ISSN 2250-3153

FRACTIONAL MELLIN INTEGRAL TRANSFORM IN (0, 1/a)

S.M. Khairnar, R.M.Pise*, J.N.Salunke**

Deparment of Mathematics
Maharashtra Academy of Engineering, Alandi-412105, Pune, India
Mobile No.: 9850330076,Fax No.: 020-27185759,
Email-smkhairnar@gmail.com

Department of Mathematics, (A.S.&H.)*
R.G.I.T. Versova, Andheri (W), Mumbai-53, India
Mobile No. 09970592504, Fax.No.022-26707026
Email.piserm@rediffmail.com

Department of Mathematics**
North Maharastra University,Jalgaon-India
Mo.bile No. 09420389908
Email.drjnsalunke@gmail.com

Abstract- The theory of integral transforms is presented a direct and systematic technique for the presentation of classical and distributional theory. In this paper ,the Laplace operators are used to define The Fractional Mellin integral transform which can be a technique for solving boundary and initial value problems This Transforms is suited in the fractional interval 0 to 1/a. This work which is put forward is to understand how Laplace operator would lead to properties , propositions, theorems and relations with Fractional Mellin integral transform The main view of our work is to give a procedure from Laplace Transform that turns out to be valid for Fractional Mellin integral transform.

The results have been modified by applying suitable functions which leads to the results in Ftactional Mellin integral transform in the interval 0 to 1/a , where a is positive. To illustrate the advantages and use of this transformation, the result of Weyl fractional transform , summation of the series and some important differential equations have been solved at the end. The graphical concept is represented by assigning different values to the parameter by using tools of Matlab, which gives a brighter view of applications of Fractional Mellin integral transforms.

Index Terms- Integral transforms, Mellin integral transform, Finite Mellin integral transform Laplace Transform,

AMS Mathematics Classification- 44A10 (200), 47D03(2001)

I. Introduction

The Laplace transform is used to obtain the Fractional Mellin integral transform in the range 0 to 1/a ,. properties like linearity property, scaling property, power property and propositions like for the functions $t^{-1} f(t^{-1})$, $\frac{d}{ds} t^{s-1} = (\log t) t^{s-1}$, e^{-t}, $\int_0^x f(u)du$, $\int_0^x u^p f(xu)g(u)du$, $\int_0^x u^p f(x/u)g(u)du$,] are satisfied by the Fractional Mellin integral transform in 0 to 1.a ..Theorems like inversion theorem, convolution theorem, parsevals theorem, first shifting theorem and second shifting theorem are valid for the Fractional Mellin integral transform in 0 to 1/a , Application of the FrMIT for Weyl Fractional Transform and summation of the series, derivatives of the Fractional Mellin integral transform in 0 to 1/a are calculated. Solution of the ordinary differential equation is exist. and it is represented graphically by using Matlab.

II. Preliminary Results

Let f(x) be a given function of x which is defined for all x ≥ 0 and's' is a parameter

$$L[f(x)] = \int_0^\infty e^{-sx} f(x)dx , \qquad (1)$$

Substituting, $x = -\log(a.t)$, $dx = -\frac{dt}{t}$, a>0 .
If x=0 then $t=1/a$ and if $x=\infty$ then $t=0$,then

$$L[f(x)] = \int_0^{1/a} a^s t^{s-1} f(t)dt ,$$

This integral is denoted by $M_2[f(t), s, 0, 1/a]$

$$M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt \quad (2)$$

a>0,s >0 is a parameter and a lies in between 1 and n, n , a is not equal to one, a is finite., the 1/a is fractional. This is a Fractional Mellin Integral Transform with its kernel a $^s t^{s-1}$, a is positive and s is a parameter. If a≠0 then 1/a takes fractional values.

III. LEMMA

3.1: Linearity property: Integral Transform (FrMIT) is a linear operation Theorem: The FrMIT is a Linear operation, that is for any function *f(t)* and *g(t)* whose Mellin Type (fractional) Integral Transforms exists and α and β are constants,

If $M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, then

$$M^{fr}[\alpha f(t)+\beta g(t),s,0,1/a] = \alpha M^{fr}[f(t),s,0,1/a] + \beta M^{fr}[g(t),s,0,1/a] \quad \textbf{(3)}$$

3.2: Scaling Property:

If $M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, then

$$M_{fr}[f(bt), s, 9, 1/a] = b^{-s} M^{fr}[f(p),s,0,b/a],$$

bt=q, b>0 **(4)**

3.3: Power Property

If $M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, then

$$M^{fr}[f(t^b), s, 0, 1/a] = \frac{1}{b} M^{fr}[f(q), s/b, 0, 1/a^b], \quad t^b = q$$

b>0, *(5)*

3.4: Propositions

3.4.1: FrMIT of $t^{-1} f(t^{-1})$

If $M^{fr}[f(t),s,0,1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$ then

$$M^{fr}[t^{-1} f(t^{-1}),s,0,1/a] = \int_0^{1/a} a^s t^{s-1} t^{-1} f(t^{-1})dt \quad (6)$$

by the substitution z=1/t, we have

$$M^{fr}[t^{-1} f(t^{-1}),s,0,1/a] = M^{fr}[f(z),-s+1,1/a,\infty]$$

where M[f(z),-s-1,1/a,∞] is the new Mellin Type Integral Transform in the range 1/a to ∞ .(future work)

3.4.2: FrMIT of $\frac{d}{ds} t^{s-1} = (\log t) t^{s-1}$

If $\frac{d}{ds} t^{s-1} = (\log t) t^{s-1}$ *then*

$$M^{fr}[\log(t)f(t),s,0,1/a] = \int_0^{1/a} a^s t^{s-1} \log(t) f(t)dt$$

$$= \int_0^{1/a} a^s [\frac{d}{ds} t^{s-1}] f(t)dt$$

$$= \frac{d}{ds} \int_0^{1/a} a^s t^{s-1} f(t)dt$$

$$M^{fr}[og(t)f(t),s,0,1/a] = \frac{d}{ds} M^{fr}[f(t),s,0,1/a] \quad \textit{(7)}$$

***3.4.3:* FrMit of** e^{-t}

If $M^{fr}[f(t),s,0,1/a] = \int_0^{a} a^{-s} t^{s-1} f(t)dt$ then

$$M^{fr}[e^{-t},s,0,1/a] = \int_0^{1/a} a^s e^{-t} t^{s-1} dt$$

$$= \int_0^{\infty} a^s e^{-t} t^{s-1} dt - \int_{1/a}^{\infty} a^s e^{-t} t^{s-1} dt$$

$$M^{fr}[e^{-t}),s,0,1/a] = a^s \Gamma(s) - M_{fr}[e^{-t}, s, 1/a, \infty] \quad (8)$$

where $M_{fr}[e^{-t}, s, 1/a, \infty]$ is the New Mellin Type Integral Transform in the range [1/a , ∞]. .

3.4.4: FrMIT of integral $\int_0^x f(u)du$

If $M^{fr}[f(t),s,0,1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$ then

$$M_{fr}[\int_0^x f(u)du, s-1,0,1/a] = \frac{-1}{(s-1)} M_{fr}[f(x),s,0,1/a] - \frac{a}{(s-1)}\phi(1/a) \quad (9)$$

where $\phi(x)=\int_0^x f(u)du$,replace s by s+1,we have

$$M_{fr}[\int_0^x f(u)du,s,0,1/a] = \frac{-1}{s}M_{fr}[f(x),s+1,0,1/a] - \frac{a}{s}\phi(1/a) \quad (9)$$

$$M_{fr}[\int_0^x dy\int_0^y f(u)du,s,0,1/a] = \frac{-1}{s(s+1)}M_{fr}[f(u),s,0.,1/a] - \frac{1}{as}\phi'(1/a) - \frac{1}{a^2 s(s+1)}\phi(1/a) \quad (10)$$

If $I_n f(x)=\int_0^x I_{n-1}f(u)du$ then

$$M_{fr}[I_n f(x),s,0,1/a] = (-1)^n \frac{\Gamma(s)}{\Gamma(s+n)} M_{fr}[f(u),s+n,0,1/a] - \frac{1}{as}\phi^{(n-1)}(1/a) - \frac{1}{a^2 s(s+1)}\phi^{(n-2)}(1/a).--\frac{1}{a^{(n-1)}s(s+1)--(s+n-1)}\phi(1/a)$$

(11)

3.4.5: : FrMIT of integral $\int_0^x u^p f(xu)g(u)du$

If

$$M_{fr}[\int_0^x u^p f(xu)g(u)du,s,0,1/a] = \int_0^{1/a} a^s x^{s-1}dx\int_0^x u^p f(xu)g(u)du$$

$$= \int_0^x u^p g(u)du\int_0^{1/a} a^s x^{s-1} f(xu)dx$$

By the substitution xu=z, we have

$$M_{fr}[\int_0^x u^p f(xu)g(u)du,s,0,1/a] = a^s M_{fr}[g(u),p-s+1,0,1/a]M_{fr}[f(z),s,0.u/a]$$

(12)

also

$$M_{fr}[x^\lambda\int_0^x u^p f(xu)g(u)du,s,0,1/a] = a^s M_{fr}[g(u),p-s+1,0,1/a]M_{fr}[f(z),s+\lambda,0.u/a]$$

(13)

3.4.6: : FrMIT of integral $\int_0^x u^p f(x/u)g(u)du$

Similarly

$$M_{fr}[\int_0^x u^p f(x/u)g(u)du,s,0,1/a] = a^s M_{fr}[g(u),p+s+1,0,a]M_{fr}[f(z),s,0.a/u]$$

(14)

also

$$M_{fr}[x^\lambda\int_0^x u^p f(x/u)g(u)du,s,0,1/a] = a^s M_{fr}[g(u),p+s+1,0,a]M_{fr}[f(z),s+\lambda,0.a/u]$$

(15)

If p= λ =0 then

$$M_{fr}[x^\lambda\int_0^x u^p f(xu)g(u)du,s,0,1/a] = a^s M_{fr}[g(u),1-s,0,1/a]M_{fr}[f(z),s,0.u/a],$$

and (16)

If p=-1 , λ =0 then from (10)

$$M_{fr}[\int_0^x f(x/u)g(u)\frac{du}{u},s,0,1/a] = a^s M_{fr}[g(u),s+1,0,1/a]M_{fr}[f(z),s,0.,u/a]$$

(16)

IV. Main Results

4.1: Inversion theorem

Theorem: The FrMIT in 0 to 1/a is

M^{fr}[$f(t), s, 0, 1/a$] = $\int_0^{1/a} a^s t^{s-1} \; f(t)dt$, a>0

Then its inversion formula is

f(t)= $\frac{1}{2\pi i}\int_{c-iN}^{c+iN} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]ds$

Proof:

Assume that M_2 [$f(t), s, 0,1/a$] is a regular equation in the strip $|Re(s)| < r$ ('r' to be real number) of the s-plane and that $0<c<\nu$,c-i∞ ≤ s ≤ c+ i∞ ,where c is constant ,

M^{fr} [(t), s, 0, 1/a] = $\int_0^{1/a} a^s t^{s-1} \; f(t)dt$

= $\int_0^{1/a} a^s t^{s-1} \; dt$ [$\frac{1}{2\pi i}\int_{c-iN}^{c+iN} t^{-s} M_{fr}[f(t),s,0,1/a]ds$]

= $\frac{1}{2\pi i}\int_{c-iN}^{c+iN} t^{-s} M_{fr}[f(t),s,0,1/a]ds \int_0^{1/a} t^{s-1} a^s dt$

= $\frac{1}{2\pi i}\int_{c-iN}^{c+iN} t^{-s} M_{fr}[f(t),s,0,1/a][\frac{t^s}{s}a^s]_0^{1/a} ds$

= $\frac{1}{2\pi i}\int_{c-iN}^{c+iN} t^{-s} M_{fr}[f(t),s,0,1/a]\frac{a^{-s}}{s}a^s ds$

M^{fr} [f (t), s, 0, 1/a] = $\frac{1}{2\pi i}\int_{c-iN}^{c+iN} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]ds$

(17)

Let $N \to \frac{1}{a}$ and assume that M^{fr}[$f(t), s, 0,1/a$] | remains unbounded as | $limit\ x| \to \infty$, when $|Re(s)| \le$ c then the integral on R.H.S of the equation (17) tends $f(t)$.Hence

f(t)= $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]ds$

(18)

It is denoted by M^{-1}_{fr} [f(t),s,0,1/a]=f(t) as $N \to \frac{1}{a}$

f(t)= $M^{-1}_{fr}[f(x),s,0,\frac{1}{a}]$ = $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]ds$

(19)

4.2: Convolution Theorem

The FrMIT in 0 to 1/a is

M^{fr} [$f(t),s,0,1/a$]= $\int_0^{1/a} a^s t^{s-1} \; f(t)dt$

then its inverses are

$M^{=1}_{fr}$ [$f(t),s,0,1/a$]= $f(x)$ = $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M[f(t),s,0,1/a]ds$

$M^{=1}_{fr}$ [$g(t),s,0,1/a$]= $g(x)$ = $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[g(t),s,0,1/a]ds$, then

M^{fr} [$f(t)\, g(x-t), s, 0, 1/a$]

= $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[g(t),s,0,1/a]ds$

M^{fr} [$g(x-t),s,0,1/a$] ds *(20)*

1.4.3: Orthogonality (Parsevals Theorem)

The FrMIT in 0 to 1/a is

M^{fr} [$f(t),s,0,1/a$]= $\int_0^{1/a} a^s t^{s-1} \; f(t)dt$,and

M^{fr} [$g(t),s,0,1/a$]= $\int_0^{1/a} a^s t^{s-1} \; g(t)dt$

then its inverses are

M^{-1}_{fr} [$(t),s,0,1/a$]= $f(x)$ = $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]ds$

M^{fr} $[g(t),s,0,1/a]= g(x) =$ $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[g(t),s,0,1/a]ds$, then

$M^{fr}[f(t)g(t), s, 0, 1/a]$ = $\frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} \frac{t^{-x}}{s} M_{fr}[f(t),s,0,1/a]M_{fr}[g(t),s,0,1/a]ds$ (21)

4.5: Definitions

(a)Uhit Step Function

If U(t)=H(t)=1,when t>0

=0,when t<0, then U(t) or H(t) is known as the Unit Step Function

(b) Heviside Unit Step Function

If U(t-a)=H(t-a)=1, when t>a

=0,when t<a,then U(t-a(or H(t-a) is known as the Heviside Unit Step Function.

4.6: First Shifting Theorem

The FrMIT in 0 to 1/a is

$M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, then

$M^{fr}[t^n f(t), s, 0,1/a] = M^{fr}[f(t), s+n, 0, 1/a]$ (10)

4.7: Second Shifting Theorem

The FrMIT in 0 to 1/a is

$M^{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)dt$, then

$M^{fr}[f(t-a)H(t-a), s, 0, 1/a] = M^{fr}[f(u), s, -b,1/a-b]$ (22)

V. FrMIT Of Weyl Fractional Transform

The Weyl transform of the function f(t(is denoted by $W^{-\alpha}[f(t)] = F(x,\alpha)$ and defined as

$$W^{-\alpha}[f(t)] = F(x,\alpha) = \frac{1}{\Gamma(\alpha)}\int_x^{\infty} (t-x)^{\alpha-1} f(t)dt$$

,0<Re(α)<1 ,x>0 , then

$$M_{fr}[F(x,\alpha),r,0,\frac{1}{a}] = \int_0^{\frac{1}{a}} a^r x^{r-1} [\frac{1}{\Gamma(\alpha)}\int_x^{\infty} (t-x)^{\alpha-1} f(t)dt]dx$$

$$M_{fr}[F(x,\alpha),r,0,\frac{1}{a}] = \frac{1}{\Gamma(\alpha)}\int_x^{\infty} (t-x)^{\alpha-1} f(t)dt \int_0^{\frac{1}{a}} a^r x^{r-1} dx)$$

$$= \frac{1}{\Gamma(\alpha)}\int_x^{\infty} (t-x)^{\alpha-1} f(t)dt\ [a^r \frac{x^r}{r}]_0^{\frac{1}{a}}$$

$$= \frac{1}{r}\frac{1}{\Gamma(\alpha)}\int_x^{\infty} (t-x)^{\alpha-1} f(t)dt$$

$$= \frac{1}{r} F(x,\alpha)$$

$$M_{fr}[F(x,\alpha),r,0,\frac{1}{a}] = \frac{1}{r} F(x,\alpha) = \frac{1}{r} W^{-\alpha}[f(t)] \quad (23)$$

VI. Application Of The FMIT To Summation Of Series

The FrMIT is

$M_{fr}[f(x),r,0,1/a] = \int_0^{\frac{1}{a}} a^r x^{r-1} f(x)dx$, and its inverse is

$$f(x) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} x^{-r} M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

The Hurvitz Zeta function is denoted by $\xi(r,a)$ and defined as

$$\xi(r,a) = \sum_{n=0}^{\infty} \frac{1}{(x+a)^r}$$,0<a<1 and Re(p)>1

It follows from the inversion of the Finite Mellin integral transform

$$f(x+a) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}} (x+a)^{-r} M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

$$\sum_{n=0}^{\infty} f(x+a) =$$

$$\frac{1}{2\pi i}\sum_{n=0}^{\infty}\int_{c-\frac{i}{a}}^{c-\frac{i}{a}a}(x+a)^{-r}M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

$$= \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}\sum_{n=0}^{\infty}(x+a)^{-ra}M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

$$= \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}M_{fr}[f(x),r,0,\frac{1}{a}]\xi(r,a)dr$$

$$\sum_{n=0}^{\infty} f(n+a) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}M_{fr}[f(x),r,0,\frac{1}{a}]\xi(r,a)dr \qquad (24)$$

6.1. The FrMIT is

1. $$M_{fr}[f(x),r,0,1/a] = \int_{0}^{a} a^{-r}x^{r-1}f(x)dx$$

, and its inverse is

$$f(x) = \frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

The Riemann Zeta function is denoted by $\xi(p,a)$ and defined as

$$\xi(p) = \sum_{n=0}^{\infty}\frac{1}{n^{p}}, \quad \text{Re}(p)>1$$

It follows from the inversion of the Finite Mellin integral transform

$$f(nx) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}(nx)^{-r}M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

$$= \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}n^{\wedge -r}x^{-r}M_{fr}[f(x),r,0,\frac{1}{a}]dr$$

$$\sum_{n=1}^{\infty} f(nx) = \frac{1}{2\pi i}\sum_{n=1}^{\infty}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}n^{-r}x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

$$= \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}(\sum_{n=1}^{\infty}n^{-r})x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

$$= \frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{fr}[f(x),r,0,1/a]\xi(r)dr$$

$$\sum_{n=1}^{\infty} f(nx) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}x^{-r}M_{fr}[f(x),r,0,1/a]\xi(r)dr$$

(25)

6.2. The FrMIT is:

$$M_{fr}[f(x),r,0,1/a] = \int_{0}^{\frac{1}{a}} a^{r}x^{r-1}f(x)dx$$

, and its inverse is

$$f(x) = \frac{1}{2\pi i}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

If $\sum_{n=1}^{\infty}(-1)^{n-1}n^{-r} = [1-2^{1-r}]\xi(r)$, then

$$\sum_{n=1}^{\infty}(-1)^{n-1}f(nx) =$$

$$\frac{1}{2\pi i}\sum_{n=1}^{\infty}\int_{c-\frac{i}{a}}^{c+\frac{i}{a}}(-1)^{n-1}n^{-r}x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

$$= \frac{1}{2\pi i}\int_{c-ia}^{c+ia}(\sum_{n=1}^{\infty}(-1)^{n-1}n^{-r})x^{-r}M_{fr}[f(x),r,0,1/a]dr$$

$$\frac{1}{2\pi i}\int_{c-}^{c+\frac{i}{a}}[1-2^{1-r}]\xi(r)M_{fr}[f(x),r,0,1/a]dr$$
=

$$\sum_{n=1}^{\infty}(-1)^{n-1}f(nx)$$ =

$$\frac{1}{2\pi i}\int_{c-}^{c+\frac{i}{a}}[1-2^{1-r}]\xi(r)M_{fr}[f(x),r,0,1/a]dr \quad (26)$$

VII. Derivatives

7.1: FrMIT of First Order Derivatives:

Theorem: Suppose that $f(t)$ is continuous for all $t \geq 0$ satisfying (1.2) for some value γ and m and has a derivative $f'(t)$ which is piecewise continuous on every finite interval in the range of $t \geq 0$. Then the Fractional Mellin Integral Transforms of the derivative

$f(t)$ exists when $s>\gamma$ and $|f(t)| \leq m\, e^{\gamma t}$ for all $t \geq 0$ for some constants

Proof:

Considering the case when $f'(t)$ is continuous for all $t \geq 0$. Then on integrating by parts, this follows

$$M^{fr}[f'(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f'(t)dt$$

$$= [a^s t^{s-1} f(t)]_0^{1/a} - \int_0^{1/a} a^s(s-1)t^{s-2}f(t)dt$$

$$= a.f(1/a)-(s-1)\int_0^{1/a} a^s f(t)t^{s-2}dt$$

$= (1\text{-}s)\, M_{fr} f(t), s\text{-}1, 0, 1/a] + a.f(1/a)$

$$M^{fr}[f'(t), s, 0, 1/a] = (1-s)\, M^{fr}[f(t), s-1, 0, 1/a] + a.f(1/a) \quad (27)$$

since $f(t)$ satisfies $|f(t)| \leq m\, e^{\gamma t}$ and thus Mellin Type (fractional) Integral Transforms for derivatives is obtained .

7.2:

1: FrMIT of nth order Derivatives:

By applying to the second-order derivative, $f''(t)$ we obtain

$$M^{fr}[f''(t), s, 0, 1/a] = \int_0^{1/a} a^{-s}t^{s-1} f''(t)dt$$

then on integrating by parts it follows that

$$M^{fr}[f''(t), s, 0, 1/a] = (1-s)(2-s)\, M^{fr}[f(t), s-2, 0, 1/a] + (1-s)\, a^2 f(1/a) + a.f'(1/a) \quad (28)$$

$$M_{fr}[f'''(t),s,0,1/a] = (1-s)(2-s)(3-s)M_{fr}^{*}[f(t),s-3,0,1/a] + af''(1/a)$$

$+(1-s)\, a^2 f'(1/a) + (1-s)(2-s)a^3 f(1/a)$, then

$$M_{fr_2}[f^n(t),s,0,1/a] = (1-s)(2-s)(3-s)...(n-s)M_{fr}[f(t),s-n,0,1/a]$$

$+a\, f^{n-1}(1/a) + (1\text{-}s)a^2 f^{n-2}(1/a)$ +-----------------

$+(1\text{-}s)(2\text{-}s)(3\text{-}s)\text{---}(n\text{-}1\text{-}s)\, a^n f(1/a)$ (29)

This is the generalized result of the of The Mellin Type (fractional) Integral Transform. nth derivative of f(t).

VIII. Applications of FrMIT

8.1. $\Delta_2 f(x) = x^2 f_{xx}(x) + xf_x(x) + f(x)$

(1) $L_2(F(t)) = t^2 f''(t) + tf'(t)$

$$M^{fr}[tf'(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1}\, tf'(t)dt$$

$= f(1/a) - s\, M_{fr} f(t), s, 0, 1/a]$

$$M^{fr}[t^2 f''(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1}\, t^2 f''(t)dt$$

$= s(s+1)\, M^{fr}[f(t), s, 1/a,] - (s+1)f(1/a) + 1/a\, f'(1/a)$, the then

$$M^{fr}[\Delta_2 f(x), s, 0, 1/a] = \int_0^{1/a} a^s x^{s-1}[x^2 f_{xx}(x) + xf_x(x) + f_{xx}(x)]dx$$

Using (14) and it is obtained as

$M^{fr}[\Delta_2 f(x), s, 0, 1/a] = [1+s^2]\, M^{fr}[f(x), s, 0, 1/a] - s\, f(1/a) + a\, f'(1/a)$ *(30)*

If $\Delta_2 f(x) = 0$ then

$$x^2 f_{xx}(x) + x f_x(x) + f_{xx}(x) = 0$$

R.H.S of (14a) gives value zero. From (14a)

$$[1+s^2]\, M^{fr}\, [f(x), s, 0, 1/a] - s\, f(1/a) + a\, f'(1/a) = 0$$

$$[1+s^2]\, M^{fr}\, [f(x), s, 0, 1/a] = s\, f(1/a) - a\, f'(1/a)$$

which indicates the higher order differential equations with constant coefficients. The complete solution is given by

$$M^{fr}\, [f(x), s, 0, 1/a] = \frac{1}{(s^2+1)}\left(s\, f(1/a) - a\, f'(1/a)\right) \quad (31)$$

where constants are evaluated by solving initial value problems and problems with boundary conditions.

IX. Functions abs Results

Sr. No.	Functions	$M_{fr}[f(t), s, 0, 1/a] = \int_0^{1/a} a^s t^{s-1} f(t)\,dt$
1)	e^t	$\frac{1}{s} + \frac{1/a}{s+1} + \frac{(1/a)^2}{2!(s+2)} + \frac{(1/a)^3}{3!(s+3)} + ---$
2)	sin(at)	$\frac{(1/a)^2}{s+1} - \frac{(1/a)^6}{3!(s+3)} + \frac{(1/a)^{10}}{5!(s+5)} ----$
3)	tan(at)	$\frac{(1/a)^2}{s+1} - \frac{(1/a)^6}{3(s+3)} + \frac{2(1/a)^{10}}{15(s+3)} + ---$
4)	$t^n \sin(at)$	$a^{-n}[\frac{(1/a)^2}{s+n+1} - \frac{(1/a)^8}{3!(s+n+3)} + \frac{(1/a)^{10}}{5!(s+n+5)} ---]$
5)	$t^n \tan(at)$	$a^{-n}[\frac{(1/a)^2}{s+n+1} - \frac{(1/a)^6}{3(s+n+3)} + \frac{2(1/a)^{10}}{15(s+n+3)} + ---]$
6)	$\sin^{-1}(t)$	$\frac{1/a}{s+1} + \frac{(1/a)^3}{6(s+3)} + \frac{(1/a)^5}{40(s+5)}$
7)	t^n	$\frac{(1/a)^n}{s+n}$
8)	log (1+t)	$\frac{1/a}{s+1} - \frac{(1/a)^2}{2(s+2)} + \frac{(1/a)^3}{3(s+3)} ----$
9)	log (1-t)	$-\frac{1/a}{s+1} - \frac{(1/a)^2}{2(s+2)} - \frac{(1/a)^3}{3(s+3)} ----$
10)	a^x	$\frac{1}{s} + \log(1/a)\frac{1/a}{s+1} + (\log(1/a))^2 \frac{(1/a)^2}{2!(s+2)} + --$

11)	$\log(1+\cos t)$	$\frac{\log(1/a)}{s}-\frac{(1/a)^2}{4(s+2)}-\frac{(1/a)^4}{48(s+4)}----$
12)	$\log(1+\tan t)$	$\frac{1/a}{s+1}-\frac{(1/a)^2}{2(s+2)}+\frac{2(1/a)^2}{3(s+3)}------$
13)	$\log\sec(\frac{\pi}{4}+t)$	$\frac{1}{2s}\log 2+\frac{1/a}{s+1}+\frac{(1/a)^2}{s+2}+\frac{2(1/a)^3}{3(s+3)}--$

X. Graphical Representation By Using Matlab

Mellin Type (fractional) Integral Transform graph plotted between x,y for various values of 's' parameter.

Here the program has been shown with one value of the parameter.

Consider the equation (15)

Part-1

$$M^{fr}[f(x), s, 0, 1/a] = \frac{1}{(s^2+1)}(s\, f(1/a) - a\; f'(1/a))$$

10.1:

% f(x)=log(1+x)

%f(x)=log(1+x),f'(x)=1/(1+x)

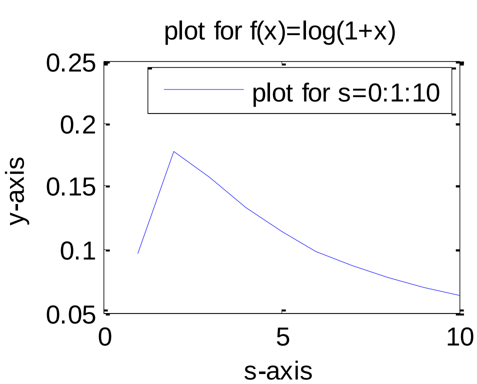
% if x=a=0 then f1=f(0)=0,f2=f'(0)=1,

% $\frac{1}{(s^2+1)}(s\, f(1/a) - a\; f'(1/a))$ =0;

%and if a=1 then f1=f(1)=0.6931,f2=f'(1)=0.5,

% $\frac{1}{s^2(s^2+1)}(s\; f\; (1/a) - a\; f'(1/a))$ $= \frac{s.*f1-f2}{(s.\wedge 2).*(s.\wedge 2+1)}$

Graphical Representation



16

Consider the equation

$$M_2[f(t), s, 0, 1/a] = (\frac{1}{s^2})(sf(1/a) - af'(1/a))$$

10. 2 % f(x)=x^2

%f(x)=x^2,f'(x)=2*x

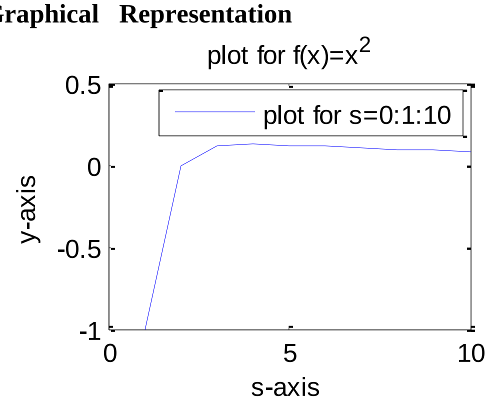
% if a=0 then f1=f(0)=0,f2=f'(0)=0,y=0 fors>0,

%f(0)=0 then $(\frac{1}{s^2})(sf(1/a) - af'(1/a))$ =0

%and if a=1 then f1=f(1)=1,f2=f'(1)=2,

% $(\frac{1}{s^2})(sf(1/a) - af'(1/a))$ = (s.^(-2)).*(s.*f1-f2)

Graphical Representation



XI. Remarks

1. Definition of the Fractional Mellin integral transform
2. Lemmas are defined and proved
3. Propositions are defined and proved
4. Theorems are stated and proved
5. Weyl transform is defined and obtained result by using FrMIT
6. Results of summation of the series are obtained by usin FrMIT
7. Derivatives are obtained by using FrMIt
8. Cauchy Linear differential equation is stated and obtained solution by using FrMIT
9. Functions and Results
10. Graphical representation

XII. CONCLUSION

This Fractional Mellin integral transform is useful to solve the ordinary differential equations by using initial and boundary conditions solution of the ordinary differential equation is shown by some examples and it is shown graphically by using Matlab.

REFERENCES

[1] Derek Naylor, On a Mellin Type Integral Transforms, Journal of Mathematics and Mechanics, (1963) vol.12, No.2

[2] C.Fox , Applications of Mellin's Transformation to the integral equations ,(1933)

[3] J.M.Mendez and J.R.Negrin , On the finite Hankel-Schwartz Transformation of Distributions, Ganita ,(1988), vol.39, No.1

[4] Ian N. Sneddon ,The use of Integral Transforms,TMH edition 1974

[5] C.Fox Application of Mellin Transformation to Integral Equation, 3rd March,1934,pp 495-502

[6] J.M.Mendez and J.R.Negrin,On the finite Hankel –Swchwartz Transformation of Distributions,Ganta.Vol.39,No.1.1908

[7] A.H.Zemanian,Generalized Integal Transformation,Interscience Publication, New York ,(1968) Sohail Ahmed,Lie Yang and Lajos Hanzo,School of EC,

[8] University of southamptons,SO17 1BJ,UK

[9] S. M. Khairnar , R. M. Pise , J. N. Salunke,Applications Of The Mellin Type Integral Transform In The Range $\{1/a, \infty)$, IJMSA,Vol. 1, No. 1,Jan. 2011.pp 1-17.

[10] D.J.Bedingham,Dimentional Regularization and Mellin summation in High-Temperature Calculations,Therotical Physics ,The Blackett laboratory , Imperial College, Prince Consort Road,SW7 2BW,UK. arXiv:ph/00110112x1 1NOV 2000.

[11] Antonic De Sena' and divide Rocchesso',A Fast Mellin and Scale Transform,Hindavi Publishing Corporation ,EURASIP ,Journal on Advances in Signal Processing,Vol.2007,Article ID 89170 ,9 pages Doc:10,1155/2007/89170

[12] J.J.Betancor ,M.T.Flores,"A Parseval equation and a generalized finite Hankel Transformation",Comment.Math.Univ.carolinae,32,4 (1991),627-638.

[13] C.Fox,"Integral transforms based upon fractional Integration",Proc.Comb.Phil.Soc.(1963),59-63.

[14] Khairnar, S. M., Pise, R. M., Salunkhe, J. N., Application of distributional Mellin integral transform in the range [0; a].