On $Sober\text{-}\mathcal{M}_X\alpha\delta\mathcal{R}_0$ spaces in $\mathcal{M}$-structures

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Abstract- The notion of $\mathcal{M}_X\alpha\delta$-closed set was introduced and studied by V. Kokilavani and P. Basker [13]. In this paper, we introduce the concept of weakly ultra-$\mathcal{M}_X\alpha\delta$-separation of two sets in a m-space using $\mathcal{M}_X\alpha\delta$-open sets. The $\mathcal{M}_X\alpha\delta$-closure and the $\mathcal{M}_X\alpha\delta$-kernel are defined in terms of this weakly ultra-$\mathcal{M}_X\alpha\delta$-separation. We also investigate some of the properties of the $\mathcal{M}_X\alpha\delta$-kernel and the $\mathcal{M}_X\alpha\delta$-closure. It is the aim of this paper to offer some weak separation axioms by utilizing $\mathcal{M}_X\alpha\delta$-open sets and the $\mathcal{M}_X\alpha\delta$-closure operator. Also we introduce $Sober\text{-}\mathcal{M}_X\alpha\delta\mathcal{R}_0$. Further, we obtain some characterizations and some properties.

Index Terms: *$\mathcal{M}_X\alpha\delta$-closed set, $\mathcal{M}_X\alpha\delta$-closure, $\mathcal{M}_X\alpha\delta$-interior, weakly ultra-$\mathcal{M}_X\alpha\delta$-separation, $\mathcal{M}_X\alpha\delta$-kernel, Sober-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$.*

I. INTRODUCTION

In 1950, H. Maki, J. Umehara and T. Noiri [3] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-cl and m_X-int operators respectively. Further they introduced m-continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [4–11]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [4], [3] and [7].

In 2010, Young Key Kim and Devi R introduced the concept of $\alpha\psi$-closure and the $\alpha\psi$-kernel via $\alpha\psi$-open sets in Topological spaces[14], In 2003, M.Caldas and D.N. Georgiou introduced $Sober\ \delta\text{-}semi\mathcal{R}_0$ spaces in Topological spaces [12]. V. Kokilavani and P. Basker introduced $\mathcal{M}_X\alpha\delta$-closed sets [13]. In this paper we introduce the $\mathcal{M}_X\alpha\delta$-closure and the $\mathcal{M}_X\alpha\delta$-kernel are defined in m-spaces. We also investigate some of the properties of the $\mathcal{M}_X\alpha\delta$-kernel and the $\mathcal{M}_X\alpha\delta$-closure. It is the aim of this paper to offer some weak separation axioms by utilizing $\mathcal{M}_X\alpha\delta$-open sets and the $\mathcal{M}_X\alpha\delta$-closure operator. Using this concept we introduce $Sober\text{-}\mathcal{M}_X\alpha\delta\mathcal{R}_0$ in minimal structures. Further, we obtain some characterizations and some properties.

Before entering to our work, we recall the following definitions, which are useful in the sequel.

2. Minimal Structures

In this section, we introduce the $\mathcal{M}$-structure and define some important subsets associated to the $\mathcal{M}$-structure and the relation between them.

Definition 2.1 [3] Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. Where m_X is an $\mathcal{M}$-structure (or a minimal structure) on X, if φ and X belong to m_X.

The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of m_X-open set is said to be m_X-closed.

Definition 2.2 [3] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For a subset A of X, m_X-closure of A and m_X-interior of A are defined as follows:

$m_X\text{-}cl(A) = \bigcap\{F : A \subseteq F,\ X - F \in \mathcal{M}\}$

$m_X\text{-}int(A) = \bigcup\{U : U \subseteq A,\ U \in \mathcal{M}\}$

Lemma 2.3 [3] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For subsets A and B of X, the following properties hold:

(a) $m_X\text{-}cl(X - A) = X - m_X\text{-}int(A)$ and $m_X\text{-}int(X - A) = X - m_X\text{-}cl(A)$.
(b) If $X - A \in m_X$, then $m_X\text{-}cl(A) = A$ and if $A \in m_X$ then $m_X\text{-}int(A) = A$ $m_X\text{-}int(A) = A$.
(c) $m_X\text{-}cl(\varphi) = \varphi$, $m_X\text{-}cl(X) = X$, $m_X\text{-}int(\varphi) = \varphi$ and $m_X\text{-}int(X) = X$.
(d) If $A \subseteq B$ then $m_X\text{-}cl(A) \subseteq m_X\text{-}cl(B)$ and $m_X\text{-}int(A) \subseteq m_X\text{-}int(B)$.
(e) $A \subseteq m_X\text{-}cl(A)$ and $m_X\text{-}int(A) \subseteq A$.
(f) $m_X\text{-}cl(m_X\text{-}cl(A)) = m_X\text{-}cl(A)$ and $m_X\text{-}int(m_X\text{-}int(A)) = m_X\text{-}int(A)$.
(g) $m_X\text{-}int(A \cap B) = (m_X\text{-}int(A)) \cap (m_X\text{-}int(B))$ and $(m_X\text{-}int(A)) \cup (m_X\text{-}int(B)) \subseteq m_X\text{-}int(A \cup B)$.
(h) $m_X\text{-}cl(A \cup B) = (m_X\text{-}cl(A)) \cup (m_X\text{-}cl(B))$ and $m_X\text{-}cl(A \cap B) \subseteq (m_X\text{-}cl(A)) \cap (m_X\text{-}cl(B))$.

Lemma 2.4 [7] Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X\text{-}cl(A)$ if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x.

Definition 2.5 [10] A minimal structure m_X on a nonempty set X is said to have the property $\mathcal{B}$ if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 2.6 A minimal structure m_X with the property $\mathcal{B}$ coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7 [5] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X satisfying the property $\mathcal{B}$. For a subset A of X, the following property hold:

(a) $A \in m_X$ iff m_X-$int(A) = A$
(b) $A \in m_X$ iff m_X-$cl(A) = A$
(c) m_X-$int(A) \in m_X$ and m_X-$cl(A) \in m_X$.

3. $\mathcal{M}_X\alpha\delta$-closed sets

Definition 3.1 [13] A subset A of an m-space $(X,\ m_X)$ is called

(a) $m_X\alpha$-open set if $A \subseteq m_X int\left(m_X cl\left(m_X int(A)\right)\right)$ and an $m_X\alpha$-closed set if $m_X cl\left(m_X int\left(m_X cl(A)\right)\right) \subseteq A$.
(b) m_X-regular open set if $A = m_X int\left(m_X cl(A)\right)$.

The $m_X\delta$-interior of a subset is the union of all m_X-regular open set of X contained in A and is denoted by m_X-$int_\delta(A)$. The subset A is called m_X-δ-open if $A = m_X$-$int_\delta(A)$, i.e. a set is $m_X - \delta$-open if it is the union of regular open sets. the complement of a $m_X - \delta$-open is called $m_X - \delta$-closed. Alternatively, a set $A \subseteq (X,\ m_X)$ is called $m_X - \delta$-closed if $A = m_X$-$cl_\delta(A)$, where $m_X - cl_\delta(A) = \{$x / x ∈ U ∈ $m_X \Rightarrow m_X - int(m_X - cl(A)) \cap A \neq \varphi\}$.

Definition 3.2 [13] A subset A of an m-space $(X,\ m_X)$ is called an

(a) $m_X\alpha g$-closed set if $m_X\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha$-open in $(X,\ m_X)$.
(b) $\mathcal{M}_X\alpha\delta$-closed set if m_X-$cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha g$-open in $(X,\ m_X)$.

4. $\mathcal{M}_X\alpha\delta$-kernel and $\mathcal{M}_X\alpha\delta$-closure

Definition 4.1 Let $(X,\ m_X)$ be an m-space and let A be a subset of X. Then

(a) The intersection of all $\mathcal{M}_X\alpha\delta$-open subsets of $(X,\ m_X)$ containing A is called the $\mathcal{M}_X\alpha\delta$-kernel of A (briefly, $\mathcal{M}_X\alpha\delta^{\sim Ker}(A)$) .i.e.,$\mathcal{M}_X\alpha\delta^{\sim Ker}(A) = \cap\{G \in \mathcal{M}_X\alpha\delta O(X): A \subseteq G\}$.
(b) Let $x \in X$. Then $\mathcal{M}_X\alpha\delta$-kernel of x is denoted by $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) = \cap\{G \in \mathcal{M}_X\alpha\delta O(X): x \in G\}$.
(c) Let X be a m-space and let $x \in X$. A subset N of X is said to be $\mathcal{M}_X\alpha\delta$-nbhd of x if there exists a $\mathcal{M}_X\alpha\delta$-open set G such that $x \in G \subset N$ which is denoted by $\mathcal{M}_X\alpha\delta$-$N$ (x).
(d) The union of all $\mathcal{M}_X\alpha\delta$-open sets that are contained in A is called the $\mathcal{M}_X\alpha\delta$-interior of A and is denoted by $\alpha\delta_{\mathcal{M}_X} int(A)$.
(e) The intersection of all $\mathcal{M}_X\alpha\delta$-closed sets containing A is called the $\mathcal{M}_X\alpha\delta$-closure of A and is denoted by $\alpha\delta_{\mathcal{M}_X} cl(A)$.

Theorem 4.2 Let X be a m-space. Then for any nonempty subset A of X, $\mathcal{M}_X\alpha\delta^{\sim Ker}(A) = \{\ x \in X :\ \alpha\delta_{\mathcal{M}_X} cl\ (\{x\}) \cap A \neq \varphi\ \}$.

Proof. Let $x \in \mathcal{M}_X\alpha\delta^{\sim Ker}(A)$. Suppose that $\alpha\delta_{\mathcal{M}_X} cl(\{x\}) \cap A = \varphi$. Then $A \subseteq X - \alpha\delta_{\mathcal{M}_X} cl(\{x\})$ and $X - \alpha\delta_{\mathcal{M}_X} cl(\{x\})$ is $\mathcal{M}_X\alpha\delta$-open set containing A but not x, which is a contradiction.

Conversely, let us assume that $x \notin \mathcal{M}_X\alpha\delta^{\sim Ker}(A)$ and $\alpha\delta_{\mathcal{M}_X} cl(\{x\}) \cap A \neq \varphi$. Then there exist an $\mathcal{M}_X\alpha\delta$-open set D containing A but not x and $y \in \alpha\delta_{\mathcal{M}_X} cl(\{x\}) \cap A$.

Hence an $\mathcal{M}_X\alpha\delta$-closed set $X - D$ contains x, and $\{x\} \subset X - D$, $y \notin X - D$. This is a contradiction to $y \in \alpha\delta_{\mathcal{M}_X} cl(\{x\}) \cap A$. Therefore $x \in \mathcal{M}_X\alpha\delta^{\sim Ker}(A)$.

Definition 4.3 In a space X, a set A is said to be *weakly ultra-$\mathcal{M}_X\alpha\delta$-separated* from a set B if there exists an $\mathcal{M}_X\alpha\delta$-open set G such that $A \subseteq G$ and $G \cap B = \varphi$ or $A \cap \alpha\delta_{\mathcal{M}_X} cl(B) = \varphi$.

By the definition 4.4 and the theorem 4.2, we have the following $x, y \in X$ of a m-space,

(a) $\alpha\delta_{\mathcal{M}_X} cl(\{x\}) = \{y : \{x\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{x\}$ }
(b) $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) = \{y : \{y\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{y\}$ }.

Definition 4.4 For any point x of a space X, is called

(a) $\mathcal{M}_X\alpha\delta$-derived (briefly,$\mathcal{M}_X\alpha\delta^{\#}D(\{x\})$) set of x is defined to be the set.
$\mathcal{M}_X\alpha\delta^{\#}D(\ \{x\}\ ) = \alpha\delta_{\mathcal{M}_X} cl(\{x\}) - \{x\} = \{\ y : y \neq x$ *and* $\{y\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{x\}\}$,
(b) $\mathcal{M}_X\alpha\delta$-shell (briefly,$\mathcal{M}_X\alpha\delta_{*Shl}(\{x\})$) of a singleton set $\{x\}$ is defined to be the set.
$\mathcal{M}_X\alpha\delta_{*Shl}(\{x\}) = \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) - \{x\} = \{\ y : y \neq x$ *and* $\{x\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{y\}$ },

Definition 4.5 Let X be a m-space. Then we define

(a) $\mathcal{M}_X\alpha\delta^{\#}N^{\#}D = \{\ x : x \in X$ and $\mathcal{M}_X\alpha\delta^{\#}D(\{x\}) = \varphi\ \}$,
(b) $\mathcal{M}_X\alpha\delta_{*N*Shl} = \{\ x : x \in X$ and $\mathcal{M}_X\alpha\delta_{*Shl}(\{x\}) = \varphi\ \}$ and
(c) $\mathcal{M}_X\alpha\delta$-$\langle x\rangle = \alpha\delta_{\mathcal{M}_X} cl(\{x\}) \cap \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$.

Theorem 4.6 Let $x,\ y \in X$. Then the following conditions hold.

(a) $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ if and only if $x \in \alpha\delta_{\mathcal{M}_X} cl(\{y\})$,
(b) $y \in \mathcal{M}_X\alpha\delta_{*Shl}(\{x\})$ if and only if $x \in \mathcal{M}_X\alpha\delta^{\#}D(\{y\})$,
(c) $y \in \alpha\delta_{\mathcal{M}_X} cl(\{x\})$ implies $\alpha\delta_{\mathcal{M}_X} cl(\{y\}) \subseteq \alpha\delta_{\mathcal{M}_X} cl(\{x\})$ and
(d) $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ implies $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\}) \subseteq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$

Proof. The proof of (a) and (b) are obvious.

(c) Let $z \in \alpha\delta_{\mathcal{M}_X} cl(\{y\})$. Then $\{z\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{y\}$. So there exists an $\mathcal{M}_X\alpha\delta$-open set G containing z such that $G \cap \{y\} \neq \varphi$. Hence $y \in G$ and by assumption $G \cap \{x\} \neq \varphi$. Hence $\{z\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{x\}$. So $z \in \alpha\delta_{\mathcal{M}_X} cl(\{x\})$. Therefore $\alpha\delta_{\mathcal{M}_X} cl(\{y\}) \subseteq \alpha\delta_{\mathcal{M}_X} cl(\{x\})$.

(d) Let $z \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$. Then $\{y\}$ *is not weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from* $\{z\}$. So $y \in \alpha\delta_{\mathcal{M}_X} cl(\{z\})$. Hence

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$\alpha\delta_{\mathcal{M}_X}cl(\{y\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{z\})$. By assumption $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ and then $x \in \alpha\delta_{\mathcal{M}_X}cl(\{y\})$. So

$\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{y\})$. Ultimately $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{z\})$. Hence $x \in \alpha\delta_{\mathcal{M}_X}cl(\{z\})$, that is $z \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$.

Therefore $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\}) \subseteq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$.

Let us recall that a subset A of X is called a degenerate set if A is either a null set or a singleton set.

Theorem 4.7 Let $x, y \in X$. Then,

(a) for every $x \in X$, $\mathcal{M}_X\alpha\delta_{*Shl}(\{x\})$ is degenerate if and only if for all $x, y \in X$, $x \neq y$, $\mathcal{M}_X\alpha\delta^{\#}D(\{x\}) \cap \mathcal{M}_X\alpha\delta^{\#}D(\{y\}) = \varphi$,

(b) for every $x \in X$, $\mathcal{M}_X\alpha\delta^{\#}D(\{x\})$ is degenerate if and only if for every $x, y \in X$, $x \neq y$, $\mathcal{M}_X\alpha\delta_{*Shl}(\{x\}) \cap \mathcal{M}_X\alpha\delta_{*Shl}(\{y\}) = \varphi$.

Proof. (a) Let $\mathcal{M}_X\alpha\delta^{\#}D(\{x\}) \cap \mathcal{M}_X\alpha\delta^{\#}D(\{y\}) \neq \varphi$. Then there exists a $z \in X$ such that $z \in \mathcal{M}_X\alpha\delta^{\#}D(\{x\})$ and $z \in \mathcal{M}_X\alpha\delta^{\#}D(\{y\})$. Then $z \neq y \neq x$ and $z \in \alpha\delta_{\mathcal{M}_X}cl(\{x\})$ and $z \in \alpha\delta_{\mathcal{M}_X}cl(\{y\})$, that is $x, y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{z\})$. Hence $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{z\})$ and so $\mathcal{M}_X\alpha\delta_{*Shl}(\{z\})$ is not a degenerate set.

Conversely, let $x, y \in \mathcal{M}_X\alpha\delta_{*Shl}(\{z\})$. Then we get $x \neq z$, $x \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{z\})$ and $y \neq z$, $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{z\})$ and hence z is an element of both $\alpha\delta_{\mathcal{M}_X}cl(\{x\})$ and $\alpha\delta_{\mathcal{M}_X}cl(\{y\})$, which is a contradiction.

(b) Obvious.

Theorem 4.8 If $y \in \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle$, then $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle = \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle$.

Proof. If $y \in \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle$, then $y \in \alpha\delta_{\mathcal{M}_X}cl(\{x\}) \cap \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$. Hence $y \in \alpha\delta_{\mathcal{M}_X}cl(\{x\})$ and $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ and so we have $\alpha\delta_{\mathcal{M}_X}cl(\{y\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{x\})$ and $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\}) \subseteq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$. Then $\alpha\delta_{\mathcal{M}_X}cl(\{y\}) \cap \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{x\}) \cap \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$. Hence $\mathcal{M}_X\alpha\delta\text{-}\langle y\rangle \subseteq \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle$. The fact that $y \in \alpha\delta_{\mathcal{M}_X}cl(\{x\})$ implies $x \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$ and $y \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ implies $x \in \alpha\delta_{\mathcal{M}_X}cl(\{y\})$. Then we have that $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle \subseteq \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle$. So $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle = \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle$.

Theorem 4.9 For all $x, y \in X$, either $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle \cap \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle = \varphi$ or $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle = \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle$.

Proof. $\mathcal{M}_X\alpha\delta\text{-}\langle x\rangle \cap \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle \neq \varphi$, then there exists $z \in X$ such that $z \in \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle$ and $z \in \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle$. So by *Theorem 4.8*, $\mathcal{M}_X\alpha\delta\text{-}\langle z\rangle = \mathcal{M}_X\alpha\delta\text{-}\langle x\rangle = \mathcal{M}_X\alpha\delta\text{-}\langle y\rangle$. Hence the result.

Theorem 4.10 For any two points $x, y \in X$, the following statements are equivalent.

(a) $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) \neq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$ and

(b) $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \neq \alpha\delta_{\mathcal{M}_X}cl(\{y\})$.

Proof. $(a) \Rightarrow (b)$ Let us assume $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) \neq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$. Then there exists a $z \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$ but $z \notin \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$. As $z \in \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$, $x \in \alpha\delta_{\mathcal{M}_X}cl(\{z\})$ and $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \subseteq \alpha\delta_{\mathcal{M}_X}cl(\{z\})$. Also we have taken $z \notin \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$, by Theorem 4.2, $\alpha\delta_{\mathcal{M}_X}cl(\{z\}) \cap \{y\} = \varphi$, so $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \cap \{y\} = \varphi$ and so $\{y\}$ is weakly ultra-$\mathcal{M}_X\alpha\delta$-separated from $\{x\}$ and hence we get that $y \notin \alpha\delta_{\mathcal{M}_X}cl(\{x\})$. Hence $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \neq \alpha\delta_{\mathcal{M}_X}cl(\{y\})$.

$(b) \Rightarrow (a)$ Suppose $\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \neq \alpha\delta_{\mathcal{M}_X}cl(\{y\})$. Then there exists a point $z \in \alpha\delta_{\mathcal{M}_X}cl(\{x\})$ but $z \notin \alpha\delta_{\mathcal{M}_X}cl(\{y\})$. So, we get an $\mathcal{M}_X\alpha\delta$-open set containing z and x but not y. That is $y \notin \mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\})$. Hence $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) \neq \mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\})$.

Theorem 4.11 Let X be a m-space and each $x \in X$, Let $\mathcal{M}_X\alpha\delta\text{-}N\,(X, m_X)$ be the collection of all $\mathcal{M}_X\alpha\delta$-nbhd of x. Then we have the following results.

(a) $\forall\, x \in X$, $\mathcal{M}_X\alpha\delta\text{-}N\,(x) \neq \varphi$

(b) $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x) \Rightarrow x \in N$.

(c) $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x), M \supset N \Rightarrow M \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$

(d) $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x) \Rightarrow$ there exists $M \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$ such $M \subset N$ and $M \in \mathcal{M}_X\alpha\delta\text{-}N\,(y)$ for every $y \in M$.

Proof. (a) Since X is $\mathcal{M}_X\alpha\delta$-open set, it is a $\mathcal{M}_X\alpha\delta$-nbhd of every $x \in X$. Hence there exists at least one $\mathcal{M}_X\alpha\delta$-nbhd (namely-$X$) for each $x \in X$. Hence $\mathcal{M}_X\alpha\delta\text{-}N\,(x) \neq \varphi$ for every $x \in X$.

(b) If $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$, then N is a $\mathcal{M}_X\alpha\delta$-nbhd of x. So by definition of $\mathcal{M}_X\alpha\delta$-nbhd, $x \in N$.

(c) Let $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$ and $M \supset N$. Then there is a $\mathcal{M}_X\alpha\delta$-open set U such that $x \in U \subset N$. Since $N \subset M, x \in U \subset M$ and M is $\mathcal{M}_X\alpha\delta$-nbhd of x. Hence $M \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$.

(d) If $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$, then there exists a $\mathcal{M}_X\alpha\delta$-open set M such that $x \in M \subset N$. Since M is a $\mathcal{M}_X\alpha\delta$-open set, it is $\mathcal{M}_X\alpha\delta$-nbhd of each of its points. Therefore $M \in \mathcal{M}_X\alpha\delta\text{-}N\,(y)$ for every $y \in M$.

Theorem 4.12 Let X be a nonempty set, for each $x \in X$, let $\mathcal{M}_X\alpha\delta\text{-}N\,(x)$ be nonempty collection of subsets of X satisfying following conditions.

(a) $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(X, m_X) \Rightarrow x \in N$.

(b) Let m_X consists of the empty set and all those non-empty subsets of U of X having the property that $x \in U$ implies that there exists an $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$ such that $x \in N \subset U$, Then m_X is a m-space for X.

Proof. (a) $\varphi \in m_X$ by definition. We now show that $x \in m_X$. Let x be any arbitrary element of X. Since $\mathcal{M}_X\alpha\delta\text{-}N\,(x)$ is nonempty, there is an $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$ and so $x \in N$. Since N is a subset of X, we have $x \in N \subset X$. Hence $X \in m_X$.

(b) Let $U_\lambda \in m_X$ for every $\lambda \in \Lambda$. If $x \in \cup\, \{U_\lambda : \lambda \in \Lambda\}$, then $x \in U_{\lambda_x}$ for some $\lambda_x \in \Lambda$. Since $U_{\lambda_x} \in m_X$, there exists an $N \in \mathcal{M}_X\alpha\delta\text{-}N\,(x)$ such that $x \in N \subset U_{\lambda_x}$ and consequently $x \in N \subset \cup\,\{U_\lambda : \lambda \in \Lambda\}$. Hence $\cup\,\{U_\lambda : \lambda \in \Lambda\} \in \tau$. It follows that m_X is a m-space for X.

5. Sober-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$ spaces

Definition 5.1 A m-space $(X,\ m_X)$ is said to be $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$ if $\bigcap_{x\in X}\alpha\delta_{\mathcal{M}_X}cl(\{x\}) = \varphi$.

Theorem 5.2 A m-space $(X,\ m_X)$ is $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$ if and only if $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) \neq X$ for every $x \in X$.

Proof. Suppose that the space $(X,\ m_X)$ be $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$. Assume that there is a point y in X such that $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{y\}) = X$. Then $y \notin O$ which O is some proper $\mathcal{M}_X\alpha\delta$-open subset of X. This implies that $y \in \bigcap_{x\in X}\alpha\delta_{\mathcal{M}_X}cl(\{x\})$. But this is a contradiction.

Now assume that $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) \neq X$ for every $x \in X$. If there exists a point y in X such that $y \in \bigcap_{x\in X}\alpha\delta_{\mathcal{M}_X}cl(\{x\})$, then every $\alpha\delta$-open set containing y must contain every point of X. This implies that the space X is the unique $\mathcal{M}_X\alpha\delta$-open set containing y. Hence $\mathcal{M}_X\alpha\delta^{\sim Ker}(\{x\}) = X$ which is a contradiction. Therefore $(X,\ m_X)$ is $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$.

Theorem 5.3 If the m-space X is $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$ and Y is any m-space, then the product $X \times Y$ is $Sober$-$\mathcal{M}_X\alpha\delta\mathcal{R}_0$.

Proof. By showing that $\bigcap_{(x,y)\in X\times Y}\alpha\delta_{\mathcal{M}_X}cl(\{x,y\}) = \varphi$ we are done. We have:

$$\bigcap_{(x,y)\in X\times Y}\alpha\delta_{\mathcal{M}_X}cl(\{x,y\}) \subset \bigcap_{(x,y)\in X\times Y}(\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \times \alpha\delta_{\mathcal{M}_X}cl(\{y\}))$$
$$=\bigcap_{x\in X}\alpha\delta_{\mathcal{M}_X}cl(\{x\}) \times \bigcap_{y\in Y}\alpha\delta_{\mathcal{M}_X}cl(\{y\}) \subset \varphi \times Y = \varphi.$$

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