200$ MeV) and its effects are not noticeable at longer distances. However, when the temperature reaches the QCD energy scale (T of order 1012 Kelvin's) or the density rises to the point where the average inter-quark separation is less than 1 fm (quark chemical potential μ around 400 MeV), the hadrons are melted into their constituent quarks, and the strong interaction becomes the dominant feature of the physics. Such phases are called quark matter or QCD matter.

The strength of the color force makes the properties of quark matter unlike gas or plasma, instead leading to a state of matter more reminiscent of a liquid. At high densities, quark matter is a Fermi liquid, but is predicted to exhibit color superconductivity at high densities and temperatures below 1012 K.

II. UNSOLVED PROBLEMS IN PHYSICS

QCD in the non-perturbative regime quark matter, the equations of QCD predict that a sea of quarks and gluons should be formed at high temperature and density. What are the properties of this phase of matter?

In early Universe, at high temperature according to the Big Bang theory, when the universe was only a few tens of microseconds old, the phase of matter took the form of a hot phase of quark matter called the quark-gluon plasma (QGP).

Compact stars (neutron stars). A neutron star is much cooler than 1012 K, but it is compressed by its own weight to such high densities that it is reasonable to surmise that quark matter may exist in the core. Compact stars composed mostly or entirely of quark matter are called quark stars or strange stars, yet at this time no star with properties expected of these objects has been observed.. Cosmic rays comprise also high energy atomic nuclei, particularly that of iron. Laboratory experiments suggest that interaction with heavy noble gas in the upper atmosphere would lead to quark-gluon plasma formation. Heavy-ion collisions at very high energies can produce small short-lived regions of space whose energy density is comparable to that of the 20-microsecond-old universe. This has been achieved by colliding heavy nuclei at high speeds, and a first time claim of formation of quark came from the SPS accelerator at CERN in February 2000. There is good evidence that the quark-gluon plasma has also been produced at RHIC.

The context for understanding the thermodynamics of quark matter is the standard model of particle physics, which contains six different flavors of quarks, as well as leptons like electrons and neutrinos. These interact via the strong interaction, electromagnetism, and also the weak interaction which allows one flavor of quark to turn into another. Electromagnetic interactions occur between particles that carry electrical charge; strong interactions occur between particles that carry color charge.

The correct thermodynamic treatment of quark matter depends on the physical context. For large quantities that exist for long periods of time (the "thermodynamic limit"), we must take into account the fact that the only conserved charges in the standard model are quark number (equivalent to baryon number),

electric charge, the eight color charges, and lepton number. Each of these can have an associated chemical potential. However, large volumes of matter must be electrically and color-neutral, which determines the electric and color charge chemical potentials. This leaves a three-dimensional phase space, parameterized by quark chemical potential, lepton chemical potential, and temperature.

In compact stars quark matter would occupy cubic kilometers and exist for millions of years, so the thermodynamic limit is appropriate. However, the neutrinos escape, violating lepton number, so the phase space for quark matter in compact stars only has two dimensions, temperature (T) and quark number chemical potential μ . A strangelet is not in the thermodynamic limit of large volume, so it is like an exotic nucleus: it may carry electric charge.

A heavy-ion collision is in neither the thermodynamic limit of large volumes nor long times. Putting aside questions of whether it is sufficiently equilibrated for thermodynamics to be applicable, there is certainly not enough time for weak interactions to occur, so flavor is conserved, and there are independent chemical potentials for all six quark flavors. The initial conditions (the impact parameter of the collision, the number of up and down quarks in the colliding nuclei, and the fact that they contain no quarks of other flavors) determine the chemical potentials.

Conjectured form of the phase diagram of QCD matter (From Wikipedia)

The phase diagram of quark matter is not well known, either experimentally or theoretically. A commonly conjectured form of the phase diagram is shown in the figure.[3] It is applicable to matter in a compact star, where the only relevant thermodynamic potentials are quark chemical potential μ and temperature T . For guidance it also shows the typical values of μ and T in heavy-ion collisions and in the early universe. For readers who are not familiar with the concept of a chemical potential, it is helpful to think of μ as a measure of the imbalance between quarks and antiquarks in the system. Higher μ means a stronger bias favoring quarks over antiquarks. At low temperatures there are no anti quarks, and then higher μ generally means a higher density of quarks.

Ordinary atomic matter as we know it is really a mixed phase, droplets of nuclear matter (nuclei) surrounded by vacuum, which exists at the low-temperature phase boundary between vacuum and nuclear matter, at $\mu = 310$ MeV and T close to zero. If we increase the quark density (i.e. increase μ) keeping the temperature low, we move into a phase of more and more compressed nuclear matter. Following this path corresponds to burrowing more and more deeply into a neutron star. Eventually, at an unknown critical value of μ , there is a transition to quark matter. At ultra-high densities we expect to find the color-flavor-locked (CFL) phase of color-superconducting quark matter. At intermediate densities we expect some other phases (labelled "non-CFL quark liquid" in the figure) whose nature is presently unknown. They might be other forms of color-superconducting quark matter, or something different.

Starting at the bottom left corner of the phase diagram, in the vacuum where $\mu = T = 0$. If we heat up the system without introducing any preference for quarks over antiquarks, this

corresponds to moving vertically upwards along the T axis. At first, quarks are still confined and we create a gas of hadrons (pions, mostly). Then around $T = 170$ MeV there is a crossover to the quark gluon plasma: thermal fluctuations break up the pions, and we find a gas of quarks, antiquarks, and gluons, as well as lighter particles such as photons, electrons, positrons, etc. Following this path corresponds to travelling far back in time (so to say), to the state of the universe shortly after the big bang (where there was a very tiny preference for quarks over antiquarks).

The line that rises up from the nuclear/quark matter T transition and then bends back towards the T axis, with its end marked by a star, is the conjectured boundary between confined and unconfined phases. Until recently it was also believed to be a boundary between phases where chiral symmetry is broken (low temperature and density) and phases where it is unbroken (high temperature and density). It is now known that the CFL phase exhibits chiral symmetry breaking, and other quark matter phases may also break chiral symmetry, so it is not clear whether this is really a chiral transition line. The line ends at the "chiral critical point", marked by a star in this figure, which is a special temperature and density at which striking physical phenomena, analogous to critical opalescence, are expected. For a complete description of phase diagram it is required that one must have complete understanding of dense, strongly interacting hadronic matter and strongly interacting quark matter from some underlying theory e.g. quantum chromodynamics (QCD). However because such a description requires the proper understanding of QCD in its non-perturbative regime, which is still far from being completely understood, any theoretical advance remains very challenging.

III. THEORETICAL CHALLENGES: CALCULATION TECHNIQUES

The phase structure of quark matter remains mostly conjectural because it is difficult to perform calculations predicting the properties of quark matter. The reason is that QCD, the theory describing the dominant interaction between quarks, is strongly coupled at the densities and temperatures of greatest physical interest, and hence it is very hard to obtain any predictions from it. Here are brief descriptions of some of the standard approaches.

LATTICE GAUGE THEORY

The only first-principles calculational tool currently available is lattice QCD, i.e. brute-force computer calculations. Because of a technical obstacle known as the fermion sign problem, this method can only be used at low density and high temperature ($\mu < T$), and it predicts that the crossover to the quark-gluon plasma will occur around $T = 170$ MeV. However, it cannot be used to investigate the interesting color-superconducting phase structure at high density and low temperature.

WEAK COUPLING THEORY

Because QCD is asymptotically free it becomes weakly coupled at unrealistically high densities, and diagrammatic methods can be used. Such methods show that the CFL phase occurs at very

high density. At high temperatures, however, diagrammatic methods are still not under full control.

MODELS

To obtain a rough idea of what phases might occur, one can use a model that has some of the same properties as QCD, but is easier to manipulate. Many physicists use Nambu-Jona-Lasinio models, which contain no gluons, and replace the strong interaction with a four-fermion interaction. Mean-field methods are commonly used to analyse the phases. Another approach is the bag model, in which the effects of confinement are simulated by an additive energy density that penalizes unconfined quark matter.

EFFECTIVE THEORIES

Many physicists simply give up on a microscopic approach, and make informed guesses of the expected phases (perhaps based on NJL model results). For each phase, they then write down an effective theory for the low-energy excitations, in terms of a small number of parameters, and use it to make predictions that could allow those parameters to be fixed by experimental observations. In this connection we write to state the following seminal and cardinal points:

Quark Description of Hadronic Phases: A first cut at applying the lessons learned from color-flavor locking and quark-hadron continuity to real QCD, which is complicated by splitting between strange and light quarks. Both classic 2-flavor color superconductivity (with the strange quark passive) and color-flavor locking are valid ground states in different parameter regimes at high density. An extremely intriguing possibility is that the 2-flavor color superconducting phase goes over into ordinary nuclear matter with no phase transition. This might qualitatively explain the small nuclear (compared to QCD) mass scale; it requires chiral symmetry restoration -- which could explain the long-standing observation $\rho_A \rightarrow 1$ in nuclear matter.

Continuity of Quark and Hadron Matter: In this work the full power of color-flavor locking became apparent. It gives us an analytically tractable realization of confinement and chiral symmetry breaking in a regime of definite physical interest. We find a detailed match between the calculable properties of the high-density (quark) phase and the properties of the low-density (nuclear) phase one has learned to expect from phenomenology, numeric's, etc.

High Density Quark Matter and the Renormalization Group in QCD with Two and Three Flavors shows how the renormalization of Fermi liquid parameters in QCD is surprisingly tractable, and identifying the favored couplings.

Color-Flavor Locking and Chiral Symmetry Breaking in High Density QCD is phase for hadronic matter at high density. Among other things, the elementary excitations are all integrally charged.

Fermion Masses, Neutrino Oscillations, and Proton Decay in the Light of SuperKamiokande: A serious attempt to decode the message of the Super Kamiokande neutrino oscillation is a discovery using all the resources of super symmetric grand unified theories.

Riemann-Einstein Structure from Volume and Gauge Symmetry is inverse to the Kaluza-Klein construction, realizing gravity as a spontaneously broken gauge theory.

A Chern-Simons Effective Field Theory for the Pfaffian Quantum Hall State is a simplified representation of the quantum Hall States exhibiting non-abelian statistics.

In his celebrated paper Adolf Haimovici (1), studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of a system of transition from classic 2 flavor color superconductivity in to ordinary nuclear matter.

In this paper we study the following systems:

- (a) Transformation of classic 2 flavor color superconductivity (CCFSC) to ordinary nuclear matter(ONM)
- (b) (QCD) and (QGP)

Axiomatic predications includes once over change, continuing change, process of change, functional relationships, predictability, cyclical growth, cyclical fluctuations, speculation theory, cobweb analyses, stagnation thesis, perspective analysis etc. Upshot of the above statement is data produce consequences and consequences produce data.

IV. CLASSIC TWO FLAVOUR COLOUR SUPER CONDUCTIVITY SYSTEM

ASSUMPTIONS:

CCFSC are classified into three categories;

Category 1: Representative of the CCFSC vis-à-vis category 1 of ONM

Category 2: (Second Interval) comprising of CCFSC corresponding to category 2 of ONM

Category 3: Constituting CCFSC which belong to higher age than that of category 1 and category 2.

This is concomitant to category 3 of ONM. In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the classification of ONM and CCFSC the fitness of things. For category 3. "Over and above" nomenclature could be used to encompass a wider range of consumption ONM. Similarly, a "less than" scale for category 1 can be used.

- a) The speed of growth of CCFSC under category 1 is proportional to the total amount of CCFSC under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between under category 1 and category 2 of CCFSC. This assumptions is made to foreclose the

necessity of addition of one more variable, that would render the systemic equations unsolvable

category, no sooner than the age of the ONM crosses the boundary of demarcation

b) The dissipation in all the three categories is attributable to the following two phenomenon :

Depletion phenomenon: Drying up of the source of CCSFSC vis-à-vis ONM dissipates the growth speed by an equivalent extent

1) **Aging phenomenon:** The aging process leads to transference of the balance of CCFSC to the next

NOTATION :

G_{13} : Quantum of CCFSC in category vis-à-vis category 1 of ONM

G_{14} : Quantum of CCFSC due to ONM in category 2

G_{15} : Quantum of CCFSC vis-à-vis category 3 of ONM

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}$: Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

- (a) The growth speed in category 1 is the sum of a accentuation term $(a_{13})^{(1)} G_{14}$ and a dissipation term $-(a'_{13})^{(1)} G_{13}$, the amount of dissipation taken to be proportional to the total quantum of CCFSC vis-à-vis ONM in the corresponding category.
- (b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(1)} G_{13}$ and $-(a'_{14})^{(1)} G_{14}$ the inflow from the category 1 dependent on the total amount standing in that category.
- (c) The growth speed in category 3 is equivalent to $(a_{15})^{(1)} G_{14}$ and $-(a'_{15})^{(1)} G_{15}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new CCFSC and concomitant ONM

GOVERNING EQUATIONS:

The differential equations governing the above system can be written in the following form

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13} \tag{1}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14} \tag{2}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15} \tag{3}$$

$$(a_i)^{(1)} > 0, \quad i = 13,14,15 \tag{4}$$

$$(a'_i)^{(1)} > 0, \quad i = 13,14,15 \tag{5}$$

$$(a_{14})^{(1)} < (a'_{13})^{(1)} \tag{6}$$

$$(a_{15})^{(1)} < (a'_{14})^{(1)} \tag{7}$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = dt \tag{8}$$

$$\frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = dt \tag{9}$$

Or we write a single equation as

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = \frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = \frac{dG_{15}}{(a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15}} = dt \tag{10}$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α, β, γ all positive we can write equation (10) as

$$\frac{\alpha dG_{13}}{\alpha((a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13})} = \frac{\beta dG_{14}}{\beta((a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14})} = \frac{\gamma dG_{15}}{\gamma((a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15})} = dt \tag{11}$$

The general solution of the CCFS system can be written in the form

$$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t} \text{ Where } i = 13, 14, 15 \text{ and } C_{13}, C_{14}, C_{15} \text{ are arbitrary constant coefficients.}$$

STABILITY ANALYSIS :

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} > 0$ all the components of the solution, ie all the three parts of the consumption of oxygen due to cellular respiration tend to zero, and the solution is stable with respect to the initial data.

2. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $(\lambda_{14} + (a'_{13})^{(1)})G_{13}^0 - (a_{12})^{(1)}G_{14}^0 \neq 0, (\lambda_{14} < 0)$, the first two components of the solution tend to infinity as $t \rightarrow \infty$, and $G_{15} \rightarrow 0$, ie. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 consumption of oxygen due to cellular respiration tends to zero.

3. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $(\lambda_{14} + (a'_{13})^{(1)})G_{13}^0 - (a_{12})^{(1)}G_{14}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i , the corresponding solution tends to infinity.

Close on the heels to equilibrium, there will be “fluxes”, “vortices”, however weak nevertheless. System shall evolve towards a stationary state in which generation of “entropy” (disorder) is as small as possible. By implication, there shall be a minimization problem mathematically, around the equilibrium state. In and around this range, linear equation would explain the characteristics of the system. On the other hand, away from “equilibrium”, the “fluxes” are more emphasized. Result is increase in “entropy”. When this occurs, the system no longer tends towards equilibrium. On the contrary, it may encounter instabilities that culminate into newer orders that move away from equilibrium. Thus, the CCFSC-ONM dissipative structures revitalize and resurrect complex forms away from equilibrium state.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of CCFSC converges to equilibrium.
2. The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point
3. Conditions 1 and 2 are independent of the size and direction of initial disturbance

4. The actual shape of the time path of oxygen consumption in the atmosphere by the ONM is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance
5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold
6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question, in the present case CCFSC-ONM-QCD-QCP systemic configuration. Such questions, whether growing system such as the one mentioned in the foregoing could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

ORDINARY NUCLEAR MATTER:

Assumptions:

- a) ONM are classified into three categories analogous to the stratification that was resorted to in CCFSC SECTOR. When ONM category is transferred to the next sector, (such transference is attributed to the aging process of ONM, ONM from that category apparently would have become qualified for classification in the corresponding category, because we are in fact classifying CCFSC based on stratification of ONM
- (1) Category 1 is representative of ONM corresponding to CCFSC consumed by ONM under category 1
 - (2) Category 2 constitutes those ONM whose age is higher than that specified under the head category 1 and is in correspondence with the similar classification of CCFSC.
 - (3) Category 3 of ONM encompasses those with respect to category 3 of ONM
- a) The dissipation coefficient in the growth model is attributable to two factors;
1. With the progress of time ONM gets aged and become eligible for transfer to the next category. Category 3 does not have such a provision for further transference
 2. ONM sector when become irretrievable(matter transformation or continuous creation and destruction of matter) are the other outlet that decelerates the speed of growth of ONM sector
- b) Inflow into category 2 is only from category 1 in the form of transfer of balance of ONM sector from the category 1. This is evident from the age wise classification scheme. As a result, the speed of growth of category 2 is dependent upon the amount of inflow, which is a function of the quantum of balance of ONM sector under the category 1.
- c) The balance of ONM sector in category 3 is because of transfer of balance from category 2. It is dependent on the amount of CCFSC sector under category 2.

NOTATION :

T_{13} : Balance standing in the category 1 of ONM

T_{14} : Balance standing in the category 2 ONM

T_{15} : Balance standing in the category 3 of ONM

$(b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$: Accentuation coefficients

$(b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

Under the above assumptions, we derive the following :

- a) The growth speed in category 1 is the sum of two parts:

1. A term $+(b_{13})^{(1)}T_{14}$ proportional to the amount of balance of ONM in the category 2
 2. A term $-(b'_{13})^{(1)}T_{13}$ representing the quantum of balance dissipated from category 1 .This comprises of ONM, which have grown old, qualified to be classified under category 2 and loss of ONM whatever may be reasons attributable and ascribable for.
- b) The growth speed in category 2 is the sum of two parts:
1. A term $+(b_{14})^{(1)}T_{13}$ constitutive of the amount of inflow from the category 1
 2. A term $-(b'_{14})^{(1)}T_{14}$ the dissipation factor arising due to aging of ONM.
- c) The growth speed under category 3 is attributable to inflow from category 2 and oxygen consumption stalled irrevocably and irretrievably due to energy transformation or continuous creation and destruction of ONM

GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the ONM portfolio

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13} \tag{12}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14} \tag{13}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - (b'_{15})^{(1)}T_{15} \tag{14}$$

$$(b_i)^{(1)} > 0 \quad , \quad i = 13,14,15 \tag{15}$$

$$(b'_i)^{(1)} > 0 \quad , \quad i = 13,14,15 \tag{16}$$

$$(b_{14})^{(1)} < (b'_{13})^{(1)} \tag{17}$$

$$(b_{15})^{(1)} < (b'_{14})^{(1)} \tag{18}$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda_i t}$, $i = 13,14,15$ where $C'_{13}, C'_{14}, C'_{15}$ are arbitrary constant coefficients and $\alpha'_{13}, \alpha'_{14}, \alpha'_{15}, \gamma'_{13}, \gamma'_{14}, \gamma'_{15}$ corresponding multipliers to the characteristic roots of the ONM system

CLSSIC TWO FLAVOUR COLOUR SUPERCONDUCTIVITY AND ORDINARY NUCLEAR MATTER –DUAL SYSTEM ANALYSIS

We will denote

- 1) By $T_i(t), i = 13,14,15$, the three parts of the ONM system analogously to the G_i of the CCFSC systems.
- 2) By $(a''_i)^{(1)}(T_{14}, t)$ ($T_{14} \geq 0, t \geq 0$) ,the contribution of the CCFSC to the dissipation coefficient of the ONM
- 3) By $(-b''_i)^{(1)}(G_{13}, G_{14}, G_{15}, t) = -(b''_i)^{(1)}(G, t)$, the contribution of the CCFSC to the dissipation coefficient of the ONM

CLASSIC TWO FLAVOUR COLOUR SUPERCONDUCTIVITY-ORDINARY NUCLEAR MATTER SYSTEM GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \tag{19}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \tag{20}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)] G_{15} \tag{21}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)] T_{13} \tag{22}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)] T_{14} \tag{23}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)] T_{15} \tag{24}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ **First augmentation factor** attributable to ONM dissipation of CCFSC

$-(b''_{13})^{(1)}(G, t) =$ **First detrition factor** contributed by CCFSC to the dissipation of ONM

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$
 $i, j = 13, 14, 15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)} \tag{25}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)} \tag{26}$$

(C) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 27

$$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)} \tag{28}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$ are positive constants
 and $i = 13, 14, 15$

They satisfy Lipschitz condition:

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\mathcal{M}_{13})^{(1)}t} \tag{29}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\mathcal{M}_{13})^{(1)}t} \tag{30}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$, the **first augmentation coefficient** attributable to ONM would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1 \tag{31}$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \tag{32}$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1 \tag{33}$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad T_i(0) = T_i^0 > 0$$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \tag{34}$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \tag{35}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \tag{36}$$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t [(a_{13})^{(1)} G_{14}(s_{(13)}) - ((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) G_{13}(s_{(13)})] ds_{(13)} \tag{37}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t [(a_{14})^{(1)} G_{13}(s_{(13)}) - ((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})) G_{14}(s_{(13)})] ds_{(13)} \tag{38}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t [(a_{15})^{(1)} G_{14}(s_{(13)}) - ((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})) G_{15}(s_{(13)})] ds_{(13)} \tag{39}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t [(b_{13})^{(1)} T_{14}(s_{(13)}) - ((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})) T_{13}(s_{(13)})] ds_{(13)} \tag{40}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t [(b_{14})^{(1)} T_{13}(s_{(13)}) - ((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})) T_{14}(s_{(13)})] ds_{(13)} \tag{41}$$

$$\bar{T}_{15}(t) = T_{15}^0 +$$

$$\int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(12)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(12)}), s_{(12)}) \right) T_{15}(s_{(12)}) \right] ds_{(12)} \tag{42}$$

Where $S_{(12)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(12)}} \right) \right] ds_{(12)} = \tag{43}$$

$$\left(1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{c_{14}^0}} + (\hat{P}_{13})^{(1)} \right] \tag{44}$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{12}, T_{14}, T_{15}$

It is now sufficient to take $\frac{(a_j)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_j)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_j^0}{c_j^0}} \right] \leq (\hat{P}_{13})^{(1)} \tag{45}$$

$$\frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \tag{46}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d \left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)} t} \right\} \tag{47}$$

Indeed if we denote

Definition of \tilde{G}, \tilde{T} :
 $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$ \tag{48}

It results

$$\begin{aligned} |G_{13}^{(1)} - G_{13}^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\hat{M}_{13})^{(1)} s_{(12)}} e^{(\hat{M}_{13})^{(1)} s_{(12)}} ds_{(12)} + \\ &\int_0^t \left\{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\hat{M}_{13})^{(1)} s_{(12)}} e^{-(\hat{M}_{13})^{(1)} s_{(12)}} + \right. \\ &\left. (a''_{13})^{(1)} (T_{14}^{(1)}, s_{(12)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\hat{M}_{13})^{(1)} s_{(12)}} e^{(\hat{M}_{13})^{(1)} s_{(12)}} + \right. \\ &\left. G_{13}^{(2)} |(a_{13})^{(1)} (T_{14}^{(1)}, s_{(12)}) - (a_{13})^{(1)} (T_{14}^{(2)}, s_{(12)})| e^{-(\hat{M}_{13})^{(1)} s_{(12)}} e^{(\hat{M}_{13})^{(1)} s_{(12)}} \right\} ds_{(12)} \end{aligned} \tag{49}$$

Where $S_{(12)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$|G^{(1)} - G^{(2)}| e^{-(\tilde{M}_{13})^{(1)}t} \leq \frac{1}{(\tilde{M}_{13})^{(1)}} ((a_{13})^{(1)} + (a'_{13})^{(1)} + (\tilde{A}_{13})^{(1)} + (\tilde{P}_{13})^{(1)} (\tilde{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)})) \quad 50$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\tilde{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)}t}$ and $(\tilde{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ . 51

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$, $i = 13,14,15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t [(a_i)^{(1)} - (a_i)^{(1)}(T_{14}(s_{13}), s_{13})] ds_{13}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i)^{(1)}t} > 0 \quad \text{for } t > 0 \quad 52$$

Definition of $((\tilde{M}_{13})^{(1)})_1, ((\tilde{M}_{13})^{(1)})_2$ and $((\tilde{M}_{13})^{(1)})_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < ((\tilde{M}_{13})^{(1)})_1 \quad \text{it follows} \quad \frac{dG_{14}}{dt} \leq ((\tilde{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \quad \text{and by integrating} \quad 53$$

$$G_{14} \leq ((\tilde{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\tilde{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\tilde{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\tilde{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13}, G_{15} and G_{13}, G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 54

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$.

Definition of $(m)^{(1)}$ and ε_1 : 55

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_1'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$$

If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1}$$

By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14}, t) + (a_{14}'')^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)} \tag{56}$$

$$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G, t) - (b_{14}'')^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

(b) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations 57

$$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$$

and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ and 58

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 59

By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the

$$\text{roots of the equations } (a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0 \tag{60}$$

$$\text{and } (b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0 \tag{61}$$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)} \tag{62}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}, \tag{63}$$

$$\text{and } (v_0)^{(1)} = \frac{\sigma_{13}^0}{\sigma_{14}^0} \tag{64}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously 65

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)} \tag{66}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } \boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$$
67

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined by 59 and 61 respectively 68

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{12}^0 e^{((S_1)^{(1)} - (P_{13})^{(1)})t} \leq G_{12}(t) \leq G_{12}^0 e^{(S_1)^{(1)}t}$$
69

where $(P_i)^{(1)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (P_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$
70

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (P_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (P_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-(a_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a_{15})^{(1)}t}$$
71

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$
72

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$
73

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$
74

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)} ((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$
75

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{12})^{(1)} (m_2)^{(1)} - (a'_{13})^{(1)}$ 76

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$
77

$$(R_1)^{(1)} = (b_{13})^{(1)} (\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{dv^{(1)}}{dt} = (a_{12})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a'_{13})^{(1)} (T_{14}, t) \right) - (a_{14})^{(1)} (T_{14}, t) v^{(1)} - (a_{14})^{(1)} v^{(1)}$$
78

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$- \left((a_{14})^{(1)} (v^{(1)})^2 + (a_{12})^{(1)} v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq$$
79

$$-\left((a_{14})^{(1)} (v^{(1)})^2 + (\sigma_1)^{(1)} v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

$$(a) \text{ For } 0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)} (v_2)^{(1)} e^{-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)} t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)} t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)} (\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

$$(b) \text{ If } 0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)} \text{ we find like in the previous case,}$$

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)} (v_2)^{(1)} e^{-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)} t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)} t}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)} (\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}} \leq (\bar{v}_1)^{(1)}$$

$$(c) \text{ If } 0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}, \text{ we obtain}$$

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)} (\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)} t}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$

and as a consequence $G_{13}(t) = (v_0)^{(1)} G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case .

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

4. STATIONARY SOLUTIONS AND STABILITY

Stationary solutions and stability curve representative of the variation of CCFSC consumption due to ONM vis-à-vis ONM curve lies below the tangent at $G = G_0$ for $G < G_0$ and above the tangent for $G > G_0$.Wherever such a situation occurs the point G_0 is called the “point of inflexion”. In this case, the tangent has a positive slope that simply means the rate of change of CCFSC is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value :

In all the cases $G = G_0$, $G < G_0$, $G > G_0$ the condition that the rate of change of CCFSC is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of (G) is that the rate of change of CCFSC function at G_0 is zero.
2. A sufficient condition for the stationary value at G_0 , to be maximum is that the acceleration of the CCFSC is less than zero.
3. A sufficient condition for the stationary value at G_0 , be minimum is that acceleration of CCFSC is greater than zero.
4. With the rate of change of G namely CCFSC defined as the accentuation term and the dissipation term, we are sure that the rate of change of CCFSC is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationariness.

We can prove the following

Theorem 3: If $(a'_i)^{(1)}$ and $(b'_i)^{(1)}$ are independent on t , and the conditions (with the notations 25,26,27,28)

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}$, $(r_{14})^{(1)}$ as defined by equation 25 are satisfied , then the system

$$(a_{13})^{(1)} G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})] G_{13} = 0$$

$$(a_{14})^{(1)} G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})] G_{14} = 0$$

$$(a_{15})^{(1)} G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})] G_{15} = 0$$

88

89

90

91

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \tag{92}$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \tag{93}$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \tag{94}$$

has a unique positive solution, which is an equilibrium solution for the system (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0 \tag{95}$$

Definition and uniqueness of T_{14}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]} \tag{96}$$

(b) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - [(b'_{13})^{(1)}(b'_{14})^{(1)}(G) + (b'_{14})^{(1)}(b'_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0 \tag{97}$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]} \tag{98}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$ belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. 99

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i \tag{100}$$

$$\frac{\partial (a_{14}^{\prime\prime})^{(1)}}{\partial T_{14}} (T_{14}^*) = (q_{14})^{(1)}, \quad \frac{\partial (b_i^{\prime\prime})^{(1)}}{\partial G_j} (G^*) = s_{ij} \tag{101}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^* \mathbb{T}_{14} \tag{102}$$

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^* \mathbb{T}_{14} \tag{104}$$

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^* \mathbb{T}_{14} \tag{105}$$

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)}) T_{13}^* G_j \tag{106}$$

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)}) T_{14}^* G_j \tag{107}$$

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)}) T_{15}^* G_j \tag{107}$$

The characteristic equation of this system is

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}) \{ ((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ & [((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)} G_{13}^*] \\ & ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^* \\ & + ((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}) (q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)} (q_{14})^{(1)} G_{14}^* \\ & ((\lambda)^{(1)} + (b'_{12})^{(1)} - (r_{12})^{(1)}) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^* \\ & ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \\ & ((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \\ & + ((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} (q_{15})^{(1)} G_{15} \\ & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)} (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (a_{15})^{(1)} (q_{13})^{(1)} G_{13}^*) \\ & ((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^* \} = 0 \end{aligned} \tag{108}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

More often than not, models begin with the assumption of 'steady state' and then proceed to trace out the path, which will be followed when the steady state is subjected to some kind of exogenous disturbance. Breathing pattern of terrestrial organisms is another parametric representation to be taken into consideration. It cannot be taken for granted that the sequence generated in this manner will tend to equilibrium i.e. a traverse from one steady state to another.

In our model, we have, using the tools and techniques by Haimovici, Levin, Volterra, Lotka have brought out implications of steady state, stability, asymptotic stability, behavioral aspects of the solution without any such assumptions, such as those mentioned in the foregoing.

IN THE FOLLOWING, WE GIVE EQUATIONS FOR THE QCD-QGP-CCFSC-ONM SYSTEM. Solutions and sine-qua-non theoretical aspects are dealt in the next paper (part II)

GOVERNING EQUATIONS

CCFSC

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15}$$

ONM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - (b'_{13})^{(1)} T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - (b'_{14})^{(1)} T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - (b'_{15})^{(1)} T_{15}$$

QCD

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - (a'_{16})^{(2)} G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - (a'_{17})^{(2)} G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - (a'_{18})^{(2)} G_{18}$$

QGP

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - (b'_{16})^{(2)} T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - (b'_{17})^{(2)} T_{17}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(2)} T_{17} - (b'_{13})^{(2)} T_{18}$$

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS

CCFSC-ONM

CCFSC

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \frac{+(a''_{13})^{(1)}(T_{14}, t)}{+(a''_{13})^{(1)}(T_{14}, t)} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \frac{+(a''_{14})^{(1)}(T_{14}, t)}{+(a''_{14})^{(1)}(T_{14}, t)} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \frac{+(a''_{15})^{(1)}(T_{14}, t)}{+(a''_{15})^{(1)}(T_{14}, t)} \right] G_{15}$$

Where $\frac{+(a''_{13})^{(1)}(T_{14}, t)}{+(a''_{13})^{(1)}(T_{14}, t)}$, $\frac{+(a''_{14})^{(1)}(T_{14}, t)}{+(a''_{14})^{(1)}(T_{14}, t)}$, $\frac{+(a''_{15})^{(1)}(T_{14}, t)}{+(a''_{15})^{(1)}(T_{14}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3 due to ONM

ONM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \frac{-(b''_{13})^{(1)}(G, t)}{-(b''_{13})^{(1)}(G, t)} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \frac{-(b''_{14})^{(1)}(G, t)}{-(b''_{14})^{(1)}(G, t)} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \frac{-(b''_{15})^{(1)}(G, t)}{-(b''_{15})^{(1)}(G, t)} \right] T_{15}$$

Where $\frac{-(b''_{13})^{(1)}(G, t)}{-(b''_{13})^{(1)}(G, t)}$, $\frac{-(b''_{14})^{(1)}(G, t)}{-(b''_{14})^{(1)}(G, t)}$, $\frac{-(b''_{15})^{(1)}(G, t)}{-(b''_{15})^{(1)}(G, t)}$ are first **detrition** coefficients for category 1, 2 and 3 due to CCFSC

QGP dissipates laws of QCD

QCD

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{16})^{(2)}(T_{17}, t)} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{17})^{(2)}(T_{17}, t)} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{18})^{(2)}(T_{17}, t)} \right] G_{18}$$

Where $\frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{16})^{(2)}(T_{17}, t)}$, $\frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{17})^{(2)}(T_{17}, t)}$, $\frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{18})^{(2)}(T_{17}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3 due to QGP

QGP

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \frac{-(b''_{16})^{(2)}(G_{19}, t)}{-(b''_{16})^{(2)}(G_{19}, t)} \right] T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \frac{-(b''_{17})^{(2)}(G_{19}, t)}{-(b''_{17})^{(2)}(G_{19}, t)} \right] T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \frac{-(b''_{18})^{(2)}(G_{19}, t)}{-(b''_{18})^{(2)}(G_{19}, t)} \right] T_{18}$$

Where $\frac{-(b''_{16})^{(2)}(G_{19}, t)}{-(b''_{16})^{(2)}(G_{19}, t)}$, $\frac{-(b''_{17})^{(2)}(G_{19}, t)}{-(b''_{17})^{(2)}(G_{19}, t)}$, $\frac{-(b''_{18})^{(2)}(G_{19}, t)}{-(b''_{18})^{(2)}(G_{19}, t)}$ are first **detrition** coefficients for category 1, 2 and 3QGP dissipating QCD

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

QCD

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{16})^{(2)}(T_{17}, t)} \frac{-(a''_{13})^{(1,1)}(T_{14}, t)}{-(a''_{13})^{(1,1)}(T_{14}, t)} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{17})^{(2)}(T_{17}, t)} \frac{-(a''_{14})^{(1,1)}(T_{14}, t)}{-(a''_{14})^{(1,1)}(T_{14}, t)} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{18})^{(2)}(T_{17}, t)} \frac{-(a''_{15})^{(1,1)}(T_{14}, t)}{-(a''_{15})^{(1,1)}(T_{14}, t)} \right] G_{18}$$

Where $\frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{16})^{(2)}(T_{17}, t)}$, $\frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{17})^{(2)}(T_{17}, t)}$, $\frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{18})^{(2)}(T_{17}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3 due to QGP

$\frac{-(a''_{13})^{(1,1)}(T_{14}, t)}{-(a''_{13})^{(1,1)}(T_{14}, t)}$, $\frac{-(a''_{14})^{(1,1)}(T_{14}, t)}{-(a''_{14})^{(1,1)}(T_{14}, t)}$, $\frac{-(a''_{15})^{(1,1)}(T_{14}, t)}{-(a''_{15})^{(1,1)}(T_{14}, t)}$ are second **detrition** coefficients for category 1, 2 and 3 due to ONM

ONM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \frac{-(b''_{13})^{(1)}(G, t)}{-(b''_{13})^{(1)}(G, t)} \frac{+(b''_{16})^{(2,2)}(G_{19}, t)}{+(b''_{16})^{(2,2)}(G_{19}, t)} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \frac{-(b''_{14})^{(1)}(G, t)}{-(b''_{14})^{(1)}(G, t)} \frac{+(b''_{17})^{(2,2)}(G_{19}, t)}{+(b''_{17})^{(2,2)}(G_{19}, t)} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \frac{-(b''_{15})^{(1)}(G, t)}{-(b''_{15})^{(1)}(G, t)} \frac{+(b''_{18})^{(2,2)}(G_{19}, t)}{+(b''_{18})^{(2,2)}(G_{19}, t)} \right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first **detrition** coefficients for category 1, 2 and 3 due to CCFSC
 $\boxed{+(b''_{16})^{(2,2)}(G_{19}, t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19}, t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19}, t)}$ are second **augmentation** coefficients for category 1, 2 and 3 due to

QCD

CCFSC:

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \right] G_{15}$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3 due to

QGP

QGP

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)} \right] T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)} \right] T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \right] T_{18}$$

Where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first **detrition** coefficients for category 1, 2 and 3 of QGP

GOVERNING EQUATIONS OF

THECCFSC- ONM-QCD-QGP(GENERALISATION OF COMMUTATIVE CONCEPT)

QGP

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)} \right] \boxed{-(b''_{13})^{(1,1)}(G, t)} T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)} \right] \boxed{-(b''_{14})^{(1,1)}(G, t)} T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \right] \boxed{-(b''_{15})^{(1,1)}(G, t)} T_{18}$$

Where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are **first detrition** coefficients for category 1, 2 and 3 due to QCD

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are **second detrition** coefficients for category 1, 2 and 3 due to CCFSC

CCFSC

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \right] \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \right] \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \right] \boxed{+(a''_{18})^{(2,2)}(T_{17}, t)} G_{15}$$

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are **first augmentation** coefficients for category 1, 2 and 3 due to ONM

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are **second augmentation** coefficients for category 1, 2 and 3 due to

QGP

QCD

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \right] G_{18}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are **first augmentation** coefficients for category 1, 2 and 3 due to

ONM

ONM

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are **first detrition** coefficients for category 1, 2 and 3 due to QCD

GOVERNING EQUATIONS OF THE SYSTEM

CCFSC-ONM-QCD-QGP

QCD

$$\begin{aligned} \frac{dG_{16}}{dt} &= (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{13})^{(1,1,1)}(T_{14}, t)} \right] G_{16} \\ \frac{dG_{17}}{dt} &= (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{14})^{(1,1,1)}(T_{14}, t)} \right] G_{17} \\ \frac{dG_{18}}{dt} &= (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{15})^{(1,1,1)}(T_{14}, t)} \right] G_{18} \end{aligned}$$

Where $\frac{+(a''_{16})^{(2)}(T_{17}, t)}{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$, $\frac{+(a''_{17})^{(2)}(T_{17}, t)}{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\frac{+(a''_{18})^{(2)}(T_{17}, t)}{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are **first augmentation** coefficients for category 1, 2 and 3 due to QGP

And $\frac{+(a''_{13})^{(1,1,1)}(T_{14}, t)}{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\frac{+(a''_{14})^{(1,1,1)}(T_{14}, t)}{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are **second augmentation** coefficient for category 1, 2 and 3 due to ONM

ONM

$$\begin{aligned} \frac{dT_{13}}{dt} &= (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \frac{-(b''_{13})^{(1)}(G, t)}{-(b''_{16})^{(2,2,2)}(G_{19}, t)} \right] T_{13} \\ \frac{dT_{14}}{dt} &= (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \frac{-(b''_{14})^{(1)}(G, t)}{-(b''_{17})^{(2,2,2)}(G_{19}, t)} \right] T_{14} \\ \frac{dT_{15}}{dt} &= (b_{15})^{(1)} T_{14} - \left[(b'_{15})^{(1)} \frac{-(b''_{15})^{(1)}(G, t)}{-(b''_{18})^{(2,2,2)}(G_{19}, t)} \right] T_{15} \end{aligned}$$

Where $\frac{-(b''_{13})^{(1)}(G, t)}{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$, $\frac{-(b''_{14})^{(1)}(G, t)}{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\frac{-(b''_{15})^{(1)}(G, t)}{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are **first detrition** coefficients for category 1, 2 and 3 due to CCFSC

$\frac{-(b''_{16})^{(2,2,2)}(G_{19}, t)}{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\frac{-(b''_{17})^{(2,2,2)}(G_{19}, t)}{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are **second detrition** coefficient for category 1, 2 and 3 due to

QCD

CCFSC

$$\begin{aligned} \frac{dG_{13}}{dt} &= (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \frac{+(a''_{13})^{(1)}(T_{14}, t)}{+(a''_{16})^{(2,2,2)}(T_{17}, t)} \right] G_{13} \\ \frac{dG_{14}}{dt} &= (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \frac{+(a''_{14})^{(1)}(T_{14}, t)}{+(a''_{17})^{(2,2,2)}(T_{17}, t)} \right] G_{14} \\ \frac{dG_{15}}{dt} &= (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \frac{+(a''_{15})^{(1)}(T_{14}, t)}{+(a''_{18})^{(2,2,2)}(T_{17}, t)} \right] G_{15} \end{aligned}$$

Where $\frac{+(a''_{13})^{(1)}(T_{14}, t)}{+(a''_{14})^{(1)}(T_{14}, t)}$, $\frac{+(a''_{14})^{(1)}(T_{14}, t)}{+(a''_{15})^{(1)}(T_{14}, t)}$ are **first augmentation** coefficients for category 1, 2 and 3 due to CCFSC

$\frac{+(a''_{16})^{(2,2,2)}(T_{17}, t)}{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$, $\frac{+(a''_{17})^{(2,2,2)}(T_{17}, t)}{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$ are **second augmentation** coefficient for category 1, 2 and 3 due to

QGP

QGP:

$$\begin{aligned} \frac{dT_{16}}{dt} &= (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \frac{-(b''_{16})^{(2)}(G_{19}, t)}{-(b''_{13})^{(1,1,1)}(G, t)} \right] T_{16} \\ \frac{dT_{17}}{dt} &= (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \frac{-(b''_{17})^{(2)}(G_{19}, t)}{-(b''_{14})^{(1,1,1)}(G, t)} \right] T_{17} \\ \frac{dT_{18}}{dt} &= (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \frac{-(b''_{18})^{(2)}(G_{19}, t)}{-(b''_{15})^{(1,1,1)}(G, t)} \right] T_{18} \end{aligned}$$

where $\frac{-(b''_{16})^{(2)}(G_{19}, t)}{-(b''_{17})^{(2)}(G_{19}, t)}$, $\frac{-(b''_{17})^{(2)}(G_{19}, t)}{-(b''_{18})^{(2)}(G_{19}, t)}$ are **first detrition** coefficients for category 1, 2 and 3 due to QCD

$\frac{-(b''_{13})^{(1,1,1)}(G, t)}{-(b''_{14})^{(1,1,1)}(G, t)}$, $\frac{-(b''_{14})^{(1,1,1)}(G, t)}{-(b''_{15})^{(1,1,1)}(G, t)}$ are **second detrition** coefficients for category 1,2 and 3 due to CCFSC

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