Analysis of electromagnetic Whistler mode instability for relativistic plasma: Lorentzian Kappa

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Abstract- Whistler mode instability in interplanetary space at 1AU(Astronomical Unit) is investigated using an anisotropic Lorentzian Kappa distribution with perpendicular AC electric field for relativistic plasma. The method of characteristic solutions using the perturbed and unperturbed particle trajectories have been used to determine the perturbed distribution function. The conductivities and dielectric tensors are then determined and used to obtain the dispersion relation. The special case of whistler mode dispersion is then use to determine the growth rates for various plasma parameters. Present studies are helpful in making estimates on high energetic plasma particles and properties of whistler mode waves and thus contribute to a better understanding of the auroral activity in the planetary atmospheres.

Index Terms- Whistler mode instability for relativistic plasma Lorentzian Kappa

I. INTRODUCTION

Whistler mode waves have been a common feature of spectrum wave observations at Earth's bow shock for many years [Heppner et al. 1967; Fair 1974]. It has been shown that whistler wave can be excited through the application of electron beams gyro phase bunching ions [Gurgiolo et al., 1993] and wave steeping [Hoppe and Russell 1980]. Whistler waves at spacecraft frequency between 1 and 7 Hz have been reported upstream of Earth [Hoppe et al., 1982]. While there are observations of whistler waves propagating parallel to the interplanetary magnetic field, most reported whistler wave observations and generation theories involve highly oblique propagation. Whistler mode waves have also been observed in commentary foreshocks and are thought to arise from the same mechanisms as above [Tsurutani et al., 1987]. It is also reported that the whistler mode can be driven by electron temperature anisotropy [Kennel and Petschek 1966].

Higher frequency whistler wave activity has also been observed upstream of the Earth's bow shock by plasma wave analysis. These waves possess spacecraft frame frequencies from approximately 10 to 100Hz and are generally synchronous with plasma oscillations at the electron plasma frequency [Anderson et al., 1981; Greenstadt et al., 1981; Toker et al., 1984]. Tokar and Gurnett [1985] argued that these waves, when observed with the shock ramp result from electron beams with high thermal anisotropy and beam velocities directed towards the magnetosheath [Feldman et al., 1983]. Similarly high frequency whistler waves have been observed by (International Sun Earth Explorer) ISEE 3 in the distant upstream plasma [Kennel et al., 1985]. These waves are also coincident with electron plasma oscillations and they possibly result from streaming electrons with a solar wind, rather than a bow shock origin in accordance with the instability analysis of Gary and Feldman [1977]. Orlowski et al [1990] suggest that whistler waves observed in planetary foreshocks may not be the result of in situ generation, but rather these observations may simply result from propagation away from the shock.

Whistler waves excited by an electron using kappa distribution and an anisotropic bi-maxwellian distribution functions in space plasma have been studied by using the one dimensional particle simulation technique and confirmed the result of linear and non-linear theory [Lu Quanming et al., 2010; 2004]. The study of thermal velocity of superathermal electron has been done by Vasyliunas [1968] using the observation of satellite data of OGO1 and OGO3 in the magnetosphere.

Electric field measurements at magnetospheric heights and shock region have given values of AC electric field along and perpendicular to Earth's magnetic field [Mozer et al., 1978 ;Wygant et al., 1987; Lindquist and Mozer 1990; Pandey et al., 2008; Misra and Pandey 1995]. Various authors have discussed the role of parallel DC and AC electric fields on the whistler mode instability in the magnetosphere by generally adopting plasma dispersion function which is based on anisotropic Maxwellian distributions to describe the resonant population [Misra and Singh 1980; Pandey et al., 2002A Pandey et al., 2002B]. However in the natural space environment, plasma is generally observed to possess a non-Maxwellian high-energy tail that can be well modeled by a generalized Lorentzian (Kappa) distribution function containing a spectral index κ. The Maxwellian and kappa distributions differ substantially in the high-energy tail but differences become less significant for higher values of Kappa [Pandey et al., 2008; Pandey et al., 2001]. Motivated by these studies whistler mode instability has been analyzed in this paper for relativistic plasma in the presence of perpendicular a.c. electric field using kappa distribution in the interplanetary space at 1AU.

II. DISPERSION RELATIONS AND GROWTH RATE

Homogeneous anisotropic collisionless plasma in the presence of an external magnetic field $\mathrm{B}_0 = \mathrm{B}_0 \hat{\mathrm{e}}_z$ and an electric field $E_{0x} = E_0 \sin \upsilon t \ \hat{e}_x$ is assumed. The in- homogeneity is assumed to be small in interaction zone. In order to obtain the particle trajectories, perturbed distribution function and dispersion relation, the linearised Vlasov-Maxwell equations are used. Separating the equilibrium and non equilibrium parts, neglecting the higher order terms and following the techniques of

Pandey et. a1 [2005] the linearized Vlasov equations are given as:

$$v.\frac{\delta f_0}{\delta r}+\frac{e_s}{m_e}\left[E_0 \sin \upsilon t+\frac{(v\times B_0)}{c}\right].\frac{\delta f_0}{\delta v}=0 \tag{1}$$

$$\frac{\delta f_1}{\delta t}+v.\frac{\delta f_1}{\delta r}+\left(\frac{F}{m_e}\right).\frac{\delta f_1}{\delta v}=S(r,v,t) \tag{2}$$

Where $f_0 =$ Unperturbed distribution function. f_1=perturbed distribution function and the force

$$F=e\left[E_0 \sin \upsilon t+\frac{(v\times B_0)}{c}\right]=m\frac{dv}{dt} \tag{3}$$

Where υ is AC field frequency, E_0 = magnitude of AC electric field and

$$S(r,v,t,)=-\left(\frac{e_s}{m_e}\right)\left[E_1+\frac{(v\times B_0)}{c}\right].\left(\frac{\delta f_1}{\delta v}\right) \tag{4}$$

where s denotes the type of electrons. Subscript '0' denotes the equilibrium values. The perturbed distribution function f_1 is determined by using the method of characteristic, which is

$$f_1(r,v,t)=\int_0^\infty S\{r_0(r,v,t,),v_0(r,v,t),t-t'\}dt$$

Transformed the phase space coordinate system for (r, v, t) to (r_0, v_0,t - t').The relativistic particle trajectories that have been obtained by solving equation (3) for given external field configuration are

$$X_0=X+\left(\frac{P_\perp \sin\theta}{\omega_c m_e}\right)-\left[P_\perp \sin\left\{\frac{\theta+\left(\frac{\omega_c t}{\beta}\right)}{\omega_c m_e}\right\}\right]+\left[\frac{\Gamma_x \sin \upsilon t}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}\right]-\left[\frac{\upsilon\Gamma_x \sin\left(\frac{\omega_c t}{\beta}\right)}{\omega_c\left\{\left(\frac{\omega_c t}{\beta}\right)-\upsilon^2\right\}}\right]$$

$$Y_0=Y-\left(\frac{P_\perp \cos\theta}{\omega_c m_e}\right)-\left[P_\perp \cos\left\{\frac{\theta+\left(\frac{\omega_c t}{\beta}\right)}{\omega_c m_e}\right\}\right]+\left(\frac{\Gamma_x}{\upsilon\omega_c}\right)-\frac{\left\{1+\upsilon^2\beta^2\cos\left(\frac{\omega_c t}{\beta}\right)-\omega_c^2\cos \upsilon t\right\}}{\beta^2\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}$$

$$z_0=z-\frac{P_z}{\beta m_e} \tag{5}$$

Where $\theta =$ angle of projection, $P_\perp$ *and* P_z are being perpendicular and parallel momentum and the velocities are

$$v_{xo}=P_\perp \cos\left\{\theta+\frac{\left(\frac{\omega_c t}{\beta}\right)}{\beta m_e}\right\}+\left[\frac{\upsilon\Gamma_x}{p\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}\right]\left\{\cos \upsilon t-\cos\left(\frac{\omega_c t}{\beta}\right)\right\}$$

$$v_{yo}=P_\perp \sin\left\{\theta+\frac{\left(\frac{\omega_c t}{\beta}\right)}{\beta m_e}\right\}+\left[\frac{\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2-\upsilon^2\right\}}\right]\left\{\left(\frac{\omega_c}{\beta}\right)\sin \upsilon t-\upsilon \sin\left(\frac{\omega_c t}{\beta}\right)\right\}$$

$$v_{zo}=\frac{P_z}{\beta m_e},\quad v_x=\frac{P_\perp \cos\theta}{\beta m_e},\quad v_y=\frac{P_\perp \sin\theta}{\beta m_e},$$

$$v_z=\frac{P_z}{\beta m_e}$$

$$\Gamma_x=\frac{eE_o}{m_e},\quad m_e=\frac{m_s}{\beta},\quad \beta=\sqrt{1-\frac{v^2}{c^2}},$$

$$\omega_c=\frac{eB_0}{m_e} \tag{6}$$

Using equation (5), (6) and the Bessel identity then performing the time integration, following the technique and method of Misra and Pandey [1995] and Pandey et al.,[2008], the perturbed distribution function is found after some lengthy algebraic simplifications as :

$$f_1=-\left(\frac{ie_s}{m_e\beta\omega}\right)\sum J_s(\lambda_3)\exp i(m-n)\theta\left[\frac{J_m J_n J_p U^* E_{1x}-iJ_m V^* E_1+J_m J_n J_p W^*}{\omega-\left(\frac{k_\parallel P_z}{\beta m_e}+p\upsilon-\frac{(n+g)\omega_c}{\beta}\right)}\right] \tag{7}$$

Due to the phase factor the solution is possible when m = n.

Here.

$$U^*=\left(\frac{c_1 P_\perp n}{\beta\lambda_1 m_e}\right)-\left(\frac{n\upsilon c_1 D}{\lambda_1}\right)+\left(\frac{p\upsilon c_1 D}{\lambda_2}\right),$$

$$V^*=\left(\frac{c_1 P_\perp J_n J_p}{\beta\lambda_1 m_e}\right)+c_1 D J_p J_n \omega_c,$$

$$W^*=\left(\frac{n\omega_c F m_e}{k_\perp P_\perp}\right)+\left(\frac{\beta m_e P_\perp \omega\partial f_0}{\partial P_z}\right)+G\left\{\left(\frac{p}{\lambda_2}\right)-\left(\frac{n}{\lambda_1}\right)\right\},$$

$$C_1 = \left\{\frac{(\beta m_e)}{P_\perp}\right\}\left(\frac{\partial f_0}{\partial P_\perp}\right)\left(\omega - \frac{k_\parallel p_z}{\beta m_e}\right) + k_\parallel \beta m_e \left(\frac{\partial f_0}{\partial P_\perp}\right),$$

$$D = \left[\frac{\Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}}\right],$$

$$F = \frac{H k_\perp P_\perp}{\beta m_e},$$

$$H = \left\{\frac{(\beta m_e)^2}{P_\perp}\right\}\left(\frac{\partial f_0}{\partial P_\perp}\right)\left(\frac{P_z}{\beta m_e}\right) + \beta m_e \left(\frac{\partial f_0}{\partial P_z}\right),$$

$$G = \frac{H k_\perp \upsilon \Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}}, \quad J_n(\lambda_1) = \frac{dJ_n(\lambda_1)}{d\lambda_1},$$

$$J_n(\lambda_2) = \frac{dJ_p(\lambda_2)}{d\lambda_2}$$

and the Bessel function arguments are defined as

$$\lambda_1 = \frac{k_\perp P_\perp}{\omega_c m_e}, \quad \lambda_2 = \frac{k_\perp \Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}},$$

$$\lambda_3 = \frac{k_\perp \upsilon \Gamma_x}{\beta\left\{\left(\frac{\omega_c}{\beta}\right)^2 - \upsilon^2\right\}}$$

The conductivity tensor $\|\sigma\|$ is found to be

$$\|\sigma\| = \frac{-i\sum\left(e^2/\beta m_e\right)^2 \omega \int d^3P J_g(\lambda_3) \| s \|}{\left[\omega - \left(\frac{k_\parallel P_z}{\beta m_e}\right) - \left((n+g)\frac{\omega_c}{\beta}\right) + p\upsilon\right]}$$

where

$$\| S \| = \begin{vmatrix} P_\perp J_n^2 J_p\left(\frac{n}{\lambda_1}\right) U^* & i P_\perp J_n V^* & P_\perp J_n^2 J_p\left(\frac{n}{\lambda_1}\right) W^* \\ P_\perp J_n J_n J_p\left(\frac{n}{\lambda_1}\right) U^* & i P_\perp J_n V^* & P_\perp J_n J_p\left(\frac{n}{\lambda_1}\right) W^* \\ P_z J_n^2 J_p\left(\frac{n}{\lambda_1}\right) U^* & i P_z J_n V^* & P_z J_n^2 J_p\left(\frac{n}{\lambda_1}\right) W^* \end{vmatrix}$$

By using these in the Maxwell's equations we get the dielectric tensor,

$$\varepsilon_{ij} = 1 + \sum\left\{\frac{4\pi e_s^2}{(\beta m_e)^2 \omega}\right\}\int \frac{d^3P J_g(\lambda_3) \| S \|}{\left(\omega - \frac{k \| Pz}{\beta m_e}\right) - \left\{\frac{(n+g)\omega_c}{\beta}\right\} + p\upsilon}$$

For parallel propagating whistler mode instability, the general dispersion relation reduces to $\varepsilon_{11} \pm \varepsilon_{12} = N^2$,

$$N^2 = \frac{k^2 c^2}{\omega^2}$$

The dispersion relation for relativistic case with perpendicular AC electric field for g= o, p = 1, n = 1 is written as:

$$\frac{k^2 c^2}{\omega^2} = 1 + \frac{4\pi e_s^2}{(\beta m_e)^2 \omega^2}\int \frac{d^3P}{\beta}\left[\frac{P_\perp}{2} - \frac{\upsilon \Gamma_x m_e}{2\left(\frac{\omega_c^2}{\beta^2} - \upsilon^2\right)}\right]\left[\left(\beta\omega - \frac{k_\parallel P_\parallel}{m_e}\right)^2 \frac{\partial f_0}{\partial P_\perp} + \frac{P_\perp k_\parallel}{m_e}\frac{\partial f_0}{\partial P_\perp}\right]\frac{1}{\beta\omega - \frac{k_\parallel P_\parallel}{m_e} - \omega_c + \beta\upsilon} \tag{9}$$

The bi-Lorentzian Kappa distribution function is given as

$$f_0 = \frac{n_0}{\pi^{3/2} k^{1/2} \theta_\perp^2 \theta_\parallel}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa+1/2)}\left(1 + \frac{P_\parallel^2}{\kappa\theta_\parallel^2}\frac{P_\perp^2}{\kappa\theta_\perp^2}\right)^{-(\kappa+1)} \tag{10}$$

$\theta_\parallel$ and $\theta\perp$ are respective thermal speeds parallel and perpendicular to the background magnetic field and is defined as

$$\theta_\perp = \left(\frac{2k-3}{k\theta_\perp^2}\right)^{1/2}\left(\frac{k_B T_\perp}{m}\right)^{1/2},$$

$$\theta_{||} = \left(\frac{2k-3}{k}\right)^{1/2}\left(\frac{k_B T_{||}}{m}\right)^{1/2}$$

Substituting and using equation (13), (14) and doing integration by parts the dispersion relation is found as:

$$\frac{k^2c^2}{\omega^2}=1+\frac{\omega_p^2}{\omega^2}\int\frac{d^3P}{\beta}\left[1-\frac{\upsilon\Gamma_x m}{P_\perp\left(\frac{\omega_c^2}{\beta^2}-\upsilon^2\right)}\right]\left[\frac{\left(\beta\omega-\frac{k_\parallel P_\parallel}{m}\right)}{\left(\beta\omega-\frac{k_\parallel P_\parallel}{m}\right)}-\frac{P_\perp^2}{2m^2c^2}\frac{\left(\omega^2-k_\parallel^2c^2\right)}{\left(\beta\omega-\frac{k_\parallel P_\parallel}{m}-\omega c+\beta\upsilon\right)^2}\right]f_0 \quad (11)$$

Doing some lengthy integrals the general dispersion relation becomes

$$\frac{k^2c^2}{\omega^2}=1+\frac{\omega_p^2}{\omega^2\beta\theta_\perp^2}\left[X_i\frac{\beta m\omega}{k_\parallel\theta_\parallel}\left(\frac{\kappa-1}{\kappa-3/2}\right)Z_{k-1}\left\{\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\xi\right\}+\left(X_2-X_5\frac{\omega^2}{k^2c^2}\right)\right.$$

$$\left.\left\{1+\xi\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\left(\frac{\kappa-1}{\kappa-3/2}\right)\right\}Z_{k-1}\left\{\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\right\}\xi\right] \quad (12)$$

Where,

$$X_1=\theta_\perp^2-\frac{\upsilon\Gamma_x m}{\left(\frac{\omega_c}{\beta}\right)-\upsilon^2}\sqrt{\pi\theta_\perp}$$

$$X_2=\theta_\perp^2A_{kT}-\frac{\upsilon\Gamma_x m}{\left(\frac{\omega_c}{\beta}\right)-\upsilon^2}\sqrt{\pi\theta_\perp}\left(\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{1}{2}-1\right)$$

$$X_3=\frac{\theta_\perp^2}{\theta_\parallel^2}\left[\theta_\perp^2-\frac{\upsilon\Gamma_x m}{\left(\frac{\omega_c}{\beta}\right)-\upsilon^2}\sqrt{\pi\theta_\perp}\left(\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{1}{2}-1\right)\right]$$

$$A_{kT}=\frac{\theta_\perp^2}{\theta_\perp^2}-1,\quad \xi=\frac{\beta m\omega-m\omega_c+\beta m\upsilon}{k_\parallel\theta_\parallel} \quad (13)$$

For real k and substituting

$$\frac{k^2c^2}{\omega^2}>>1$$

And using the expression of modified dispersion function Summers &Thorne [1991] $Z^*_k(\xi)$ in the limit

$$Z_\kappa^*(\xi)=\frac{\kappa!\kappa^{\left(\kappa-\frac{1}{2}\right)}\sqrt{\pi}}{\Gamma_{(\kappa-1/2)}\xi^2(\kappa+1)}\left\{1-\frac{\kappa(\kappa+1)}{\xi^2}+........\right\}-\left(\frac{2\kappa-1}{2\kappa}\right)\frac{1}{\xi}\left(1+\frac{\kappa}{2\kappa-1}\frac{1}{\xi}+......\right) \quad (14)$$

For $|\xi| \to \infty$

The expression for growth rate for real frequency ω_r in dimensionless form is found to be

$$\frac{\gamma}{\omega_c}=\frac{\frac{\sqrt{\pi}(k-1)\kappa^{\kappa-1/2}}{\beta(k-3/2)!\tilde{k}}\left(\frac{X_2}{X_1}-k_3\right)k_4^3\left\{-\left(\frac{k_4}{\tilde{k}}\right)\right\}^{-2k}}{1+\beta X_4+\frac{\kappa}{\kappa-3/2}\frac{\tilde{k}^2}{2}\left(\frac{1+\beta X_4}{k_4^2}+\frac{2X_4X_3\theta_\parallel^2}{X_1\tilde{k}^2c^2}\right)-\frac{\tilde{k}^2}{k_4}\left(\frac{x_2}{x_1}-k_3\right)} \quad (15)$$

$$X_3=\frac{\tilde{k}^2}{\delta\beta_1}\left[(1+\beta X_4)+\frac{X_2}{X_1}\frac{\beta_1}{1+\beta X_4}\right] \quad (16)$$

Where

$$k_3=\frac{\beta X_3}{k_4}+\frac{X_4}{X_1}\frac{X_3^2}{\tilde{k}^2}\frac{\theta_\parallel^2}{c^2},$$

$$\tilde{k}=\frac{k_\parallel\theta_\parallel}{m_e\omega_c},$$

$$k_4=1-\beta X_3+\beta X_4,$$

$$X_3=\frac{\omega_r}{\omega_c},\quad X_4=\frac{-\beta\upsilon}{\omega_c},\quad \beta_1=\frac{k_BT_\parallel\mu_0n_0}{B_0^2} \quad (17)$$

When the relativistic factor is not considered, that is when the velocity of plasma does not approach velocity of light, Then m_s = m_e and the expressions for Growth rate and real frequency reduce to Pandey et al.,[2005].

III. RESULTS AND DISCUSSION

For numerical evaluation of normalized growth rate and real frequency of relativistic whistler mode in the presence of perpendicular AC electric field has been analyzed for Kappa distribution function of electron density for inter planetary space at 1AU.

Following plasma parameters have been taken from Pandey et al [2005] $B_0 = 8 \times 10^{-9}$T, $A_T= [(T_\perp/T_\parallel) - 1] = 0.25, 0.5, 0.75$, κ= 2,3,4, relativistic factor b_1 = v/c= 0.3, 0.6, 0.9, E_0 = 20 mV/m. AC field frequency υ varies from zero to 400 Hz. According to this choice of plasma parameters, the explanations and details of results are given as follows. Fig. 1 depicts the variation of normalized growth rate and real frequency with respect to normalized $\bar{k}$ for various values of temperature anisotropy for kappa distribution index κ= 2. At this location the growth rate as well as the bandwidth increases with the increase of the temperature anisotropy and maxima is shifted towards the higher $\bar{k}$ values. It is clear from the figure that the temperature anisotropy is the main source of energy to drive the excitation of the wave. Lorentzian (Kappa) plasma series expansion brings change in perpendicular thermal velocity $\theta_\perp$. Therefore any change in $\theta_\perp$ shall affect marginally $T_\perp$, affecting temperature anisotropy terms. Temperature anisotropy being the primary source of instability gets further modified by Lorentzian (Kappa) distribution function, giving rise to further increase in growth rate. Recently it was found that suprathermal electron in Kappa distribution modifies the intensity and Doppler frequency of electron plasma lines. The inclusion of temperature anisotropy in Lorentzian (Kappa) plasma can explain the observed higher frequencies spectrum of whistler waves [Pandey et al.,2008]. Figure 2 shows the variation of growth rate and real frequency for various values of number density and other fixed plasma parameters. The growth rate increases as density increases. The maxima shifts to higher values of k. Fig. 3 exhibits variation of normalized growth rate and real frequency versus $\bar{k}$ for various values of the thermal energy of electron at other fixed plasma parameters. The depressive's properties of the whistler waves are known to dependent sensitivity on the density and composition of thermal energy of plasma it is clear as the growth rate as well as the band width increases with the increase of thermal velocity of electron. Fig. 4 shows the variation of normalized growth rate and real frequency with $\bar{k}$ for variation of the relativistic factor b_1 = (v/c). With the increase of the relativistic factor the growth rate increases and the bandwidth widens. This shows that the velocity of the energetic electrons have triggering effect on the growth of the wave.

Fig. 5 Shows the variation of normalized growth rate and real frequency with $\bar{k}$ for various values of spectral index κ. The growth rate increases and bandwidth shrinks towards higher wave number for increasing the value of κ. For $\kappa \rightarrow \infty$ the value of normalized growth rate approaches the value of growth rate for Maxwellian distribution function. This effect remains basically applicable to the Lorentzian (Kappa) plasma also, except that the limit of temperature anisotropy in this case is little higher because of series solution involving κ. Fig. 6 Shows variation of normalized growth rate and real frequency versus $\bar{k}$ for various values of AC electric field frequency for other fixed plasma parameters. The growth rate increases with increase of the value of a.c. frequency, maxima shifts to lower values of $\bar{k}$. it means that the a.c. frequency modifies resonance frequency. The increase of AC frequency increases the growth rate due to the negative exponential of Landau damping. The perpendicular electric field that modifies the perpendicular velocity contributing to the energy exchange contributes significantly to the emission of VLF signals and can explain the low frequency side of the spectrum. The energy exchange between electrons, the components of the wave electric field and the impressed AC field perpendicular to the magnetic field mainly contributes to the cyclotron growth or the damping of the waves.

Thus the frequency of the perpendicular AC electric field brings the maxima to different $\bar{k}$ as if the resonant charged particles were oscillating at different cyclotron frequencies and absorbing energy and thus growing.

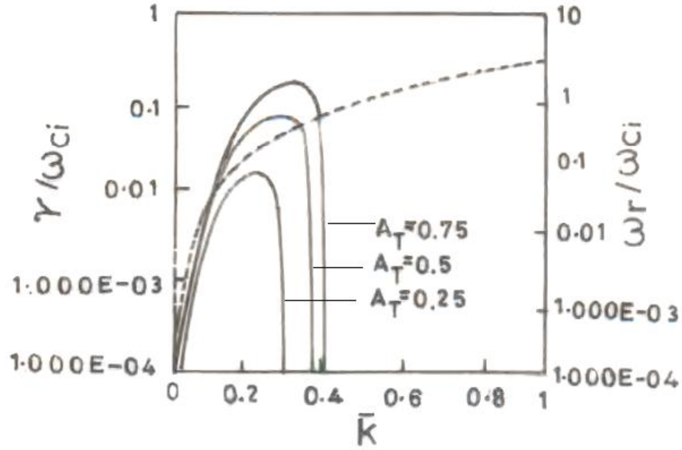

Fig. 1 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various values of temperature anisotropy at other fixed plasma parameters.

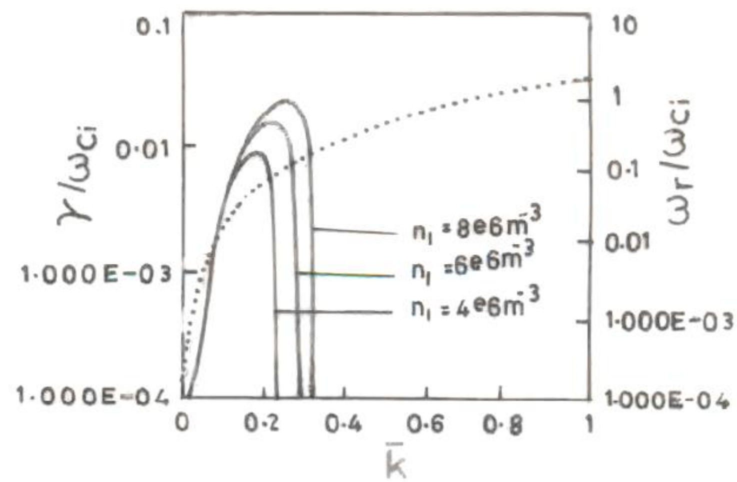

Fig. 2 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various values of number density anisotropy at other fixed plasma parameters.

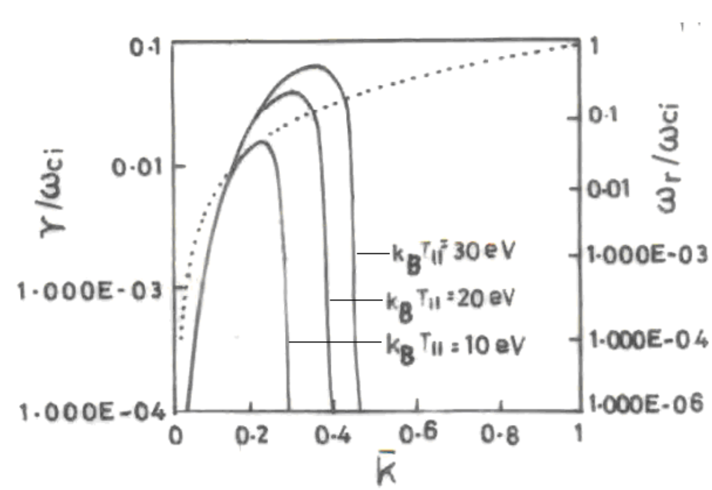

Fig. 3 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various values of thermal energy at other fixed plasma parameters.

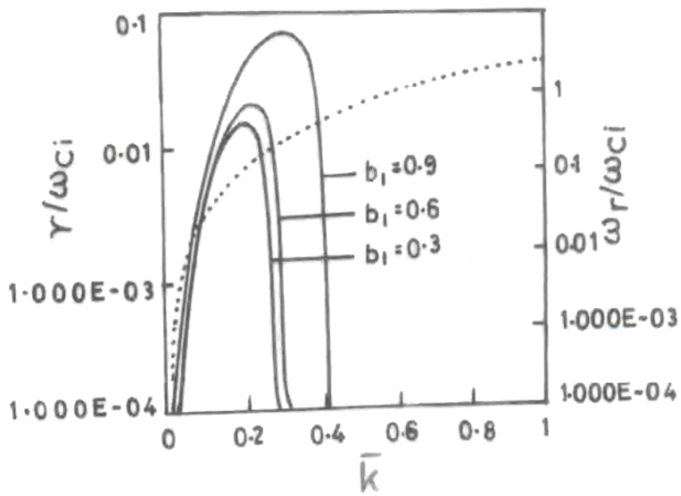

Fig. 4 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various values of relativistic factor at other fixed plasma parameters.

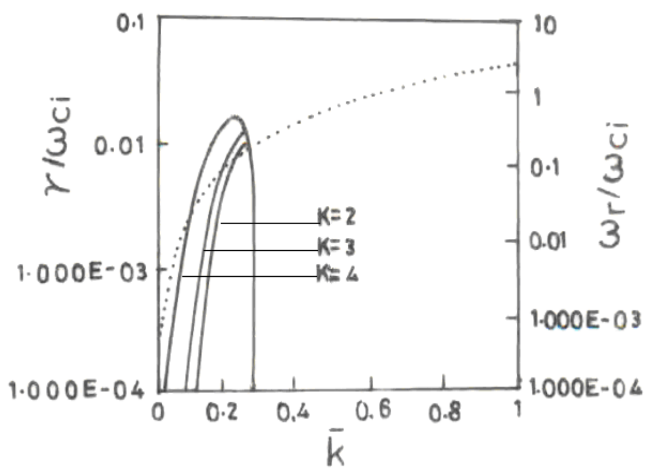

Fig. 5 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various values of spectral index at other fixed plasma parameters.

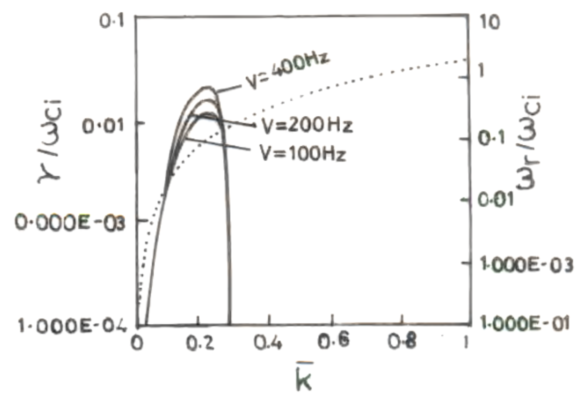

Fig. 6 Variation of growth rate (solid line) and real frequency (dotted line) with respect to $\bar{k}$ for various valuesof frequency of a.c. field at other fixed plasma parameters.

IV. CONCLUSION

The normalized growth rates have been evaluated for plasma parameters suited in interplanetary space at 1AU. The method of characteristics solution and kinetic approach has been used for the derivation of dispersion relation and growth rate. The effects of AC. frequency and relativistic factor will discuss in the light of Kappa distribution function. The velocity of background plasma has been considered in the order of velocity of light, so the relativistic approach of mass changing with velocity has been taken in account. Thus changing the mathematical treatment from velocity to momentum form in detail, an expression for the growth rate of the system has been calculated and the results for representative values of the parameters suited to bow shock region at 1AU has been obtained. It is inferred that A.C. field frequency modifies the resonance criteria, which influences the growth rate. Also the growth rate increases by increasing the number density of cold plasma and temperature anisotropy. Plasma particles having higher Kappa spectral index provide additional source of energy. In addition to the other factors the relativistic plasma modifies the growth rate and also shifts the wave band significantly. The relativistic electrons by increasing the growth rate and widening the bandwidth may explain a wide frequency range of whistler emissions in the Earth's magnetosphere.

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