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Effect of in-plane forces on frequency parameters

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Abstract- Vibration and buckling characteristics of stiffened plates subjected to in-plane uniform and non-uniform edge loading at the plate boundaries are investigated using the finite element method. Rectangular stiffened plates possessing different boundary conditions, aspect ratios, varying mass and stiffness properties and varying number of stiffeners have been analyzed for buckling and vibration studies. The characteristic equations for the natural frequencies, buckling loads and their corresponding mode shapes are obtained from the equation of motion. The effects of the position of stiffeners and number of stiffeners, aspect ratios, boundary conditions, stiffeners parameters upon the buckling load parameter and fundamental frequency of the stiffened plates are investigated. The results are obtained considering the bending displacements of the plate and the stiffener. Eccentricity of the stiffeners gives rise to axial and bending displacement in the middle plane of the plate. Comparison with published results indicates good agreement. In the structure modelling, the plate and the stiffeners are treated as separate elements where the compatibility between these two types of elements is maintained.

Index Terms- Finite element method, Stiffened plate, Buckling and frequency parameter, Stiffeners parameters

Notations

a - Plate dimension in longitudinal direction
b - Plate dimension in the transverse direction
t - Plate thickness
E, G - Young's and shear moduli for the plate material

b^s, d^s - web thickness and depth of a x-stiffener

ξ, η - Non –dimensional element coordinate

A^S - Cross sectional area of the stiffener

I_S - Moment of inertia of the stiffener cross-section about reference axis

$\{q\}^r$ - Vector of nodal displacement a r^{th} node

$[D^P]$ - Rigidity matrix of plate

$[D^S]$ - Rigidity matrix of stiffener

$[K^e]$ - Elastic stiffness matrix of plate

$[K^S]$ - Elastic stiffness matrix of stiffener

$\lfloor M_p \rfloor, [M_S]$ - Consistent mass matrix of plate, stiffener

$[K^G]$ - Geometric stiffness matrix

$[N]^r$ - Matrix of a shape function of a node r

P^{cr} - Critical buckling load
P (t) - In plane load
T_S - Torsional constant

P_S - Polar moment of inertia of the stiffener element

I. INTRODUCTION

The dynamic behaviour of stiffened plates has been the subject of intensive study for many years. For aerodynamic considerations, stiffener will be provided inside the hull of aircraft and for space structures the stringer can be provided outside if that is more structurally efficient. Stiffened plates are structural components consisting of plates reinforced by a system of ribs to enhance their load carrying capacities. These structures are widely used in aircraft, ship, bridge, building, and some other engineering activities. In many circumstances these structures are found to be exposed to in-plane loading.

The buckling and vibration characteristics of stiffened plates subjected to uniform and non-uniform in-plane edge loading are of considerable importance to aerospace, naval, mechanical and structural engineers. Aircraft wing skin panels, which are made of thin sheets are usually subjected to non-uniform in-plane stresses caused by concentrated or partial edge loading at the edges, and due to panel stiffener support conditions

Diez *et al.* [1] studied the effect of combined normal and shear in-plane loads by the Galerkin method. Transverse vibration of rectangular plates subjected to in-plane forces under various combinations has been studied by Singh and Dey [2] by a difference based variational approach.

The vibration and buckling of partially loaded simply supported plates were studied by Deolasi et al. [3]. Recently Sundersan et al. [4] have studied the influence of partial edge compression on buckling behaviour of angle ply plates for a few orientations.

A brief literature survey reveals that a variety of methods have been proposed to study the vibration of stiffened plates. The most common method used in early literature was to approximate the stiffened plate as equivalent orthotropic plates, using the smeared stiffener approach. In more recent literature, with the help of high-speed computers, the plate and stiffeners were treated separately. Such numerical methods as the finite element method and finite difference method are widely used.

Aksu [5] has presented a variational principle in conjunction with the finite difference method for analysis of free vibration of uni-directionally and cross-stiffened plates considering in-plane inertia and in plane displacements in both directions.

Shastry and Rao [6] have used the 3 noded conforming element and refined beam-bending element for arbitrary oriented stiffeners.

Olson and Hazell [7] have presented a critical study on clamped integrally stiffened plate by the finite element method. The mode shapes and frequencies have been determined experimentally using the real time holographic technique. The effect of change in rib stiffness on various modes has been studied.

Bapu Rao *et al.* [8] have also reported their work on experimental determination of frequencies with real time holographic technique for skew stiffened cantilever plates. Mukhopadhyay [9] has applied the semi-analytic finite difference method to the stability analysis of rectangular stiffened plates based on the plate beam idealization.

Sheikh and Mukhopadhyay [10] applied the spline finite strip method to the free vibration analysis of stiffened plates of arbitrary shapes. They analyzed the plate of rectangular, skew and annular shapes with concentric as well as eccentric stiffeners. Large amplitude, free flexural vibration of stiffened plates has been investigated by the spline finite strip method by Sheikh and Mukhopadhyay [11]. The stiffener has been elegantly modelled so that it can be placed anywhere within the plate strip.

Harik and Guo [12] have developed a compound finite element model to investigate the stiffened plates in free vibration where they have treated the beam and plate element as the integral part of a compound section, and not as independent bending components.

Bedair [13] has studied the free vibration characteristics of stiffened plates due to plate/stiffener proportions. He has considered the plate and the stiffener as the discrete elements rigidly connected at their junctions and the nonlinear strain energy function of the assembled structure has been transformed into an unconstrained optimization problem to which Sequential quadratic programming has been applied to determine the magnitudes of the lowest natural frequency and the associated mode shape

Allman [14] has carried out the analysis of buckling loads of square and rectangular stiffened plates using triangular element. He has presented the results both by including and neglecting the torsional stiffness of the stiffeners. Vibration of stiffened plates with elastically restrained edges has been analyzed by Wu and Liu [15] using Rayleigh-Ritz method. The first four lower frequencies for restrained plates up to six intermediate stiffeners are calculated.

Mukhopadhyay [16] has extended the static and vibration analysis of plates to analyse the stability of ship plating and allied plated structures using the semi-analytic method.

An isoparametric stiffened plate bending element for the buckling analysis of stiffened plate has been presented by Mukherjee and Mukhopadhyay [17]. Here the stiffener can be positioned anywhere within the plate element and need not necessarily be placed on the nodal lines.The general spline finite strip method has been extended by Sheikh and Mukhopadhyay [18] to analyse stiffened plate of arbitrary shape. Stiffened plates having various shapes, boundary conditions and also possessing various dispositions of stiffeners have been analyzed by the proposed method. The stability of partially stiffened, simply supported and clamped square plates is studied by Roy *et al.* [19]. A high precision triangular finite element and a compatible stiffener element are used in the finite element analysis. Buckling of stiffened plates has been studied by Bedair [20]. An investigation on stiffened plates has been conducted to determine the elastic parameters as well as the cross-sectional dimensions of rectangular stiffeners from experimental modal data and finite element prediction, using model-updating technique by Chakraborty and Mukhopadhyay [21].

A differential quadrature analysis for the free vibration of eccentrically stiffened plates is studied by Zeng and Bert [22]. The plate and the stiffeners are separated at the interface with equilibrium and continuity condition satisfied. Vibration and dynamic stability of stiffened plates subjected to in-plane uniform harmonic edge loading is studied using finite element analysis by Srivastava *et al*. [23] considering and neglecting in-plane displacements. Further Srivastava *et al* [24] extended their work to study the principal dynamic instability behaviour of stiffened plates subjected to non-uniform harmonic in-plane edge loading. Various methods used for vibration analysis such as Ritz technique, Levy's solution, finite difference method, finite element method, Galerkin method, differential quadrature method and method using boundary characteristics orthogonal polynomials (BCOP) have been reviewed extensively by Leissa [25]. The effect of the gap between the stiffener tip and the supporting edge on the natural frequencies has been investigated by Nair and Rao [26]. The panel is represented by triangular plate bending elements and the stiffener by beam elements.

The applied load is seldom uniform and the boundary conditions may be completely arbitrary in practice. The problem becomes complicated when the numbers of stiffeners are increasing regardless of the position of stiffeners not necessary along the nodal lines. Loading is non-uniformly distributed over the edges and along the stiffeners thus affecting the boundary conditions. Analysis of stiffened plate is carried out normally by energy method by adding energies due to plate and stiffener. The energy stored in the stiffener will depend on its cross section and if a thin walled open section, the effect of twisting as well as warping have to be included. These studies for most part being concerned with the numerical analysis of the theoretical buckling load and also mostly related to unstiffened plates.

The present paper deals with the problem of vibration and buckling of rectangular stiffened plates subjected to in-plane uniform and non-uniform edge loading. Finite element formulation is applied for obtaining the non-uniform stress distribution in the plate and also to solve the buckling load and frequency parameters in various modes with different boundary conditions, aspect ratios and various parameters of stiffened plates. The analysis presented determines the stresses all over the region. In the present analysis, the plate is modeled with the nine nodded isoperimetric quadratic element where the contributions of bending and membrane actions are taken into account. Thus the analysis can be carried out for both thin and thick plates. Moreover it can be applied to a structure having irregular boundaries. The formulation of the stiffener is done in such a manner so that it may lie anywhere within a plate element. In order to maintain compatibility between plate and stiffener, the interpolation functions used for the plate are used for the stiffeners also.

II. Mathematical formulation

The governing equations for the buckling and vibration of stiffened plates subjected to in-plane harmonic edge loading are developed. The presence of uniform and non-uniform external in-plane loads, boundary conditions, stiffeners locations and cutouts if any in the plate induce a non-uniform stress field in the structures. This necessitates the determination of the stress field as a prerequisite to the solution of the problems like vibration, buckling and vibration behaviour of stiffened plates. As the thickness of the structure is relatively smaller, the determination of stress field reduces to the solution of a plane stress problem in the plate skin and stiffeners (where the thickness and breath are small compared to length). The stiffened plates are modeled and the governing equations are solved by finite element method. In the present analysis, the plate is modelled with nine noded isoparametric quadratic elements where the contributions of bending and membrane actions are taken into account. One of the advantages of the element is that it includes the effect of shear deformation and rotary inertia in its formulation. Thus the analysis can be carried out for both thin and thick plates. Moreover it can be applied to a structure having irregular boundaries. Also it can handle any position of cutout, different boundary and position of in-plane concentrated loads or loading conditions. In order to maintain compatibility between plate and stiffener, the interrelation functions used for the plate are used for the stiffeners also. Numerical methods like finite element method (FEM) are preferred for problems involving complex in plane loading and boundary conditions as analytical methods are not easily adaptable. The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation. The plate skin and the stiffeners/composite are modelled as separate elements but the compatibility between them is maintained. The nine noded isoparametric quadratic elements with five degrees of freedom (u, v, w, θ_X and θ_y) per node have been employed in the present analysis.

The in-plane displacements u and v need to be considered only when the stiffeners are connected eccentrically to the plate. If the plate and stiffeners are connected concentrically, no in plane stresses develop The effect of in-plane deformations is taken into account in addition to the deformations due to bending, which will help to model the stiffener eccentricity conveniently. The element matrices of the stiffened plate element consist of the contribution of the plate and that of the stiffener.
in the power of the thickness co-ordinate as:

The explicit evaluation of integrals involved in the evaluation of element stiffness and mass matrices of the plate is tedious and as such is not attempted. A Gaussian integration technique has been adopted for this purpose for its high accuracy and also it can be implemented easily. A exact integration needs an order of 3 x 3. However, a reduced integration proves to be more effective and cheaper. To integrate the element matrices a 2 x 2 Gaussian integration has been adopted, however, the order of integration has been mentioned.

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The displacement at any point within the element can be expressed as:

$$\begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix} = \sum_{r=1}^{9} N_r \left[I_5 \right] \begin{Bmatrix} u_r \\ v_r \\ w_r \\ \theta_{xr} \\ \theta_{yr} \end{Bmatrix} \tag{1}$$

Strain displacement relation can be written as:

$$\{\varepsilon\} = \sum \left[B_p\right]_r \{q\}_r = \left[B_p\right] \{q\} \tag{2}$$

$$[B_P]_r = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} & -\frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix} \quad (3)$$

and

The generalized stress-strain relationship for a plate element is

$$\{\sigma_p\} = \lfloor D_p \rfloor \{\varepsilon_p\} \quad (4)$$

where the stress resultant vector is

$$\{\sigma_p\}^T = \{N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy}\} \quad (5)$$

Using the isoparametric coordinates, the element stiffness matrix is expressed as:

$$[K_b]_p = \int_{-1}^{+1}\int_{-1}^{+1} [B_p]^T [D_p] [B_p] \, |J_p| \, d\xi \; d\eta \quad (6)$$

The element mass matrix can be expressed in iso parametric coordinate as:

$$[M_p]_e = \int_{-1}^{1}\int_{-1}^{1} [N]^T [m_p] [N] \, |J_p| \, d\xi \; d\eta \quad (7)$$

Geometric stiffness matrix expressed in isoparametric coordinates as:

$$[K_G]_P = \int_{-1}^{+1}\int_{-1}^{+1} [B_{Gp}]^T [\sigma_P] [B_{Gp}] \, |J_p| \, d\xi \, d\eta \quad (8)$$

$$[\sigma_P] = \begin{bmatrix} \sigma_x t & \tau_{xy} t & 0 & 0 & 0 & 0 \\ \tau_{xy} t & \sigma_y t & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\sigma_x t^3}{12} & 0 & \frac{\tau_{xy} t^3}{12} & 0 \\ 0 & 0 & 0 & \frac{\sigma_y t^3}{12} & 0 & \frac{\tau_{xy} t^3}{12} \\ 0 & 0 & \frac{\tau_{xy} t^3}{12} & 0 & \frac{\sigma_x t^3}{12} & 0 \\ 0 & 0 & 0 & \frac{\tau_{xy} t^3}{12} & 0 & \frac{\sigma_y t^3}{12} \end{bmatrix} \qquad (9)$$

Geometric stiffness of stiffener

The strain matrix can be expressed as

$$\{\varepsilon_S\}_G = [B_{GS}]\{q\} = \sum_{r=1}^{9} [B_{GS}]_r \{q\}_r \qquad (10)$$

$$[B_{GS}]_r = \begin{bmatrix} 0 & 0 & \frac{\partial N_r}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial x} & 0 \end{bmatrix}$$

and (11)

$$[\sigma_S] = \begin{bmatrix} \sigma_x A_S & 0 \\ 0 & \sigma_X S_S \end{bmatrix}$$

where (12)

where A_S is the area, F_S the first moment of area about reference plane, S_S the second moment of area about reference plane, T_S the torsional constant and P_S the polar moment of area of the stiffener cross-section.

The expression for the geometric stiffness matrix can be formed by equating the internal work done by the stresses to the external work done by the nodal forces.

The geometric stiffness of the stiffener element can be expressed in iso-parametric co- ordinate as:

$$[K_G]_S = \int_{-1}^{+1} [B_{GS}]^T [\sigma_S] [B_{GS}] \, |J_S| \, d\xi \qquad (13)$$

The derivatives of x and y with respect to x $^{/}$ are given by

$$\frac{\partial x}{\partial x^{/}} = \cos\phi$$

$$\frac{\partial y}{\partial x^{/}} = \sin\phi \qquad (14)$$

The components of generalized strain vectors are obtained as follows:

$$\frac{\partial u^{/}}{\partial x^{/}} = \frac{\partial u^{/}}{\partial x}\frac{\partial x}{\partial x^{/}} + \frac{\partial u^{/}}{\partial y}\frac{\partial y}{\partial x^{/}} \qquad (15)$$

$$\frac{\partial \theta_x^{/}}{\partial x^{/}} = \frac{\partial \theta_x^{/}}{\partial x}\frac{\partial x}{\partial x^{/}} + \frac{\partial \theta_x^{/}}{\partial y}\frac{\partial y}{\partial x^{/}} \qquad (16)$$

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$$\frac{\partial \theta y^{/}}{\partial x^{/}} = \frac{\partial \theta_y{}^{/}}{\partial x}\frac{\partial x}{\partial x^{/}} + \frac{\partial \theta_y{}^{/}}{\partial y}\frac{\partial y}{\partial x^{/}} \tag{17}$$

We can substitute the values and finally we get as:

$$\left\{\varepsilon_{x^{/}}\right\} = \left\{\begin{array}{c} \frac{\partial u}{\partial x}\cos^2\phi + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\sin 2\phi + \frac{\partial v}{\partial y}\sin^2\phi \\ -\frac{\partial \theta_x}{\partial x}\cos^2\phi + \frac{1}{2}\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta y}{\partial x}\right)\sin 2\phi + \frac{\partial \theta_y}{\partial y}\sin^2\phi \\ \left(\frac{\partial w}{\partial x} - \theta_x\right)\cos\phi + \left(\frac{\partial w}{\partial y} - \theta_y\right)\sin\phi \\ -\left(\frac{1}{2}\left(-\frac{\partial \theta_x}{\partial x} + \frac{\partial \theta_y}{\partial y}\right)\sin 2\phi + \frac{\partial \theta_x}{\partial y}\sin^2\phi + \frac{\partial \theta_y}{\partial x}\cos^2\phi\right) \end{array}\right\} \tag{18}$$

The generalized strain components in the stiffeners in $x^{/}$ and $y^{/}$ coordinates are given by

$$\left\{\bar{\varepsilon}_{x}{}^{/}\right\} = [T]\left\{\bar{\varepsilon}_x\right\} \tag{19}$$

Where [T] is the transformation matrix and is given by

$$[T] = \begin{bmatrix} \cos^2\phi & 0 & 0 & 0 \\ \sin^2\phi & 0 & 0 & 0 \\ 0.5\sin 2\phi & 0 & 0 & 0 \\ 0 & \cos^2\phi & 0 & -0.5\sin 2\phi \\ 0 & \sin^2\phi & 0 & 0.5\sin 2\phi \\ 0 & 0.5\sin 2\phi & 0 & \cos^2\phi \\ 0 & 0.5\sin 2\phi & 0 & -\sin^2\phi \\ 0 & 0 & \cos\phi & 0 \\ 0 & 0 & \sin\phi & 0 \end{bmatrix} \tag{20}$$

$$\left\{\bar{\varepsilon}_x\right\} = \left\{\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), -\frac{\partial \theta_x}{\partial x}, -\frac{\partial \theta_y}{\partial y}, -\frac{\partial \theta_y}{\partial x}, \frac{\partial \theta_x}{\partial y}, \left(\frac{\partial w}{\partial x} - \theta_x\right), \left(\frac{\partial w}{\partial y} - \theta_y\right)\right\} \tag{21}$$

$$\left\{\bar{\varepsilon}_x\right\} = \sum_{r=1}^{9}[B_S]_r\{\delta\}_r = [B_S]\{\delta\} \tag{22}$$

where

$$[B_S] = [\,[B_S]_1 \quad [B_S]_2 \quad \mathrm{K} \quad [B_S]_r \quad \mathrm{K} \quad [B_S]_9\,] \tag{23}$$

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$$[B_S]_r = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} \end{bmatrix} \quad (24)$$

and

$$[\overline{D}_S]_{x'} = \begin{bmatrix} EA_S & EF_S & 0 & 0 \\ EF_S & ES_S & 0 & 0 \\ 0 & 0 & GT_S & 0 \\ 0 & 0 & 0 & GA_S/1.2 \end{bmatrix} \quad (25)$$

$$[B_S]_{x'r} = \begin{bmatrix} 0 & \frac{\partial N_r}{\partial x}\cos^2\phi - \frac{\partial N_r}{\partial x}\frac{\sin 2\phi}{2} & \frac{\partial N_r}{\partial x}\sin^2\phi - \frac{\partial N_r}{\partial x}\frac{\sin 2\phi}{2} \\ \frac{\partial N_r}{\partial x}\cos\phi + \frac{\partial N_r}{\partial x}\sin\phi & -N_r\cos\phi & -N_r\sin\phi \\ 0 & \frac{\partial N_r}{\partial x}\frac{\sin 2\phi}{2} - \frac{\partial N_r}{\partial x}\cos^2\phi & -\frac{\partial N_r}{\partial x}\frac{\sin 2\phi}{2} + \frac{\partial N_r}{\partial x}\sin^2\phi \end{bmatrix} \quad (26)$$

The equivalent nodal forces are given by

$$\{F_e\} = \iint [N]^T P_o(t) \, |J| \, d\xi \, d\eta \quad (27)$$

The intensity of loading within the patch is assumed to be uniform. In such situations it becomes necessary to obtain the equivalent nodal forces when a concentrated load is acting within the element. Again, the equivalent nodal forces are expressed as:

$$P_r = P_o [N]^T |J| \quad (28)$$

2.1 Governing Equations

The governing equations for specified problems like vibration, static and dynamic stability are as:.

Free vibration**:**

$$[[K_b] - \omega^2 [M]]\{q\} = 0 \quad (29)$$

Vibration without in-plane load:

$$[M]\{\ddot{q}\} + [K_b]\{q\} = \{0\} \tag{30}$$

Vibration with in-plane load**:**

$$[M]\{\ddot{q}\} + [[K_b] - P[K_G]]\{q\} = \{0\} \tag{31}$$

Or
$$[[K_b] - \alpha P_{cr}[K_G] - \omega^2[M]]\{q\} = 0 \tag{32}$$

Static stability or buckling

$$[[K_b] - P[K_G]]\{q\} = \{0\} \tag{33}$$

where $[K_b]$, $[K_G]$, $[M]$ are overall elastic stiffness, geometric stiffness, and mass matrices respectively, $\{q\}$ is the displacement vector. To evaluate the overall elastic stiffness, geometric stiffness, and mass matrices $[K_b]$, $[K_G]$, $[M]$ respectively, it is necessary to use the same shape functions for both plate and stiffener elements The element matrices for the plate and stiffener are generated separately and then added up to form overall matrices.

The equations are solved using the technique proposed by Corr and Jennings [28] where the matrices [*K*], [*M*] and [*KG*] are stored in single array according to skyline storage algorithm. In all the cases, the stiffness matrix [*K*] is factorized according to Cholesky's decomposition technique. With this, the solution for displacement is simply obtained by its forward elimination and backward substitution techniques. These displacements components are used to find out the stress field. These stresses are used to calculate the geometric stiffness matrices. The solutions of equations go through a number of operations. Moreover it requires a number of iterations to get the solution since these equations come under the category of eigenvalue problem. In such cases, the solution of eigen vector and eigen value is more than one where the different solutions correspond to different modes of vibration or different modes of buckling. The mode which gives lowest value of the eigen value is quite important and it is known as fundamental mode.

2.3 Non-dimensionalisation of Parameters

Majority of the model parameters and results are presented in non-dimensional form to make them independent of the plate size, thickness, material properties, etc for the convenience of the analysis. The non-dimensionalisation of different parameters like vibration, buckling and excitation frequency for dynamic stability analysis is taken as given below:

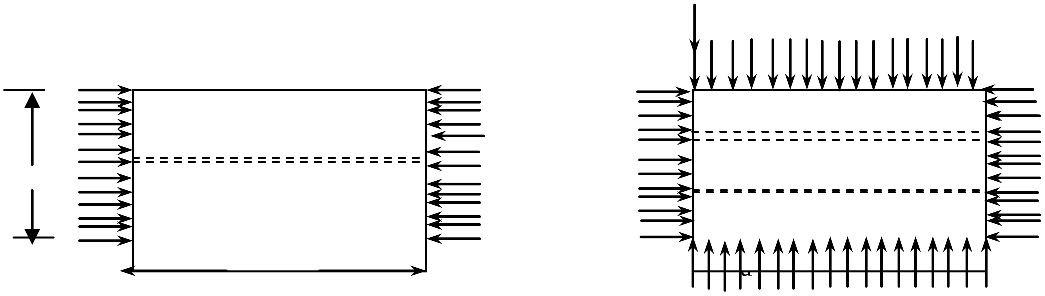
- Frequencies of vibration (ω) $\varpi b^2 \sqrt{\rho t/D}$
- Buckling load (λ)
  - (1) Distributed load $N_X b^2/\pi^2 D$
  - (2) Concentrated load $P_{cr} b/D$

Where D is the plate flexural rigidity, D = $Et^3/12(1-\nu^2)$, P is the applied load, P_{cr} is the buckling load, ρ is the density of the plate material and t is the plate thickness. In addition, certain quantities are expressed as the ratio of that quantity to some reference quantity. Assuming a general case of several longitudinal ribs and denoting

- EI_S the flexural rigidity of a stiffener at a distance (D_x) from the edge y = 0, the stiffener parameter terms
- δ and γ are defined as: $\delta = A_S/bt$ = Ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener.
- $\gamma = EI_S/bD$ = Ratio of bending stiffness rigidity of stiffener to the plate, where I_S is the moment of inertia of the stiffener cross-section about reference axis.

2.4 Problem Identification

The basic configuration of the problem considered here is a unstiffened and stiffened plate subjected to various uniform and non-uniform edge loadings as shown in figures 1 (a-d). The cross-section of the stiffened plate is shown in figure 4.2 for rectangular cross-section and figure 4.3 for Skew rectangular plate cross section under uniaxially loading.



(a) Uni-axial loading (b) Bi-axial loading

(c) In-plane shear loading (d) In-plane Shear loading

Figure 1 (a-d): Plate subjected to inplane uniform edge loading

The problem considered here consists of a rectangular plate (a x b) with stiffener subjected to various types of loading. In Figures 1 (a-d), the plates are subjected to uniformly distributed in plane uniaxial, biaxial, shear edge loading. The length (a) of the stiffened plate considered above is varied keeping its other parameters unchanged. The cross-section of the stiffener is shown in figure 2 and 3. Rectangular skew plates under general uniformly distributed in-plane edge loading are considered also as shown in figure 4.3 to study the vibration and dynamic stability behaviour of skew stiffened plates.

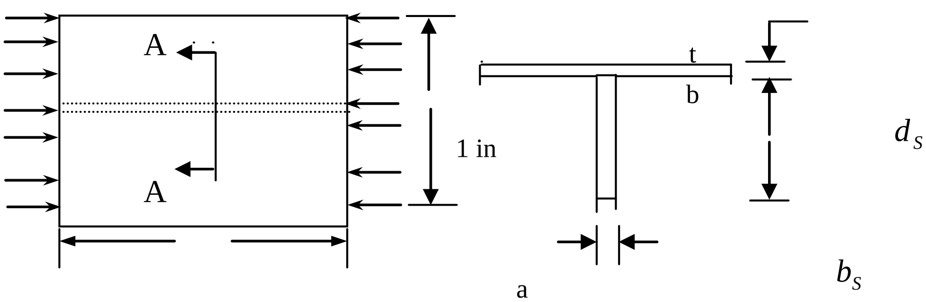


Figure 2: Stiffened plate cross section

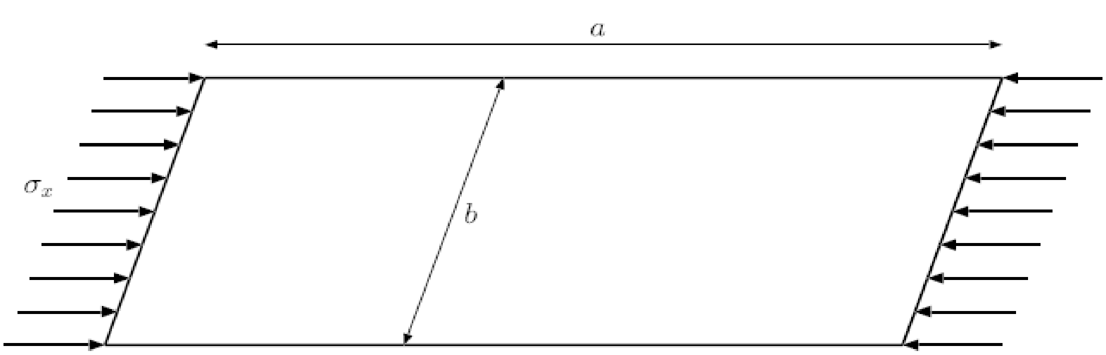


Figure 3: Skew rectangular plate under uniaxially loading

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A parametric study is carried out here for the plates to present some new results in the present studies.on the topic using the present finite element approach. Different kind of loading cases as written below are applied for buckling, vibration of stiffened plates.

1. In-plane Uniaxial compression
2. In-plane biaxial Compression
3. In-plane shear load
4. Biaxial compression and in-plane shear

III. Result and Discussion

3.1 Convergence and validation studies with previous results

In a finite element analysis, it is desired to have the convergence studies to estimate the order of mesh size to be necessary for the numerical solution. The problem of isotropic rectangular unstiffened plates with uniform loading is investigated in table 1 for buckling and vibration for a/b = 1, 0.5, 2, 2.5 and validated with available results of Leissa [25]. As the convergence study shows that a mesh size of 10x10 is sufficient enough to get a reasonable order of accuracy. The analysis in the subsequent problems is carried out with this mesh size.

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Table 1: Buckling and Vibration studies of rectangular unstiffened plate

a/b	Boundary Condition	Buckling Parameter	Non dimensional frequency parameter				
			Reference	Mode No			
				1	2	3	4
1	SSSS	3.99	Present	19.73	49.35	49.35	78.96
			Leissa [25]	19.732	49.438	49.438	78.95
	CCCC	10.07	Present	35.98	73.43	--	108.21
			Leissa [25]	35.992	73.221	73.221	108.27
	SCSS	4.84	Present	23.64	51.12	58.23	86.34
0.5	SSSS	6.24	Present	49.33	78.94	128.76	167.45
	CCCC	19.31	Present	98.29	127.56	179.97	254.56
	SCSS	10.37	Present	69.23	94.789	140.09	207.45
2.5	SSSS	4.13	Present	11.643	16.97	24.356	35.98
			Leissa[25]	11.44	16.18	24.08	35.135
	CCCC	7.85	Present	23.64	27.81	35.46	46.93
			Leissa [25]	23.648	27.817	35.446	46.770
	SCSS	4.116	Present	11.74	17.123	25.56	38.76

A square plate clamped in all edges having a centrally placed concentric stiffener as presented by Nair & Rao [26] using a package stift1, Mukharjee [17] , Mukhopadhyay [16], and Seikh [18] using FEM , semi analytical method, and spline finite strip method respectively has been analyzed presently in table 2. Seikh [18] has given results neglecting and including mass moment of inertia which has been validated in present results marked as Present (1) for M.I. Neglecting and Present (2) as mass moment of inertia including. The first six frequencies are compared. The agreement is excellent. In Mukhopadhyay [16] in-plane displacement is not considered in the analysis so results cause slightly varying. Table2 also present convergence study showing good convergence of results.

Plate size = 600mm x 600 mm

Plate thickness = 1.0 mm,

Poisson's ratio = 0.34

Mass density = 2.78e-6 Kg / mm^3

E = 6.87 $X10^{-7} N/mm^2$,

As = 67.0 mm^2 , Is = 2290 mm^4

Js = 22.33 mm^4

Table 2: Frequency in (rad / s) of clamped stiffened plate with a concentric Stiffener

Source	Mode No	1	2	3	4	5	6
Present	6 x 6	318.62	404.53	474.30	541.56	727.81	771.47
	8 x 8	317.36	401.66	472.34	538.26	719.23	763.23
	10 x 10	317.00	400.84	471.74	537.32	716.35	759.54
Present	10 x 10	317.00	400.84	471.74	537.32	716.35	759.54

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Nair and Rao [26]	317.54	400.12	472.23	537.14	714.14	760.17
Mukharjee [17]	322.34	412.23	506.87	599.34	772.15	860.93
Mukhopadhyay [16]	305.12	382.34	454.76	519.17	696.18	741.15
(1) Seikh [18]	316.85	400.35	471.69	536.95	716.04	759.14
(2) Seikh [18]	316.85	400.35	471.68	536.94	716.02	759.12

The Effect of the stiffener for the same plate but with a stiffener of 20 mm by 3 mm size has been solved and presented in table 3. Dimension of the stiffened plate is shown in figure 4. Seikh [18] solved this problem by spline finite strip method and finite element method respectively. The results are compared and are found to agree well. Nair also solved placing the stiffener at various eccentricities.

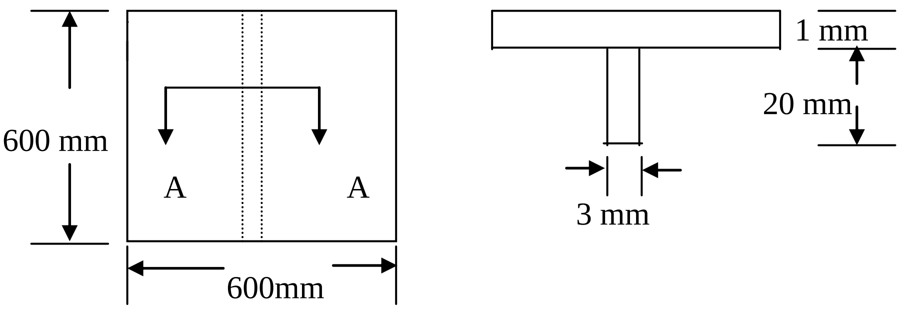

Figure 4: Eccentrically clamped square stiffened plate

Table 3: Convergence of the frequency with Mesh Division

		Mesh Division					
		3 x 3	4 x 4	6 x 6	8 x 8	9 x 9	10 x 10
Sheikh [18]	Mode 1	324.703	319.921	317.433	316.949	316.94	316.799
	Mode 2	403.147	403.147	401.627	400.611	400.369	400.327
Present	Mode 1	342.473	325.62	318.64	317.66	317.25	316.79
	Mode 2	410.06	420.62	404.53	401.66	403.10	400.32

3.2 Buckling of longitudinally stiffened plate under uniaxial load

3.2.1 Validation Studies for Buckling Studies of a Central concentric stiffened plate

The present formulation is validated for buckling analysis of rectangular stiffened plate. The buckling load parameters have been obtained for various aspect ratios, bending stiffness rigidity and stiffener area ratios for one central longitudinal stiffener in table 4. The plate thickness ratio (*a*/*h*) and isotropic plate and stiffener material (ν) are taken as 100 and 0.3 respectively. The dimension '*a*' is varied keeping *b* as constant to get different values of aspect ratio (*a*/*b*). The plate is subjected to uniform compression in the x-direction. The mesh division chosen for the whole plate is 10 x 10. A good agreement is observed with the result obtained by Timoshenko and Gere [27]. In table 4 A is for Timoshenko and Gere [27].

Table 4: Comparison of buckling load parameters of a rectangular stiffened plate with one longitudinal central stiffener subjected to uniform normal edge loading along x- direction.

a/b	$\gamma = 5$				$\gamma = 10$			
	$\delta = 0.05$		$\delta = 0.1$		$\delta = 0.05$		$\delta = 0.10$	
	Present	A [27]	Present	A [27]	Present	A [27]	Present	A [27]

0.6	16.54	16.5	16.54	16.5	16.54	16.50	16.54	16.5
1.0	11.98	12.0	11.03	11.11	15.99	16.00	15.99	16.0
1.2	9.85	9.83	9.02	9.06	15.34	15.30	14.09	14.2
1.6	8.17	8.01	7.35	7.38	11.43	11.40	10.48	10.5
2.0	8.01	7.95	7.28	7.29	10.20	10.20	9.34	9.35
3.0	8.31	8.31	7.67	7.62	12.03	12.0	11.10	11.1

After the validation of free vibration analysis and buckling analysis of stiffened plates, detailed studies has been done for variation of buckling and vibration load parameters of stiffened plates for different aspect ratios, various boundary conditions with varying load P/P_{cr} with different number of stiffeners.

The variation of buckling and frequency parameters with P/P_{cr} for rectangular eccentric stiffened plate with 1, 2, 3 longitudinal equispaced stiffeners subjected to in-plane uniform uni-axial for various aspect ratios (a/b = 1, 1.5, 2) and boundary conditions (SSSS, CCCC, SCSS) are studied in details and presented in table 5-8 for different boundary conditions and aspect ratios for the sake of getting various interlinking results of number of stiffeners and aspect ratios. Here n is number of stiffeners.

It is observed from the above studies that frequency parameters at any value of P/P_{cr} will be more for square stiffened plate. These values decrease with the increase of aspect ratios. Similarly it is also concluded that buckling loads parameters will decrease, as the aspect ratios will increase. It may be concluded from above observation that buckling load parameter and frequency parameter will increase with increase of number of stiffeners. The value will also increase with the increase of degree of restraints. Thus the buckling and frequency parameter of CCCC is more than SSSS and SCSS. In the same manner the buckling and frequency parameter of SCSS is more than SSSS.

Table 5: Variation of frequency parameter and buckling load parameter (λ) in the case of different boundary condition and various aspect ratios for one longitudinal stiffener at the centre having (δ = 0.1 and γ = 5).

a/b	Boundary Condition	λ	ϖ Frequency parameter				
			-1	-0.5	0	0.5	0.9
1	SSSS CCCC SCSS	11.03 24.90 17.02	46.39 87.89 62.70	40.17 79.47 54.91	32.80 69.76 45.60	23.19 57.64 33.24	2.31 7.95 1.81
1.5	SSSS CCCC SCSS	7.56 23.73 11.07	25.60 52.48 33.47	22.17 46.41 29.20	18.10 39.17 24.09	12.84 29.53 17.29	1.28 ---- 1.77
2	SSSS CCCC SCSS	7.285 22.47 8.89	18.84 37.97 22.49	16.32 33.54 19.64	13.32 28.22 16.22	9.41 21.04 11.667	0.94 2.20 1.20

Table 6: Variation of frequency parameter and buckling load parameter (λ) in the case of different boundary condition and aspect ratios for two longitudinal stiffener having (δ = 0.1 and γ = 5)

a/b	Boundary Condition	λ	ϖ Frequency parameter				
			-1	-0.5	0	0.4	0.6	0.9
1	SSSS CCCC SCSS	14.44 43.50 26.10	53.07 --- 77.26	45.96 94.46 67.35	37.52 79.78 55.46	29.07 64.95 43.34	23.73 55.48 35.57	2.65 8.73 1.37
1.5	SSSS CCCC SCSS	8.72 29.54 14.0	27.49 56.88 37.65	23.81 49.89 32.79	19.44 41.28 26.96	15.06 32.51 21.06	12.29 26.83	1.37 3.07 1.94

2	SSSS CCCC SCSS	7.68 25.11 10.27	19.35 39.46 24.18	16.79 34.64 21.07	13.69 28.81 17.35	10.60 22.80 13.56	8.66 18.89 11.13	0.96 2.18 1.26

Table 7: Variation of frequency parameter and buckling load parameter (λ) in the case of different boundary condition and aspect ratios for three longitudinal stiffener having (δ = 0.1 and γ = 5).

a/b	BoundaryCondition	λ	ϖ Frequency parameter					
			-1	-0.5	0	0.4	0.6	0.9
1	SSSS CCCC SCSS	17.04 60.64 32.08	57.67 --- 85.63	49.94 --- 74.55	40.78 88.85 61.27	31.58 69.93 47.76	25.79 57.79 39.15	2.88 6.61 4.41
1.5	SSSS CCCC SCSS	9.69 34.12 16.30	28.97 60.95 40.63	25.09 53.25 35.37	20.49 43.95 29.05	15.87 34.42 22.64	12.96 28.29 18.36	1.45 3.20 2. 09
2	SSSS CCCC SCSS	8.03 27.2 11.38	19.78 40.92 25.46	17.13 35.82 22.18	14 29.67 18.24	10.83 23.35 14.24	8.85 19.25 11.68	0.99 2.19 1.32

Table 8: Variation of frequency parameter and buckling load parameter (λ) for stiffened plate (Equispaced, one stiffener, two stiffener, three stiffener) with different aspect ratio and boundary condition having (δ = 0.1, γ = 5)

No of Stiffener		1		2		3	
a/b	Boundary Condition	λ	ϖ	λ	ϖ	λ	ϖ
1	SSSS CCCC SCSS	11.03 24.90 17.02	32.80 69.76 45.60	14.40 43.50 26.70	37.52 79.78 55.46	17.04 60.64 32.08	40.78 88.85 61.25
2	SSSS CCCC SCSS	7.285 22.47 8.89	13.32 28.22 16.22	7.68 25.11 10.27	13.69 28.81 17.35	8.03 27.2 11.38	14.00 29.67 18.24
1.5	SSSS CCCC SCSS	7.56 23.73 11.07	18.10 39.17 24.09	8.72 29.54 14.03	19.44 41.28 26.96	9.69 34.12 16.30	20.49 43.95 29.05

3.3 Unstiffened/stiffened plates under in-plane biaxial load

3.3.1 Convergence and validation study

The accuracy of the proposed method for unstiffened plates under in-plane biaxial load are first established by comparing the results of various problems with those of earlier investigators' available in the literature. The analysis has been done for buckling load factor for square plates having various boundary conditions (SSSS, CCCC, CSCS) under bi-axial load considering compressive in-plane load and validated in table 9. To study the aspect of convergence, a simply supported square plate subjected to in-plane compressive load in both directions has been undertaken in table 10. In this case, the result is compared with finite difference solution by Singh and Dey [2]. The result is observed to converge satisfactorily at 8 x 8 and 10 x 10 mesh divisions.

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Further frequency parameter (ω) for a simply supported square plate subjected to bi-axial load. ($N_x = N_y = N$ and $N_{xy} = 0$), ν = 0.3, P/P_{cr} = 0.5 is analyzed and validated in table 9 with the result of Singh and Dey [2] and analytical solution of Diez [1].

Table 9: Buckling load factor for square plate under biaxial load

Boundary Conditions	Non-dimensional buckling load			
	Present		Ref [2]	
	Mode1	Mode 2	Mode1	Mode 2
SSSS	1.99	4.999	2.0	5.0
CCCC	5.60	9.333	5.61	9.42
CSCS	3.82	5.948	3.83	5.92

Table 10: Convergence and validation study of frequency parameter ω for a simply supported square plate subjected to bi-axial load. ($N_x = N_y = N$, $N_{xy} = 0$), ν = 0.3, P/P_{cr} = 0.5

Mode No.	Frequency Parameter ω					
	Present				Singh and Dey [2]	Analytical [1]
	4 x 4	6 x 6	8 x 8	10 x 10		
1	13.974	13.959	13.957	13.956	13.96	13.96
2	44.729	44.253	44.148	44.144	44.13	43.35
3	44.729	44.253	44.148	44.144	44.13	43.35
4	75.015	74.078	73.892	73.865	73.86	73.87

After convergence and validation study for buckling and free vibration of unstiffened plates in well condition for bi-axial edge loading, the analysis is now extended to unstiffened/stiffened plates subjected to bi-axial load for buckling and vibration analysis.

3.4 Vibration and buckling studies

The variation of frequency parameter with in-plane load intensity factor of a plate of various aspect ratio and edge condition compressed uniformly on all edges ($N_x = N_y = N$ & $N_{xy} = 0$) is studied in this section. The fundamental frequency parameter for plate having different boundary conditions (SSSS, CCCC, SCSS) are studied in table 11.

The trend of the results for a simply supported plate shown in table 11 is almost similar for CCCC plates and SCSS, except for its gradient of rise. For CCCC plate, there is a steep increase in frequencies with increasing aspect ratios. The natural frequencies are found to increase with decreased magnitude of compressive in-plane forces.

Table 11: Variation of frequency parameter with in-plane load intensity factor of a late of various aspect ratio and edge condition compressed uniformly on all edges ($N_x = N_y = N$ & $N_{xy} = 0$), ν = 0.3.

a/b	Boundary Condition	Buckling Parameter λ	Frequency parameter at P/P_{cr}				
			0	0.2	0.5	0.8	0.995
1	SSSS	1.99	19.73	17.65	13.95	8.82	0.438
	CCCC	5.30	35.98	32.28	25.65	16.32	0.815
	SCSS	2.66	23.64	21.26	16.76	10.62	0.53
0.5	SSSS	4.99	43.33	44.13	34.88	22.06	0.109
	CCCC	15.67	98.29	88.22	70.17	44.73	0.224
	SCSS	8.61	69.31	62.10	49.29	31.24	0.115
2	SSSS	1.24	12.34	11.03	8.72	5.56	0.270
	CCCC	3.92	24.57	22.06	17.54	11.18	0.558
	SCSS	1.34	12.91	11.55	9.14	5.78	0.286

Now the study is extended to stiffened plates subjected to bi-axial in-plane edge loading. The variation of buckling and frequency parameters with P/P_{cr} for rectangular stiffened plate with 1, 2, 3 longitudinal equispaced stiffeners (Stiffener parameters δ = 0.1 and γ = 5) subjected to in-plane uniform bi-axial for various aspect ratios (a/b= 1, 1.5, 2) and boundary conditions (SSSS, CCCC, SCSS) are studied in detail and shown in table 12-14 for the sake of getting various interlinking results of number of stiffeners and aspect ratios.

Table 12: Variation of frequency parameter with in-plane load intensity factor of stiffened Plate with one central stiffener having (δ = 0.1 and γ = 5) compressed uniformly on all edges ($\sigma_x = \sigma_y = \sigma, \tau_{xy} = 0$), $\upsilon = 0.3$

a/b	Boundary Condition	λ	Frequency parameter at Load intensity factor P/P_{cr}						
			-2	-1	-0.5	0	0.25	0.5	0.8
1	SSSS	5.00	53.86	44.65	39.21	32.80	29.03	24.69	17.87
	CCCC	9.33	95.28	83.89	77.32	69.75	63.78	52.22	33.18
	SCSS	5.36	65.79	56.86	51.64	45.60	42.11	36.56	23.13
0.5	SSSS	7.99	131.57	111.94	100.09	78.98	68.12	55.098	35.34
	CCCC	17.23	221.34	186.09	154.12	127.22	110.76	89.98	56.78
	SCSS	10.64	163.97	137.97	115.56	94.87	81.76	67.45	42.34
2	SSSS	1.67	23.06	18.126	15.99	13.21	11.23	9.01	5.12
	CCCC	6.18	48.12	38.98	33.98	28.43	23.98	20.34	11.98
	SCSS	2.38	27.98	22.54	19.12	16.34	14.09	11.23	7.12

Table 13: Variation of frequency parameter with in-plane load intensity factor of stiffened plate with two equispaced stiffener having (δ = 0.1 and γ = 5) compressed uniformly on all edges ($\sigma_x = \sigma_y = \sigma, \tau_{xy} = 0$), $\upsilon = 0.3$

a/b	Boundary Condition	λ	Frequency parameter at Load intensity factor P/P_{cr}						
			-2	-1	-0.5	0	0.25	0.5	0.8
1	SSSS	7.35	62.89	51.80	45.23	37.52	33.0	27.73	19.61
	CCCC	14.21	---	97.82	89.60	79.78	73.62	65.33	46.50
	SCSS	10.24	83.50	71.09	63.70	55.46	50.00	45.12	31.09
2	SSSS	1.88	23.70	19.35	16.76	13.69	11./85	9.68	6.12
	CCCC	6.39	48.32	39.95	34.89	28.81	25.13	20.70	13.26
	SCSS	2.91	29.99	24.51	21.24	17.35	15.03	12.28	7.77

Table 14: Variation of frequency parameter with in-plane load intensity factor of stiffened plate with three equispaced stiffeners having (δ = 0.1 and γ = 5) compressed uniformly on all edges. ($\sigma_x = \sigma_y = \sigma, \tau_{xy} = 0$)

a/b	Boundary Condition	λ	Frequency parameter At Load intensity factor P/P_{cr}						
			-2	-1	-0.5	0	0.25	0.5	0.8
1	SSSS	8.27	66.59	52.21	48.54	40.78	36.28	31.13	23.52
	CCCC	18.96	--	---	99.40	88.85	82.80	75.88	54.01
	SCSS	13.24	92.66	78.63	70.51	61.27	56.05	50.27	32.53

2	SSSS	2.08	24.42	19.78	17.13	13.99	12.11	9.89	6.26
	CCCC	7.19	49.84	41.18	35.96	29.67	25.87	21.29	13.69
	SCSS	3.39	31.53	25.77	22.33	18.24	15.80	12.91	8.17

The analysis is also presented below in figure 5 for simply supported stiffened plate which has been above for 1, 2, 3 stiffeners to show as a combined effects and overview.

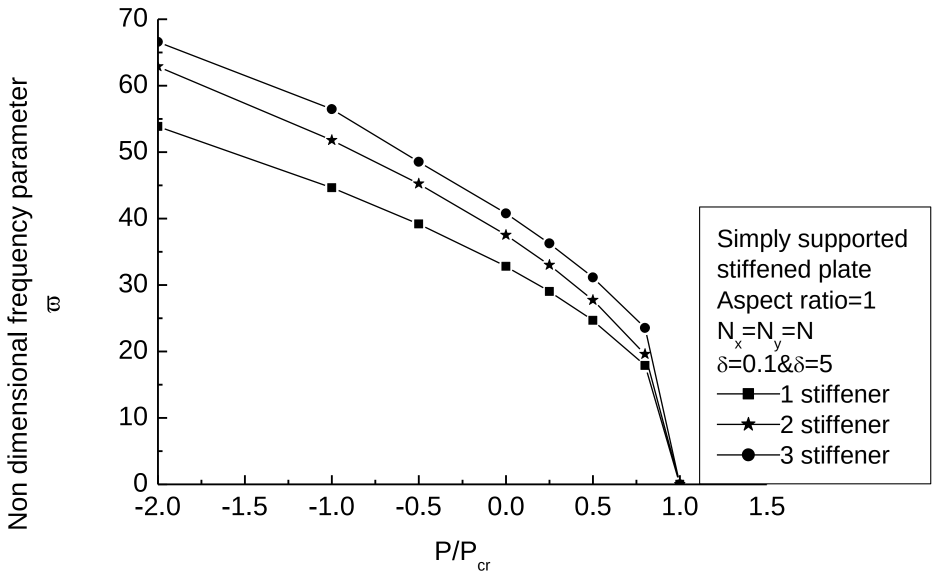

Figure 5: Variation of Non-dimensional frequency parameter vs P/P_{cr} for stiffened plate subjected by uniform compression on all edges.

3.5 Unstiffened and stiffened plates uunder shear edge loading

3.5.1 Validation studies with unstiffened plate

The variation of frequency parameter with various in-plane load intensity (P/P_{cr}) of unstiffened square plate having boundary conditions (SSSS, CCCC, CSCS) subjected to inplane shear load at all edges have been analyzed in various modes and the results are obtained in table 15 and validated with the result of Singh and Dey [2].

Table 15: Buckling load factors for square plate under shear load.

Boundary Condition	Buckling Load Parameter			
	Present		Ref [2]	
	Mode 1	Mode 2	Mode1	Mode 2
SSSS	9.32	11.568	9.33	11.56
CCCC	14.65	16.958	14.66	16.96
CSCS	12.58	14.228	12.58	14.23

3.6 Buckling and Vibration Studies of stiffened pates subjected to Shear Load

After validating the results for buckling of unstiffened plates under shear, it is extended for further study for stiffened plates in order to get details insight and with the view to get some new numerical results under in-plane shear load.

The variation of frequency parameters with P/P_{cr} for simply supported rectangular stiffened plate with 1, 2, 3 longitudinal equispaced stiffeners (Stiffener parameters δ = 0.1 and γ = 5) subjected to uniform shear has been studied for (a/b= 1, 1.5, 2) in table 16-19 4.27- 4.30. Variation of non-dimensional frequency parameter (ϖ) vs P/Pcr for simply supported stiffened plate having (δ =0.1 and γ =5) subjected by in-plane uniform shear may be drawn. This is for the sake of getting various interlinking results of number of stiffeners and aspect ratios. It is observed from the above studies that frequency parameters at any value of P/P_{cr} will be more for square stiffened plate. These values decrease with the increase of aspect ratios. Similarly it is also concluded that buckling loads parameters will decrease, as the aspect ratios will increase.

Table 16: Variation of frequency parameter with in-plane load intensity factor (P/P_{cr}) of a simply supported stiffened plate with one central stiffener having (δ = 0.1, γ = 5) subjected to edge shear only. ($\sigma_x = \sigma_y = 0, \tau_{xy} = \sigma$), $\upsilon = 0.3$

a/b	λ	Frequency parameter at P/P_{cr}					
		0	0.2	0.4	0.5	0.6	0.8
1	20.38	32.80	32.10	29.71	27.58	24.42	9.44
1.5	11.41	18.10	17.73	16.53	15.54	14.19	9.27
2	9.44	13.32	13.06	12.23	11.55	10.64	7.72

Table 17: Variation of frequency parameter with in-plane load intensity factor (P/P_{cr}) of a simply supported stiffened plate with two equispaced stiffeners (δ = 0.1, γ = 5) subjected to edge shear only. ($\sigma_x = \sigma_y = 0, \tau_{xy} = \sigma$), $\upsilon = 0.3$.

a/b	λ	Frequency parameter at P/P_{cr}					
		0	0.2	0.4	0.5	0.6	0.8
1	35.76	37.52	36.75	34.21	32.05	29.04	--
1.5	17.22	19.44	19.07	17.88	16.89	15.53	10.7
2	11.82	13.44	13.44	12.66	12.01	11.13	8.20

Table 18: Variation of frequency parameter with in-plane load intensity factor (P/P_{cr}) of a simply supported stiffened plate with three equispaced stiffeners (δ = 0.1, γ = 5) subjected to edge shear only. ($\sigma_x = \sigma_y = 0, \tau_{xy} = \sigma$), $\upsilon = 0.3$

a/b	λ	Frequency parameter at P/P_{cr}					
		0	0.2	0.4	0.5	0.6	0.8
1	57.78	40.78	39.77	36.36	33.19	14.56	---
1.5	21.39	20.49	21.12	18.93	17.93	16.56	11.8
2	13.58	13.99	13.74	12.95	12.30	11.42	8.52

Table 19: Variation of frequency parameter with in-plane load intensity factor (P/P_{cr}) and buckling load parameter of a simply supported stiffened plate with equispaced stiffeners (δ =0.1 and γ =5) subjected to edge shear only. ($\sigma_x = \sigma_y = 0, \tau_{xy} = \sigma$), $\upsilon = 0.3$.

No of stiffeners	a/b	Buckling Parameter	Frequency parameter at P/P_{cr}					
			0	0.2	0.4	0.5	0.6	0.8
1	1	20.38	32.80	32.10	29.71	27.58	24.42	9.44
	1.5	11.41	18.10	17.73	16.53	15.54	14.19	9.27
	2	9.44	13.32	13.06	12.23	11.55	10.64	7.72

2	1 1.5 2	35.76 17.23 11.82	37.52 19.44 13.69	36.75 19.07 13.44	34.21 17.88 12.66	32.05 16.89 12.01	29.04 15.53 11.13	-- 10.77 8.20
3	1 1.5 2	57.78 21.39 13.58	40.78 20.49 13.99	39.77 20.12 13.74	36.36 18.93 12.95	33.19 17.93 12.30	14.56 16.56 11.42	--- 11.81 8.52

IV. CONCLUSION

The results from the study of the compressive buckling and vibration behaviour of a stiffened plate subjected to in-plane uniform and non-uniform edge loading can be summarized as follows.

The stability resistance increases with increase of restraint at the edges for all types of loading, stiffener parameters and plate aspect ratios. The stability resistance increases with increase of number of stiffeners. The variation of buckling load with the position of the concentrated load on the edges is more pronounced for the stiffened plates of the smaller aspect ratios.

The buckling load parameter of unstiffened plates simply supported along all the edges increase as the loads are nearer to the support. For plates with small aspect ratios, the boundary condition on the loaded edge has the significant effect on the load required to cause elastic stability. Natural frequencies of stiffened plates always decrease with the increase of the in-plane compressive load. The fundamental frequency becomes zero at the respective values of the buckling load**.** The inclusion of in-plane displacements reduces the frequency of the stiffened plates. The buckling load of the stiffened plates reduces with the eccentricity of the stiffeners. The eccentricity of the stiffened plate element should not be neglected, especially for higher modes of vibration.

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