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FUZZY GRAPHS ON COMPOSITION, TENSOR AND NORMAL PRODUCTS

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Abstract

Special fuzzy graph can be obtained from two given fuzzy graphs using the operations, Cartesian product, composition, tensor and normal products. In this paper, we find the degree of a vertex in fuzzy graphs formed by these operations in terms of the degree of vertices in the given fuzzy graphs in some particular cases.

Index Terms- Cartesian product, Composition, Degree of a vertex, Tensor product, Normal product.

1. Introduction

Fuzzy graphs introduced by Rosenfeld in 1975[1-2,9,10]. The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Moderson.J.N. and Peng.C.S [3-8]. In this paper, we study about the degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using the operations Cartesian product and composition of two fuzzy graphs, tensor and normal product of two fuzzy .In general, the degree of vertices in Cartesian product and composition of two fuzzy graphs, tensor and normal product of two fuzzy graphs G_1 andG_2 cannot be expressed in terms of those in G_1 andG_2. In this paper, we find the degree of vertices in Cartesian product, composition, tensor and normal product of G_1 andG_2 in some particular cases.

2. Research Elaborations

Definition 2.1: A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of G: (σ, μ) is denoted by G^*: (V, E) where $E \subseteq V \times V$.

Definition 2.2: Let G: (σ, μ) be a fuzzy graph. The degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u \neq v} \mu(uv)$$
$$= \sum_{uv \in E} \mu(uv)$$

Definition 2.3: The order of a fuzzy graph G is defined by $O(G) = \sum_{u \in V} \sigma(u)$.

Note: Throughout this paper G_1: (σ_1, μ_1) and G_2 :(σ_2, μ_2) denote two fuzzy graphs with underlying crisp graphs G_1^*: (V_1, E_1) and G_2^* :(V_2, E_2) with $|V_i| = P_i$, i = 1, 2. Also $d(G_i^*)(u_i)$ denotes the degree of u_i in G_i^*

Definition 2.4: The cartesian product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G = G_1 \times G_2 : (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ on G^*: (V, E) where $V = V_1 \times V_2$ and
$E = \{((u_1,u_2)(v_1,v_2)) / u_1=v_1, u_2v_2 \in E_2 \text{ or } u_2=v_2, u_1v_1 \in E_1\}$ with

$$(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2), \quad \text{for all } (u_1,u_2) \in V_1 \times V_2$$

$$\& \quad (\mu_1 \times \mu_2)((u_1,u_2)(v_1,v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2), & \text{if } u_1 = v_1 \text{ and } u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1v_1), & \text{if } u_2 = v_2 \text{ and } u_1v_1 \in E_1 \end{cases}$$

Definition 2.5: The Composition of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G = G_1[G_2] : (\sigma_1 \circ \sigma_2, \mu_1 \circ \mu_2)$ on G^*:(V,E) where $V = V_1 \times V_2$ and
$E = \{((u_1,u_2)(v_1,v_2)) / u_1=v_1, u_2v_2 \in E_2 \text{ or } u_2 \neq v_2, u_1v_1 \in E_1\}$ with

$$(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2), \quad \text{for all } (u_1,u_2) \in V_1 \times V_2$$

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& $(\mu_1 \times \mu_2)((u_1,u_2)(v_1,v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2v_2), & if\ u_1 = v_1 \quad and\ u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1v_1), & if\ u_2 = v_2 \quad and\ u_1v_1 \in E_1 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & if\ u_2 = v_2 \quad and\ u_1v_1 \in E_1 \end{cases}$

3. Results

Definition 3.1: **T**he normal product of two fuzzy graphs (σ_i, μ_i) on $G_i = (V_i, X_i)$, i=1,2 is defined as a fuzzy graph $(\sigma_1 \circ \sigma_2, \mu_1 \circ \mu_2)$ on G = (V,X) where V= V_1 X V_2 and

$X = \{((u,u_2)(u,v_2)) \setminus u \in V_1, (u_2v_2) \in X_2\} \cup \{((u_1,w)(v_1,w)) \setminus (u_1,v_1) \in X_1, w \in V_2\} \cup \{(u_1,u_2)(v_1,v_2) \setminus (u_1v_1) \in X_1, (u_2v_2) \in X_2\}$

Fuzzy sets $\sigma_1 \circ \sigma_2$ and $\mu_1 \circ \mu_2$ are defined as

$(\sigma_1 \circ \sigma_2)(u_1, u_2) = (\sigma_1(u_1) \wedge \sigma_2(u_2))$ $(u_1, u_2) \in V_1 x V_2$

$(\mu_1 \circ \mu_2)((u, u_2)(v_1, v_2)) = \{\sigma_1(u) \wedge \mu_2(u_2v_2)\}$ $u \in V_1$ and $(u_2v_2) \in X_2$

$(\mu_1 \circ \mu_2)((u_1, w)(v_1, w)) = \{\mu_1(u_1v_1) \wedge \sigma_2(w)\}$ $w \in V_2$ and $(u_1v_1) \in X_1$

$(\mu 1 \circ \mu_2)((u_1,u_2)(v_1,v_2)) = \{\mu_1(u_1v_1) \wedge \mu_2(u_2v_2)\}$ $(u_1u_2) \in X_1$ and $(v_1v_2) \in X_2$

Definition 3.2: The tensor Product of two fuzzy graphs (σ_i, μ_i) on $G_i = (V_i, X_i)$, i=1,2 is defined as a fuzzy graph $(\sigma_1 \otimes \sigma_2, \mu_1 \otimes \mu_2)$ on G = (V,X) where V= V_1 X V_2 and $X = \{(u_1, u_2), (v_1, v_2) \setminus (u_1,v_1) \in X_1, (u_2,v_2) \in X_2\}$.

Fuzzy sets $\sigma_1 \otimes \sigma_2$ and $\mu_1 \otimes \mu_2$ are defined as

$(\sigma_1 \otimes \sigma_2)(u_1, u_2) = \{(\sigma_1(u_1) \wedge \sigma_2(u_2)\}$ for all $(u_1, u_2) \in V$

$(\mu_1 \otimes \mu_2)\{(u_1, u_2), (v_1, v_2)\} = \{\mu_1(u_1, v_1) \wedge \mu_2(u_2, v_2)\}$ $(u_1, u_2) \in X_1$ and $(v_1, v_2) \in X_2$

3.1 Degree of a vertex in Cartesian product

By the definition, for any vertex $(u_1, u_2) \in V_1 \times V_2$,

$d_{G1\ xG_2}(u_1,u_2) = \sum_{(u1,u2)(v1,v2)\in E}(\mu_1 \text{ x } \mu_2)((u_1,u_2)(v_1.v_2))$

$= \sum_{u1=v1,u2v2\in E2} \sigma_1(u_1) \wedge \mu_2(u_2v_2) + \sum_{u2=v2,u1v1\in E1} \sigma_2(u_2) \wedge \mu_1(u_1v_1)$

In the following theorems, we find the degree of $(u_1, u2)$ in G_1 X G_2 in terms of those in G_1 and G_2 in some particular cases.

Theorem 3.1: Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$ then $d_{G_1 \text{ x } G_2}(u_1,u_2) = d_{G_1}(u_1) + d_{G_2}(u_2)$.

Proof: From the definition of a degree of a vertex in cartesian product

$d_{G_1 \text{ x } G_2}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \sigma_1(u_1) \wedge \mu_2(u_2v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1)$

$= \sum_{u_2v_2\in E_2} \mu_2(u_2v_2) + \sum_{u_1v_1\in E_1} \mu_1(u_1v_1)$ (since $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$)

$= d_{G_1}(u_1) + d_{G_2}(u_2)$.

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Example 3.1

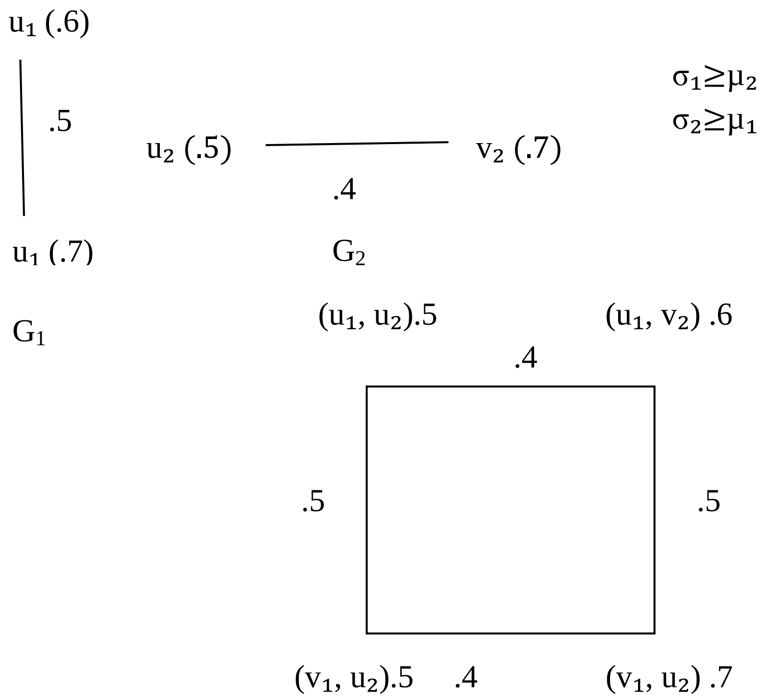

Figure. 1. Cartesian product of G_1 & G_2 (G_1 x G_2)

Here G_1, G_2 are two fuzzy graphs

$d_{G_1 \times G_2}(u_1,u_2) = d_{G_1}(u_1) + d_{G_2}(u_2)$. (since $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ & by theorem 3.1)

$= .5 + .4$

$= .9$

$d_{G_1 \times G_2}(u_1,v_2) = .5 + .4$

$= .9$

Similarly we can find the degrees of all the vertices in G_1 x G_2. This can be verified in the figure 1.

3.2 Degree of a vertex in composition

By the definition, for any vertex $(u_1,u_2) \in V_1 \times V_2$,

$$d_{G_1 \times G_2}(u_1,u_2) = \sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_1 \circ \mu_2)((u_1,u_2)(v_1.v_2))$$

$$= \sum_{u_1=v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2v_2) + \sum_{u_2=v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1 v_1)$$

Theorem 3.2 Let G_1: (σ_1, μ_1) and G_2 :(σ_2,μ_2) be two fuzzy graphs. If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$ then $d_{G_1[G_2]}(u_1,u_2) = P_2\, d_{G_1}(u_1) + d_{G_2}(u_2)$.

Proof:

$$d_{G_1[G_2]}(u_1,u_2) = \sum_{u_1=v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2v_2) + \sum_{u_2=v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1)$$

$$= \sum_{u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2=v_2, u_1v_1 \in E_1} \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \mu_1(u_1v_1) \quad \text{(since } \sigma_1 \geq \mu_2 \text{ and } \sigma_2 \geq \mu_1)$$

$$= d_{G_2}(u_2) + |V_2| \sum_{u_1v_1 \in E_2} \mu_1(u_1v_1)$$

$$= d_{G_2}(u_2) + p_2\, d_{G_1}(u_1).$$

Example 3.2

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Consider the fuzzy graphs G_1, G_2 and G_1 [G_2] in figure 2 .

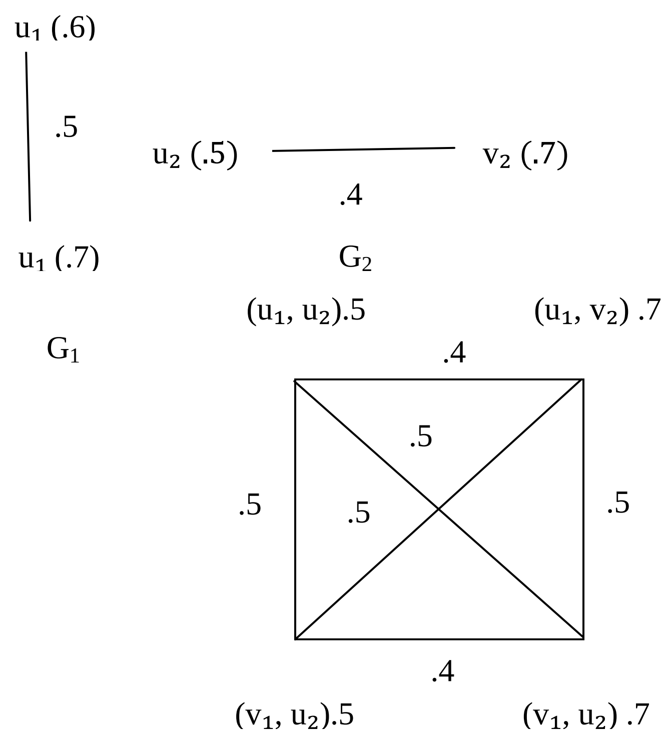

Figure .2. Composition of G_1 & G_2 (G_1 [G_2])

Here $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$ and by theorem 3.2

$$d_{G_1[G_2]}(u_1,u_2) = d_{G_2}(u_2) + P_2 d_{G_1}(u_1).$$
$$= .4 + 2\,(.5)$$
$$= 1.4$$
$$d_{G_1[G_2]}(u_1,v_2) = d_{G_2}(v_2) + P_2 d_{G_1}(u_1).$$
$$= .4 + 2(.5)$$
$$= 1.4$$

Similarly we can find the degrees of all the vertices in $G_1[\,G_2]$.

This can be verified in the figure 2.

3.3 Degree of a vertex in tensor product

By definition, for any $(u_1, u_2) \in V_1 \times V_2$

$$d_{G_1 \otimes G_2}(u_1, u_2) = \sum (\mu_1 \otimes \mu_2)((u_1, u_2)(v_1, v_2))$$
$$= \sum_{u_1 v_1 \in E_1} \mu_1(u_1, v_2) \wedge \mu_2(u_2, v_2)$$

Theorem 3.3: Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\mu_2 \geq \mu_1$ then $d_{G_1 \otimes G_2}(u_1, u_2) = d_{G_1}(u_1)$. And if $\mu_1 \geq \mu_2$ then $d_{G_1 \otimes G_2}(u_1, u_2) = d_{G_2}(u_2)$.

Proof:

$$d_{G_1 \otimes G_2}(u_1, u_2) = \sum_{u_1 v_1 \in E_1} \mu_1(u_1, v_2) \wedge \mu_2(u_2, v_2)$$
$$= \sum \mu_1(u_1, v_1)$$
$$= d_{G_1}(u_1)$$

Example 3.3

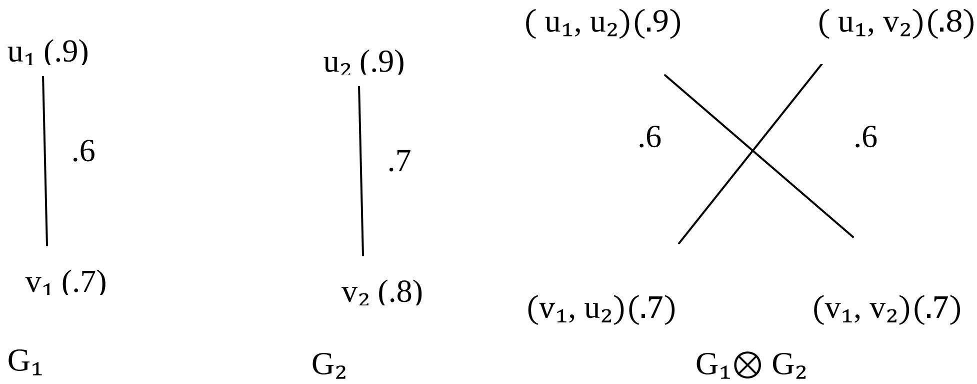

Figure. 3. Tensor product of G_1& G_2 ($G_1 \otimes G_2$)

Consider the fuzzy graphs G_1 and G_2 in fig 3. Here $\mu_2 \geq \mu_1$ and by theorem 3.3

$d_{G_1 \otimes G_2}(u_1, u_2) = d_{G_1}(u_1)$

$= .6$

$d_{G_1 \otimes G_2}(v_1,v_2) = d_{G_1}(v_1)$

$= .6$

This can be verified in the fig 3.

3.4 Degree of a vertex in normal product

By definition, for any $(u_1, u_2) \in V_1 \times V_2$

$$d_{G_1 \circ G_2}(u_1, u_2) = \sum_{((u_1,v_1)(u_2,v_2))\in E} (\mu_1 \circ \mu_2)((u_1,v_1)(u_2,v_2))$$

$$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2,v_2) + \sum_{u^2 = v^2, u^1v^1 \in E^1} \sigma_2(u_2) \wedge \mu_1(u_1,v_1) + \sum_{u_2v_2\in E_2, u_1v_1 \in E_1} \mu_2(u_2v_2) \wedge \mu_1(u_1v_1)$$

Theorem 3.4 Let G_1: (σ_1,μ_1) and G_2:(σ_2,μ_2) be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$ then $d_{G_1 \circ G_2}(u_1,u_2) = P_2\, d_{G1}(u_1) + d_{G2}(u_2)$.

Proof:

$$d_{G_1 \circ G_2}(u_1, u_2) = \sum_{(u_1,v_1)(u_2,v_2)\in E} (\mu_1 \circ \mu_2)(u_1,v_1)(u_2,v_2)$$

$$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2,v_2) + \sum_{u^2 = v^2, u^1v^1 \in E^1} \sigma_2(u_2) \wedge \mu_1(u_1,v_1) + \sum_{u_2v_2\in E_2, u_1v_1 \in E_1} \mu_2(u_2v_2) \wedge \mu_1(u_1v_1)$$

$$= \sum_{u_2v_2\in E_2} \mu_2(u_2v_2) + \sum_{u_2=v_2, u_1v_1\in E_1} \mu_1(u_1v_1) + \sum_{u_1v_1 \in E_1} \mu_1(u_1v_1)$$ (since $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$)

$= d_{G_2}(u_2) + |V_2|\, d_{G_1}(u_1)$.

$= d_{G_2}(u_2) + p_2\, d_{G_1}(u_1)$.

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Example 3.4

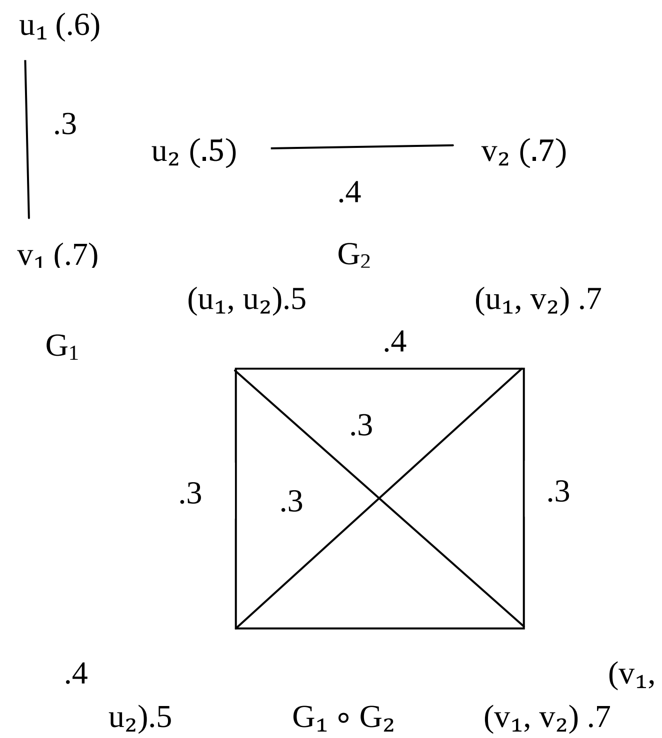

Figure.4. Normal product of G_1 & G_2 ($G_1 \circ G_2$)

Consider the fuzzy graphs G_1 and G_2 in figure 4. Here $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$ and by theorem 3.4

$d_{G_1 \circ G_2}(u_1,u_2) = d_{G_2}(u_2) + P_2 d_{G_1}(u_1)$

$= .4 + 2(.3)$

$= 1$

$d_{G_1 \circ G_2}(v_1,v_2) = d_{G_2}(v_2) + P_2 d_{G_1}(v_1)$

$= .4 + 2(.3)$

$= 1$

Similarly we can find the degrees of all the vertices in G_1 x G_2**.** This can be verified in the figure 3.

Conclusion

In this paper ,we have found the degree of vertices in G_1 x G_2, $G_1[G_2]$, $G_1 \otimes G_2$, $G_1 \circ G_2$ in terms of the degree of vertices in G_1 and G_2 under some conditions and illustrated them through examples. This will be helpful when the graphs are very large. Also they will be very useful in studying various properties of cartesian product, composition, tensor product, normal product of two fuzzy graphs.

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