ISSN 2250-3153

Mathematical Modeling for Demand and Supply Estimation for Cotton in Maharashtra

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Abstract- The output supply and input demand are closely interlinked to each other. Therefore, any change in input and output prices affect the input demand and output supply simultaneously. The present study used cross sectional cum time series data of Vidarbha region of Maharashtra state for cotton crop for the ten years from 1999-00 to 2008-09 were collected from the Agricultural Prices and costs Scheme, Department of Agricultural Economics and Statistics, Dr. PDKV, Akola by keeping in view to estimate input demand and output supply by using normalized Cobb-Douglas profit function. The analysis of input demand equations has shown that the demand elasticities with respect to own prices have the expected negative signs indicating that the results are in accordance with theory of demand. One per cent increase in own price, holding other prices constant, will reduce human labour employment at 1.296 per cent, bullock labour demand 1.343 per cent, fertilizer demand 2.093 per cent and seed demand 1.665 per cent in cotton crop. The absolute value of own price elasticity of human labour, bullock labour and seed price have been found greater than unity, indicating an elastic response of input utilization to their own price. It can be also observed from the study that own prices had negative Impact on supply of cotton. The profit function for cotton was decreased in prices of Farm yard manure and prices of seed. Among the variable factors, normalized seed price in general had the highest negative impact on variable profit for cotton crop followed by farm yard manure price. The study also shows that assuming no change in fixed factors or the level of technology, the input-output price structure has resulted in an appreciable change in human labour employment, bullock labour employment, fertilizer, farm yard manure and seed demand in the production of cotton.

Index Terms - cotton, input demand, Maharashtra, profit function

I. INTRODUCTION

Cotton is a very important fibre crop of global importance with a significant role in Indian Agriculture, Industrial development, employment generation and improving national economy. Cotton known as white gold or king of fibre and plays a prominent role in Indian economy. In India it is grown annually on 9.41 million hectares with 23.16 million bales of an average production. Even though India ranks first in area in the world, it occupies third position in production and low position in productivity. The average productivity of cotton in India was 419 kg lint ha^{-1}, (Anonymous [a] 2009), while in Maharashtra it is grown in an area of 3.14 million hectares with production of 4.94 million bales and productivity is only 267 kg lint ha^{-1}.

Cotton is one of the important cash crops of Vidarbha. In Vidarbha it is grown in an area of 11.24 lakh hectares with production of 18.58 lakh bales of an average production and productivity is 262 kg lint ha^{-1} which is comparatively lower than India's cotton productivity. (Anonymous [b] 2009)

Technological change and positive price policy can play a significant role in stimulating agricultural production through the desired allocation of resources. At these stages, the policy planners face the challenge to formulate suitable agricultural policy by which the desired growth of agricultural output can be achieved. In order to formulate effective price policy, one needs reliable empirical knowledge about the degree of responsiveness of demand for factors and supply of products, to reliable prices and technological changes. The output supply and factor demand are closely interlinked to each other. Therefore, any change in factor and product prices affects the factor demand and output supply simultaneously. Rising cost of inputs discourages the input use and reduces the output supply. The decline in output supply raises food prices. The rapid increase in population and the increase in money income as a result of economic development create a strong pressure on demand which also leads to increase in food grain prices. These cause hardship to the consumers. This can be corrected only by a large and adequate supply of agricultural output and a greater attention is, therefore, required to be focused for matching the demand for foodgrains and agricultural commodities with the supply thereof. The rise in foodgrains prices should be sufficiently high not-only to counteract the rising cost of inputs but also to leave a rate of profit conductive for investment in agriculture and thereby accelerating supply of agricultural output. In this context, one needs detailed knowledge about the net effect of price and non-price factors like factor and product prices, technology, irrigation, capital use, acreage etc. so that required adjustment needed in price and non-price factors could be worked out to attain the specific goals of prices, production and crop income.

II. METHODOLOGY

The Agricultural Prices and Costs (APC) scheme under the guidance of government of Maharashtra provides valuable data about Agriculture in Maharashtra. The present study used cross sectional cum time series data of Vidarbha region for the cotton crop for the ten years i.e. from 1999-00 to 2008-09. Every year's 100 farmers were selected purposively for the present study. The scheme is involved in the collection of representative data on input use and yield and there upon estimation of cost of cultivation of principle crops grown in the region.

ISSN 2250-3153

Data is collected every year and for all the enterprises. Although, the sample for particular year are selected with respect to specified principle crop. The data were collected for all the crops grown on the sample holdings.

Definitions of variables

A brief description and abbreviation of the variables used in this study are as follows

Q = Physical output of particular crops measured in quintal per hectare. It includes main product as well as by-product. By product were converted into quintals of crop equivalent output by dividing the total value of by-product by the price of the main product.

N = Human labour hours used per hectare for particular crop. It includes both the hired and family labour. Woman hours were converted into man hour's equivalent by treating 1.5 women hours equal to one man hour.

B = Bullock labour in hours of the bullock pairs used per hectare for particular crop. It includes both owned and hired bullock pair labour.

X = Total quantity of plant nutrients ($N + P_2O_5 + K_2O$) measured in kilogram per hectare for particular crop.

F = Farm yard manure per hectare measured in quintals.

S = Total quantity of seed measured in kilogram per hectare for particular crop.

K = A measure of flow of capital services. It includes machinery and depreciation charges, imputed value of interest, seed cost and certain operating expenses not consider else-where.

L = Area planted under the crop measured in hectares.

w.N = Total wage bill in rupees per hectare. It includes actual payment made to hired human labour and the imputed value of service of family labour

b.B = Total bullock labour wage bill in rupees per hectare for particular crop.

r.X = Total fertilizer bill in rupees per hectare for particular crop.

s.S = Total cost of seed per hectare for particular crop

m.F = The Total cost of farm yard manure valued.

w = Wage rate in rupees per man hour. It was obtained by dividing total wage bill (w.N) by total human labour hours.

b = Total bullock labour rate for a bullock labour hour. It was obtained by dividing total wage bill (b B) by total bullock labour hours.

r = Price of plant nutrients in rupees per kilogram. It was obtained by dividing the total cost of fertilizer (r.X) by total quantity of plant nutrients.

s = Total rate of the seed in rupees per kilogram. It was obtained by dividing the total cost of seed (s.S) by total quantity of seed.

m = Price of FYM per quintal in rupees. It was obtained by dividing the total Cost of farm yard manure by the total quantity of (M) used per hectare for particular crop.

A. Mathematical Model

Profit Function Analysis: The theory of profit function, developed to helps in overcoming the problem of simultaneous equation bias, if present. Another distinct advantage of this approach over production function is that with the help of duality theorem (Shephard, 1953), the variable factor demand function and supply function of products can be derived directly from the estimated profit function. Econometric application of this production theory based on duality between production function and variable profit function is a breakthrough in the theory of production. Shepherd's Lemma (1953) applies equally to profit functions, which states that the partial derivative of profit function with respect to output and input prices give the supply and demand function, respectively.

Let the Cobb-Douglas production function with usual neo-classical properties be written as

$$Q = A\ N^{\alpha 1} B^{\alpha 2}\ X^{\alpha 3} F^{\alpha 4} S^{\alpha 5} K^{\beta 1} L^{\beta 2}\ U$$

Where, (Q) is output of crop, human labour (N), Bullock labour (B), chemical plant nutrients (X) , farm yard manure (F) and seed (S) are the variable input and capital input (K) and Land (L) are fixed input, and U is error term.

When working with profit function one has to choose functional forms which are homogenous of degree one in all prices, whereas this is not necessary for normalized profit function. The profit function formulation suggested by Lau and Yotopoulos (1972) enables us to derive factor demand as a function of normalized input rates and the quantities of fixed inputs.

Invoking the theory of profit function, the normalized profit function for the above production function can be written as below.

$$\frac{\pi}{P} = A^* (w/P)^{\alpha_1^*}(b/P)^{\alpha_2^*}(r/P)^{\alpha_3^*}(m/P)^{\alpha_4^*}(s/P)^{\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}\ U$$

OR

$$\pi^* = A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} S^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}\ U \qquad (1)$$

Where $\pi^* = \pi/p$ =normalized profit or output price (UOP) profit, w* is the normalized wage rate, b* is the normalized bullock labour price, r* is the normalized fertilizer price, m* is the normalized farm yard manure price and s* is the normalized seed price.

From the estimated parameters of normalized profit function, the production elasticities of inputs and intercept were derived.

B. Input Demand Function

Shepherd's Lemma (1953) asserts that the first order negative derivative of the normalized profit equation with respect to normalized wage rate, bullock labour price, fertilizer price, farm yard manure price and seed price respectively, gives the derived factor demand function. The factor demand equation on case of Cobb-Douglas type normalized profit function was given as

i. Human labour demand equation

$$-\frac{\partial \pi^*}{\partial w^*} = -\alpha_1^* \left(\frac{\pi^*}{w^*}\right) = N \text{ or}$$

$$\frac{w^* N}{\pi^*} = -\alpha_1^* \qquad (a)$$

ii. Bullock labour demand equation

$$-\frac{\partial \pi^*}{\partial b^*} = -\alpha_2^* \left(\frac{\pi^*}{b^*}\right) = B \text{ or}$$

$$\frac{b^* B}{\pi^*} = -\alpha_2^* \qquad \text{(b)}$$

iii. Fertilizer demand equation

$$-\frac{\partial \pi^*}{\partial r^*} = -\alpha_3^* \left(\frac{\pi^*}{r^*}\right) = X \text{ or}$$

$$\frac{r^* X}{\pi^*} = -\alpha_3^* \qquad \text{(c)}$$

iv. Farm yard manure demand equation

$$-\frac{\partial \pi^*}{\partial f^*} = -\alpha_4^* \left(\frac{\pi^*}{f^*}\right) = F \text{ or}$$

$$\frac{f^* F}{\pi^*} = -\alpha_4^* \qquad \text{(d)}$$

v. Seed demand equation

$$-\frac{\partial \pi^*}{\partial s^*} = -\alpha_5^* \left(\frac{\pi^*}{s^*}\right) = S \text{ or}$$

$$\frac{s^* S}{\pi^*} = -\alpha_5^* \qquad \text{(e)}$$

Substituting π* from identity (1) into (a) to (e), the demand equation can be written as:

Labour demand equation

$$N = -\alpha_1^* A^* (w^*)^{\alpha_1^* - 1} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

Bullock labour demand equation

$$B = -\alpha_2^* A^* w^{*\alpha_1^*} (b^*)^{\alpha_2^* - 1} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

Fertilizer demand equation

$$X = -\alpha_3^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} (r^*)^{\alpha_3^* - 1} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

Farm yard manure demand equation

$$F = -\alpha_4^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} (m^*)^{\alpha_4^* - 1} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

Seed demand equation

$$S = -\alpha_5^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} (s^*)^{\alpha_5^* - 1} K^{\beta_1^*} L^{\beta_2^*}$$

C. Output supply Function

Shepherd's Lemma (1953) asserts that first order derivative of profit function with respect to output price gives output supply function.

$$\frac{\partial \pi}{\partial P} = \theta \left(\frac{\pi}{P}\right) = Q \quad \text{or} \quad \frac{P.Q}{\pi} = \theta$$

The output supply function in the form of Cobb-Douglas production function was written as

$$Q = A\,\theta P^{\theta - 1} w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

The above equation was giving the output supply with respect to output prices, wage rate, bullock labour price, fertilizer price, farm yard manure price, seed price and price of capital input.

D. Joint Estimation of Cobb-Douglas profit functions and input demand.

The normalized profit function and factor demand functions for human labour, bullock labour, fertilizer farmyard manure and seed were jointly estimated using Zellner's method (1962) for estimating 'Seemingly Unrelated Regression Equation (SURE)' by imposing the restriction that α^{1*}, α^{2*}, α^{3*}, α^{4*} and α^{5*} are equal in both the normalized profit function and relevant factor demand equations.

By using SURE method the coefficient were estimated as

$$\hat{\alpha}_{SURE} = (X' V^{-1} X)^{-1} X' V^{-1} Y$$

Where, X is independent variable and Y is dependent variable

$$V = \sum \otimes I_N$$

Where, ∑ representing the covariance of residual between the equations, ⊗ is the Kronecker product and I_N is the identity matrix of number of observations.

Impact of observed changes

The theory of profit function provides a set of factor demand and output supply equations. The factor demand of cotton crop for i[th] variable factor is as:

$Xi = X (P_Q, w, b, r, m, s)$ i = 1, 2, 3, 4, 5.

The output supply equation for cotton is as

$Q = Q (P_Q, w, b, r, m, s)$

Where, P_Q is the output price Suffix i = 1 for human labour, 2 = bullock labour, 3 = Fertilizer, 4 = Farmyard Manure and 5 = Seed.

Taking the total differential of above equations and writing in terms of growth rates.

$$\dot{X}_i = E_{Xi}^{PQ}\, \dot{PQ} + E_{Xi}^{W}\, \dot{w} + E_{Xi}^{b}\, \dot{b} + E_{Xi}^{r}\, \dot{r} + E_{Xi}^{f}\, \dot{f} + E_{Xi}^{s}\, \dot{s} + E_{Xi}^{K}\, \dot{K} + E_{Xi}^{L}\, \dot{L}$$

$$\dot{Q} = E_{Q}^{PQ}\, \dot{PQ} + E_{Q}^{W}\, \dot{w} + E_{Q}^{b}\, \dot{b} + E_{Q}^{r}\, \dot{r} + E_{Q}^{f}\, \dot{f} + E_{Xi}^{s}\, \dot{s} + E_{Q}^{K}\, \dot{K} + E_{Q}^{L}\, \dot{L}$$

The dot on the variable indicates the rates of change (growth rate) in the variable. E is the parameters of the elasticities of factor demand and output supply. (i.e. E_X^Z denotes elasticity of X with respect to changes in exogenous variable Z). P. Kumar and Mruthyunjay (1989).

III. RESULTS AND DISCUSSION

A. Input Demand Function

A system of factor demand equations were derived from the estimated normalized profit function. The results of human labour, bullock labour, fertilizer, farm yard manure and seed demand equation for cotton are presented below and the degree of responsiveness of input and output price movements on the use of inputs are discussed. This information is of crucial importance in the formulation of effective price policies for crops to reach specified production goals.

Input Demand Function for cotton

Table 1 revealed that demand elasticities with respect to own price had anticipated negative signs indicating that the results were in accordance with the theory of demand.

The absolute value of own price elasticity of human labour, bullock labour, fertilizer and seed were greater than unity indicating there by an elastic response of input utilization to their own price. One per cent increase in own price, holding other prices constant, will reduce human labour employment at 1.296 per cent, bullock labour demand 1.343 per cent, fertilizer demand 2.093 per cent and seed demand 1.665 per cent in cotton crop.

Table 1: Input demand function for cotton

Variables	Human Labour	Bullock Labour	Fertilizer	F.Y.M.	Seed
Intercept	0.501	0.571	1.838	0.379	-0.625
Output price	1.102	1.557	2.415	0.750	0.847
Wage rate	-1.296	-0.079	-0.602	-0.558	0.260
Bullock labour price	0.134	-1.343	0.005	-0.014	0.057
Fertilizer price	0.018	0.046	-2.093	-0.042	-0.006
F.Y.M. price	-0.019	-0.008	-0.041	-0.155	0.0004
Seed Price	0.034	0.092	0.101	-0.002	-1.665
Capital	0.227	-0.112	0.243	0.218	0.513
Land	-0.167	-0.153	-0.028	-0.196	-0.006

A negative sign of cross price elasticity with respect to the price of other variable inputs shows that the pair is complement and a positive sign is an indicator of substitutive relationship. However, the positive sign of cross price elasticity with respect to quantities of fixed inputs indicates complementarity and negative sign indicates substitutive relationship.

Table 1 shows that the human labour and bullock had a substitutive relationship for cotton (Kumar *etal* 2010 reported that there was a substitutive relationship between human labour and bullock labour for wheat and sugarcane) while there was a complementarity between human labour and farm yard manure.

B. Output supply Function

The output supply equations for cotton in Vidarbha region of Maharashtra state were derived from the estimated profit function. The output supply equation given in Table 2 gives the estimates of the responses of own output price, variable prices and fixed factors on output supply of selected crops.

It can be observed from the study that the own price had negative impact on supply of cotton. However, the output supply response was inelastic to cotton. (i.e. -0.260).

Table 2: Output supply function for cotton

Variables	Elasticity
	Cotton
Intercept	-0.798
Output price	-0.260
Wage rate	0.415
Bullock Labour price	0.174
Fertilizer price	0.009
Farm yard Manure price	-0.001
Seed Prices	0.074
Capital Input	0.291
Land	-0.122

Table 2 reveals that for the cotton, variable inputs responded positively to the output price, except farm yard manure. The input response elasticities were highly inelastic, nearly zero. The elasticity with respect to farm yard manure price was -0.001, resulted that a 1 per cent increase in farm yard manure price were associated with about 0.001 per cent per cent decline in crop output. Among the variable factors, human wage rate, bullock labour price, fertilizer price and seed price had positive impact on the supply of cotton, while among the fixed factor, capital was found to be effective in increasing the supply of cotton. The output supply elasticity with respect to capital was (0.291). Capital input had positive impact on the supply of cotton.

C. Joint estimation of the Normalized profit functions and factor share for variable inputs

Lau and Yotopoulas (1972) pointed out that due to the presence of common parameters in profit and factor demand equation; they should be estimated jointly imposing the restriction that common parameters in both equations are equal. The five equations - UOP profit function, human labour, bullock labour, fertilizer, farm yard manure and seed demand functions were estimated jointly using Zellner's method (1962) for estimating 'Seemingly Unrelated Regression Equation (SURE)' by imposing appropriate restrictions.

Table 3 reveals that for the cotton crop, the profit function was decreased in prices of Farm yard manure and prices of seed. Among the variable factors, normalized seed price in general had the highest negative impact on variable profit for cotton crop followed by farm yard manure price.

Table 3: Joint estimation of the Normalized profit function and factor share for variable inputs

Variables	Parameters	SURE Estimated Values
Normalized profit (dependent variable)	In $\frac{\pi}{p}$	
Wage rate	In $\frac{w}{p}$ (α_1^*)	0.525
Bullock labour price	In $\frac{b}{p}$ (α_2^*)	0.004
Fertilizer price	In $\frac{r}{p}$ (α_3^*)	0.089
Farm yard manure price	In $\frac{m}{p}$ (α_4^*)	-0.089
Seed	In $\frac{s}{p}$ (α_5^*)	-0.119
Capital input	InK (β_1)	0.562
Land	In L (β_2)	-0.198

Impact of Observed Changes

The impacts of observed changes in input-output price structure on factor demand and output supply were presented in Table 4. Under the assumption that the input output prices continue to change in future at the same rate as was observed in the last decade and also there is no change in the endowment of fixed factors. The partial effects of price changes on the growth of factor demand and output supply were computed.

Table 4 shows that assuming no change in fixed factors or the level of technology, the input-output price structure has resulted in an appreciable change in human labour employment, bullock labour employment, fertilizer, farm yard manure and seed demand in the production of cotton crop.

Table 4: Impact of observed changes in input output price structure on factor demand and output supply

Particulars	**Per cent Growth**
Human Labour	-7.23
Bullock Labour	-3.25
Fertilizer	-9.84
Farm Yard Manure	-4.95
Seed	-34.64
Out put	5.04

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