

A GENERAL THEORY OF FOOD WEB CYCLE - PART ONE

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ABSTRACT: *Social systems, and ecosystems, its sustentative limits, singular pauses, logical attributes, real relations, expressional proposition, heterogeneous complications, - has been developed at the forefront of science. Key concepts of this new way of understanding complex systems –are theories of chaos, attractors, fractals, dissipative structures, self-organization, and autopoietic networks. The intellectual tradition of systems thinking, and the models of living systems developed during the early decades of the century, forms the conceptual and historical roots of the new scientific framework, current models and theories may be seen as an outline of an emerging theory of living systems. What is now emerging at the forefront of science is a coherent scientific theory that offers, for the first time, a unified view of mind, matter, and life. In the following we give a general theory for three pairs: (a) Terrestrial Organism (b)Oxygen Consumption (c)Dead Organic Matter (d)Decomposer Organisms (e) Green Plants (f) Nutrients. Expediency and contingency of such studies are of imminent nature, in consideration to the fact that ozone layer depletions by aerosols have been at the centre stage and the incorporation of green house effects in the cycle is of utmost importance.*

INDEX TERMS : Terrestrial Organism ,Oxygen Consumption ,Dead Organic Matter ,Decomposer Organisms , Green Plants ,Nutrients, Governing Equations, Dissipative Structures, Holism, Ecosystem, Complex Systems, Theories Of Chaos, Ozone Layer Depletions

INTRODUCTION:

Since industrial society has been predominated by the Cartesian split between mind and matter and by the ensuing mechanistic paradigmatic line of thought and way of thinking for the past three hundred years, this new vision that finally overcomes the Cartesian split will have not only important scientific and philosophical consequences, but will also have tremendous practical implications. It will change the way we relate to each other and to our living natural environment, the way we deal with our health, the way we perceive our business organizations, our educational systems, and many other social and political institutions.

In particular, the new vision of life will help us build and nurture sustainable communities - the great challenge of our time - because it will help us understand how nature's communities of plants, animals, and microorganisms - the ecosystems - have organized themselves so as to maximize their ecological sustainability. Basic principles of ecology are the, the language of nature, the stormy petrel that can cause pernicious ramifications, in the eventuality of the fact that it is not taken care of with proper ecological thought and practice. . Principles of ecology are also the basic principles of organization of all living systems. I believe therefore that The Web of Life provides a solid basis for ecological thought and practice.

HOLISM

In the following ‘e’ and ‘eb’ are used to represent ‘encompasses’ and ‘encompassed by’ towards the end of consummation of better understanding of the abstract concepts. It is done only to facilitate the reader so that any subtleties and nuances in writing becomes revelatory and comprehension much easier and familiarized. Holism (from ὅλος *holos*, a Greek word meaning *all, whole, entire, total*), is the idea that natural systems (physical, biological, chemical, social, economic, mental, linguistic, etc.) and their properties, should be viewed as wholes, not as e(e) collections of parts. This often includes (e) the view that systems somehow function as wholes and that their functional dynamics (e&eb) cannot be fully understood solely in terms of their component parts. By knowing how to open an account in a Bank, you cannot understand the entire mode of transactional ties in the Banking system. Geometrical interpretation of the differential equations, for instance, clearly places, in testimony two distinct realities: (a)there is field of directions (b) topological accidents which may suddenly crop up in it, such as such as the

plane of singular points to which no 'directions' has been attached. And there are 'integral curves' with the form, they take on in the vicinity of 'singularities' of the field of directions. The existence and notion of 'singularities' are notions relative to the field of vectors, defined by the differential equation. The form of 'integral curves' is relative to the solution of this equation. The forms are assuredly complimentary, since the nature of singularities of the field is defined by the forms of the curves in the vicinity. But it is to be remembered that the fact 'field of vectors' on one hand and 'integral curves' on the other are essentially two distinct mathematical realities.

The term holism was coined in 1926 by Jan Smuts. Reductionism is sometimes seen as the opposite of holism. Reductionism in science says that a complex system can be explained by(e) reduction to its fundamental parts. For example, the processes of biology are reducible to chemistry and the laws of chemistry are explained by physics. Social scientist and physician Nicholas A. Christakis explains that "for the last few centuries, the Cartesian in science has been to break(e&eb) matter down into ever smaller bits, in the pursuit of understanding. And this works, to some extent...but putting things back together in order to understand them is harder, and typically comes later in the development of a scientist or in the development of science." Inherently, the definition systematizes the usual procedure that is adopted in science, one namely the case studies and the second the General Theories. The term holism was coined in 1926 by Jan Smuts, a South African statesman, in his book, *Holism and Evolution*. Smuts defined holism as "The tendency in nature to form (eb) wholes that are greater (e) than the sum of the parts through creative evolution."

The idea has ancient roots. Examples of holism can be found throughout human history and in the most diverse socio-cultural contexts, as has been confirmed by many ethnological studies. The French Protestant missionary, Maurice Leenhardt coined the term cosmomorphism to indicate the state of perfect sybiosis (e&eb) with the surrounding environment which characterized the culture of the Melanesians of New Caledonia. For these people, an isolated individual is totally indeterminate, indistinct and featureless until he can find his position within the natural and social world in which he is inserted. The confines between the self and the world are annulled to the point that the material body itself is no guarantee of the sort of recognition of identity which is typical of our own culture. However, the concept of holism also played a pivotal role in Spinoza's philosophy and more recently in that of Hegel and Husserl.

HOLISM IN SCIENCE

In the latter half of the 20th century, holism led to *systems thinking* and its derivatives, like the sciences of chaos and complexity, convolution and complication. Systems in biology, psychology, or sociology are frequently so complex that their behavior is, or appears, "new" or "emergent, embryonic and evolutionary": it cannot be deduced from the properties of the elements alone.

Holism has thus been used as a catchword. This contributed to the resistance encountered by the scientific interpretation of holism, which insists that there are ontological reasons that prevent reductive models in principle from providing efficient algorithms for prediction and prognostication of system behavior in certain classes of systems. Holism has also been described as "a soapy term which evades necessary conflict, variance and deferential differentiation," if not obsequial compliance, conformity and observance thereto the reductionism, Scientific holism holds that the behavior of a system cannot be perfectly predicted, no matter how much data is available. Natural systems can **produce** surprisingly unexpected behavior, and it is suspected that behavior of such systems might be computationally irreducible, which means it would not be possible to even approximate the system state without a full simulation of all the events **occurring** in the system. Key properties of the higher level behavior of certain classes of systems may be mediated by rare "surprises" in the behavior of their elements due to the principle of interconnectivity, thus evading predictions and dodging forecasts except by brute force simulation. Stephen Wolfram has provided such examples with simple cellular automata, whose behavior is in most cases equally simple, but on rare occasions highly unpredictable.

Complexity theory (also called "science of complexity"), is a contemporary heir and legatee of systems thinking. It comprises both computational and holistic, **relational** approaches towards understanding complex adaptive systems and, especially in the latter, its methods can be seen as the polar opposite, conflictingly contrarian to reductive methods. **General theories** of complexity have been proposed, and numerous complexity institutes and departments have sprung up around the world. The Santa Fe Institute is arguably the most famous of them.

IN ANTHROPOLOGY

There is an ongoing dispute as to whether anthropology is intrinsically holistic. Supporters of this concept consider anthropology holistic in two senses. First, it is **concerned with** all human beings across times and places, and with all dimensions of humanity (evolutionary, biophysical, sociopolitical, economic, cultural, psychological, etc.). Further, many academic programs following this approach, take a **"four-field"** approach to anthropology **that encompasses** physical anthropology, archeology, linguistics, and cultural anthropology or anthropology. Some leading anthropologists disagree, and consider anthropological holism to be an artifact, an antiquated object d'art from 19th century social evolutionary thought that inappropriately imposes scientific positivism upon cultural anthropology. The term "holism" is additionally used within social and cultural anthropology to refer to an analysis of a society as a whole which refuses to break society into component parts. One definition says: "as a methodological ideal, processual principle holism implies ... that one does not permit oneself to believe that our own established institutional boundaries (e.g. between politics, sexuality, religion, and economics) necessarily may be found also in foreign societies."

IN BUSINESS

A holistic brand (also holistic branding) trade mark, is considering the entire brand or image of the company. For example a universal brand image across all countries, including everything from advertising styles to the stationery the company has made, to the company colors..Ecologyistheleading and most important approach to holism, as it tries to include biological, chemical, physical and economic views in a given area. The complexity **grows** with the area, so that it is necessary to **reduce the** characteristic of the view in other ways, for example to a specific time of duration. John, Scots born early conservationist wrote, "When we try to pick out anything by itself we find it hitched to everything else in the Universe". More information is to be found in the field of **systems ecology**, a cross-disciplinary field influenced by general systems theory, with all its wide ranging ramifications and broad spectral applications..

IN ECONOMICS

With roots in Schumpeter, the evolutionary approach might be considered the holist theory in economics. They share certain language from the biological evolutionary approach. They take into account how the innovation system, advanced novelty **evolves** over time. Knowledge and know-how, know-who, know-what and know-why **are part of the** whole business economics. Knowledge can also be tacit, as described by Michael Polanyi. These models are open, and consider that it is hard to predict exactly the **impact** of a policy measure. They are also less mathematical.

Semantic holism and confirmation holism

In philosophy, any doctrine, didactic and dogmatic canons that emphasizes the priority of a **whole over its parts** is holism. Some suggest that such a definition owes its origins to a non-holistic view of language and places it in the reductivist camp. Alternately, a 'holistic' definition of holism **denies** the necessity of a **division** between the function of separate parts and the **workings of** the 'whole'. It suggests that the key recognizable characteristic of a concept of holism is a sense of the fundamental truth of any particular experience. This exists in contradistinction to what is perceived as the **reductivist reliance on (e&eb) inductive method** as the key to verification of its concept of how the parts function within the whole.

In the philosophy of language this becomes the claim, called semantic holism, that the meaning of an individual word or sentence can only be understood in terms of its **relations** to a larger body of language, even a whole theory or a whole language. In the philosophy of mind, a mental state may be identified only in terms of its **relations with** others. This is often referred to as "**content holism**" or "holism of the mental". This notion involves the philosophies of such figures as Frege, Wittgenstein and Quine.

Epistemological and confirmation holism are mainstream ideas conventional line of thinking in contemporary philosophy. Ontological holism was espoused by David Bohm in his theory on The Implicate Order.

HEGEL'S HOLISM

Hegel rejected "the fundamentally atomistic conception of the object," (Stern, 38) arguing that "individual objects exist as **manifestations** of indivisible substance-universals, which **cannot be reduced** to a set of properties or attributes; he therefore holds that the object should be treated as an ontologically primary whole." (Stern, 40). In direct opposition to Kant, therefore, "Hegel insists that **the unity we** find in our experience of the world is **not constructed** by us out of a plurality of intuitions." (Stern, 40) In "his ontological scheme a concrete individual is **not reducible** to a plurality of sensible properties, but rather **exemplifies and illustrates** a substance universal." (Stern, 41) His point is that it is "a mistake to treat an organic substance like blood as nothing more than a compound of unchanging chemical elements that can be **separated and united** without being fundamentally altered." (Stern, 103) In Hegel's

view, a substance like blood is thus "more of an organic unity and cannot be understood as just an external composition of the sort of distinct substances that were discussed at the level of chemistry." (Stern, 103) Thus in Hegel's view, blood is blood and cannot be successfully reduced to what we consider are its component parts; we must view it as a whole substance entire unto itself. This is most certainly a fundamentally holistic view.

GEMEINSCHAFT AND GESELLSCHAFT

Émile Durkheim developed a concept of holism which he set as opposite to the notion that a society was nothing more than a simple collection of individuals. In more recent times, Louis has contrasted "holism" (e&eb) to "individualism" as two different forms of societies. According to him, modern humans live in an individualist society, whereas ancient Greek society, for example, could be qualified as "holistic", because the individual found identity in the whole society. Thus, the individual was ready to sacrifice himself or herself for his or her community, as his or her life without the *polis* had no sense whatsoever.

Scholars such as David Bohm and M. I. Sanduk consider the society through the Plasma Physics. From physics point of view, the **interaction of individuals** within a group may lead a continuous model. Therefore for M. I. Sanduk "The nature of fluidity of plasma (ionized gas) arises (eb) from the interaction of its free interactive (e&eb) charges, so the society may behave as a fluid owing to the free interactive individuals. This fluid model may explain many social phenomena like social instability, diffusion, flow, viscosity...*So the society behaves as a sort of intellectual fluid*".

IN PSYCHOLOGY OF PERCEPTION

A major holist movement in the early twentieth century was Gestalt psychology. The claim was that perception is not an aggregation(e&eb) of atomic sense data but a field, in which there is a figure and a ground. Background has holistic effects(e&eb)-some thing like augmented or dissipated reality one talks of in quantum mechanics- on the perceived figure. Gestalt psychologists included Wolfgang Kohler, Max Wertheimer, and Kurt Koffka. Kohler claimed the perceptual fields corresponded to (e&eb) electrical fields in the brain. Karl Lashley did experiments with gold foil pieces inserted in monkey brains purporting to show that such fields did not exist. However, many of the perceptual illusions and visual phenomena, illustrational chart exhibited by the Gestaltists were taken over (often without credit) by later perceptual psychologists. Gestalt psychology had influence on Fritz Perls' gestalt therapy, although some old-line Gestaltists **opposed the association** with counter-cultural and New Age trends later associated with gestalt therapy. Gestalt theory was also **influential** on phenomenology. Aron Gurwitsch wrote on the role of the field of consciousness in gestalt theory in **relation** to phenomenology. Maurice Merleau-Ponty made much use of holistic psychologists such as work of Kurt Goldstein in his "Phenomenology of acuity and perception."

IN TELEOLOGICAL PSYCHOLOGY

Alfred Adler believed that the individual (an integrated whole expressed through a self-consistent unity of *thinking, feeling, and action, moving toward an unconscious, fictional final goal*), must be understood within the larger wholes of society, from the groups to which he **belongs** (starting with his face-to-face relationships), to the larger whole of mankind. The recognition of our social embeddedness and the need for developing an interest in the welfare common well being of others, as well as a respect for nature, is at the heart of Adler's philosophy of living and principles of psychotherapy Edgar Morin, the French philosopher and sociobiologist, can be considered a holist based on the **transdisciplinary** nature of his work. Mel Levine, M.D., author of *A Mind at a Time*, and co-founder (with Charles R. Schwab) of the not-for-profit organization **All Kinds of Minds**, can be considered a holist based on his view of the '**whole child**' as a product (eb) of many systems and his work supporting the educational needs of children through the management of a child's educational profile as a whole rather than isolated weaknesses in that profile. In theological anthropology, which belongs to theology and not to anthropology, holism is the belief that the nature of humans consists of an ultimately divisible¹ union of components such as body, soul and spirit.

IN THEOLOGY

Holistic concepts are strongly represented within the thoughts expressed within Logos (per Heraclitus), Pantheism and Pantheism.

IN NEUROLOGY

A lively debate has run since the end of the 19th century regarding the **functional organization of the well designed brain**. The holistic tradition (e.g., Pierre Marie) maintained that the brain was a homogeneous organ with **no specific subparts** whereas the localizationists (e.g., Paul Broca) argued that the brain was organized in functionally distinct cortical areas which were each specialized to process a given type of information or implement specific mental operations. The controversy was epitomized with the existence of a language area in the brain, nowadays known as the Broca's area. Although Broca's view has gained acceptance, the issue isn't settled insofar as the brain as a whole

is a highly connected(e&eb) organ at every level from the individual neuron to the hemispheres. Architecture is often argued by design academics and those practicing in design to be a holistic enterprise. Used in this context, holism tends to imply an all-inclusive design perspective. This trait is considered exclusive to architecture, distinct from other professions involved in design projects. The Taxonomy of Educational Objectives identifies many levels of cognitive functioning, which can be used to create a more holistic education. In authentic assessment, rather than using computers to score multiple choice tests, a standards based assessment uses trained scorers to score open-response items using holistic scoring methods. In projects such as the North Carolina Writing Project, scorers are instructed not to count errors, or count numbers of points or supporting statements. The scorer is instead instructed to judge holistically whether "as a whole" is it more a "2" or a "3". Critics question whether such a process can be as objective as computer scoring, and the degree to which such scoring methods can result in different scores from different scorers.

MEDICINE

In primary care the term "holistic," has been used to describe approaches that take into account social considerations and other intuitive judgments. The term holism, and so called approaches, appears in psychosomatic medicine in the 1970s, when they were considered one possible way to conceptualize psychosomatic phenomena. Instead of charting one-way causal links from psyche to soma, or vice-versa, it aimed at a systemic model, where multiple biological, psychological and social factors were seen as interlinked. Other, alternative approaches in the 1970s were psychosomatic and somatopsychic approaches, which concentrated on causal links only from psyche to soma, or from soma to psyche, respectively. At present it is commonplace in psychosomatic medicine to state that psyche and soma cannot really e(e&eb)be separated for practical or theoretical purposes. A disturbance on any level - somatic, psychic, or social - will radiate (a tensorial way of representation and configuration essential production and spread) to all the other levels, too. In this sense, psychosomatic thinking is similar to the biopsychosocial model of medicine.

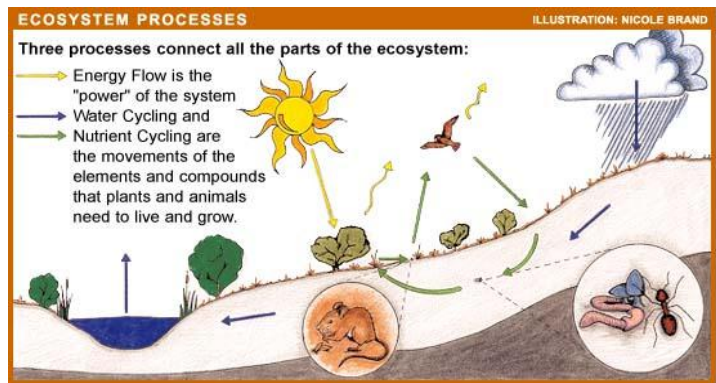
THE FOOD WEB AND OTHER VITAL CYCLES

The energy necessary for all life processes reaches the earth in the form of sunlight.

By photosynthesis green plants convert (e&eb) the light energy into chemical energy, and carbon dioxide and water are transformed(e&eb) into sugar and stored in the plant. Herbivorous animals acquire some of the stored energy by eating the plants; those animals in turn serve as food for, and so pass the energy to, predatory animals. Such sequences, called food chains, overlap at many points, forming so-called food webs. For example, insects are food for reptiles, which are food for hawks. But hawks also feed directly on insects and on other birds that feed on insects, while some reptiles prey on birds. Since a severe loss of the original energy occurs with each transfer from species to species, the ecologist views the food (energy) structure as a pyramid: Each level supports a smaller number and mass of organisms. Thus in a year's time it would take millions of plants weighing tons to feed the several steer weighing a few tons that could support one or two people. The ecological conclusion is that if human beings would eat more plants and fewer animals, food resources would stretch much further. Once the energy for life is spent, it cannot be replenished except by the further exposure of green plants to sunlight.

The chemical materials extracted from the environment and elaborated into living tissue by plants and animals are continually recycled within the ecosystem by such processes as photosynthesis, respiration, nitrogen fixation, and nitrification. These natural processes of withdrawing and returning materials are variously called the carbon cycle, the oxygen cycle, and the nitrogen cycle. Water is also cycled. Evaporation from lakes and oceans forms clouds; the clouds release rain that is taken up by the soil, absorbed by plants, and passed on to feeding animals—which also drink directly from pools and lakes that catch the rain. The water in plant and animal wastes and dead tissue then evaporates and can be recycled. Interference with these vital cycles by disturbance of the environment—for example, by pollution of the air and water—may disrupt the workings of the entire ecosystem. The cycles are facilitated when an ecosystem has a sufficient biological diversity of species to fill its so-called ecological niches, the different functional sites in the environment where organisms can act as producers

of energy, consumers of energy, or decomposers of wastes. Such diversity tends to make a community stable and self-perpetuating.



We acknowledge the following:

MicroID Microbe Web Viewer (By Dr. Steve Spilatro) -
MicroID Microbe Web Quizzer (By Dr. Steve Spilatro) -

A BIRD'S EYE VIEW:

The main concepts we are trying to get across in this section concern how energy moves through an ecosystem. If you can understand this, you are in good shape, because then you have an idea of how ecosystems are balanced, how they may be affected by human activities, and how pollutants will move through an ecosystem.

Roles of Organisms

Organisms can be either **producers or (e&e)bconsumers** in terms of energy flow through an ecosystem. Producers convert energy from the environment into carbon bonds, such as those found in the sugar glucose. *Plants* are the most obvious examples of producers; plants take energy from sunlight and use it to convert carbon dioxide into glucose (or other sugars). Algae and cyan bacteria are also *photosynthetic* producers, like plants. Other producers include bacteria living around deep-sea vents. These bacteria take energy from chemicals coming from the Earth's interior and use it to make sugars. Other bacteria living deep underground can also produce sugars from such inorganic sources. Another word for producers is *autotrophs*.

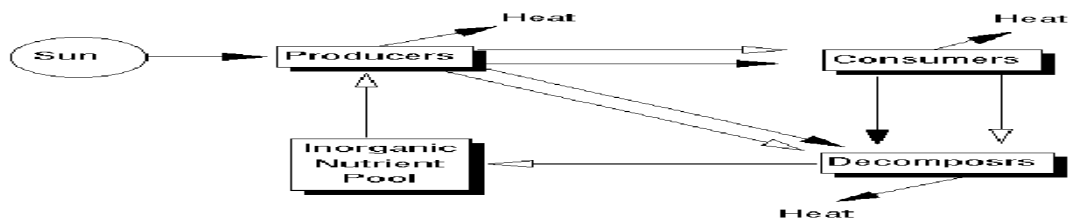
Consumers get their energy from the carbon bonds made by the producers. Another word for a consumer is a *heterotroph*. Based on what they eat, we can distinguish between 4 types of heterotrophs:

| | | |
|--------------|---------------------|------------------|
| consumer | trophic level | food source |
| Herbivores | primary | plants |
| Carnivores | secondary or higher | animals |
| Omnivores | all levels | plants & animals |
| Detritivores | ----- | detritus |

A *trophic level* refers to the organisms position in the food chain. Autotrophs are at the base. Organisms that eat autotrophs are called *herbivores* or *primary consumers*. An organism that eats herbivores is a *carnivore* and a *secondary consumer*. A carnivore which eats a carnivore which eats a herbivore is a *tertiary consumer*, and so on. It is important to note that many animals do not specialize in their diets. *Omnivores* (such as humans) eat both animals and plants. Further, except for some specialists, most carnivores don't limit their diet to organisms of only one trophic level. Frogs, for instance, don't discriminate between herbivorous and carnivorous bugs in their diet. If it's the right size, and moving at

the right distance, chances are the frog will eat it. It's not as if the frog has brain cells to waste wondering if it's going to mess up the food chain by being a secondary consumer one minute and a quaternary consumer the next.

Energy Flow through the Ecosystem



The diagram above shows how both energy and inorganic nutrients flow through the ecosystem. We need to define some terminology first. Energy "flows" through the ecosystem in the form of carbon-carbon bonds. When respiration occurs, the carbon-carbon bonds are broken and the carbon is combined with oxygen to form carbon dioxide. This process releases the energy, which is either used by the organism (to move its muscles, digest food, excrete wastes, think, etc.) or the energy may be lost as heat. The dark arrows represent the movement of this energy. Note that all energy comes from the sun, and that the ultimate fate of all energy in ecosystems is to be lost as heat. Energy does not recycle!!

The other component shown in the diagram is the inorganic nutrients. They are inorganic because they do not contain carbon-carbon bonds. These inorganic nutrients include the phosphorous in your teeth, bones, and cellular membranes; the nitrogen in your amino acids (the building blocks of protein); and the iron in your blood (to name just a few of the inorganic nutrients). The movement of the inorganic nutrients is represented by the open arrows. Note that the autotrophs obtain these inorganic nutrients from the inorganic nutrient pool, which is usually the soil or water surrounding the plants or algae. These inorganic nutrients are passed from organism to organism as one organism is consumed by another. Ultimately, all organisms die and become detritus, food for the decomposers. At this stage, the last of the energy is extracted (and lost as heat) and the inorganic nutrients are returned to the soil or water to be taken up again. The inorganic nutrients are recycled, the energy is not.

Many of us, when we hear the word "nutrient" immediately think of calories and the carbon-carbon bonds that hold the caloric energy. IT IS VERY IMPORTANT that you be careful in your use of the word nutrient in this sense. When writing about energy flow and inorganic nutrient flow in an ecosystem, you must be clear as to what you are referring. Unmodified by "inorganic" or "organic", the word "nutrient" can leave your reader unsure of what you mean. This is one case in which the scientific meaning of a word is very dependent on its context. Another example would be the word "respiration", which to the layperson usually refers to "breathing", but which means "the extraction of energy from carbon-carbon bonds at the cellular level" to most scientists (except those scientists studying breathing, who use respiration in the lay sense).

To summarize: In the flow of energy and inorganic nutrients through the ecosystem, a few generalizations can be made:

The ultimate source of energy (for most ecosystems) is the sun

The ultimate fate of energy in ecosystems is for it to be lost as heat.

Energy and nutrients are passed from organism to organism through the food chain as one organism eats another.

Decomposers remove the last energy from the remains of organisms.

Inorganic nutrients are cycled, energy is not.

FOOD CHAINS AND WEBS:

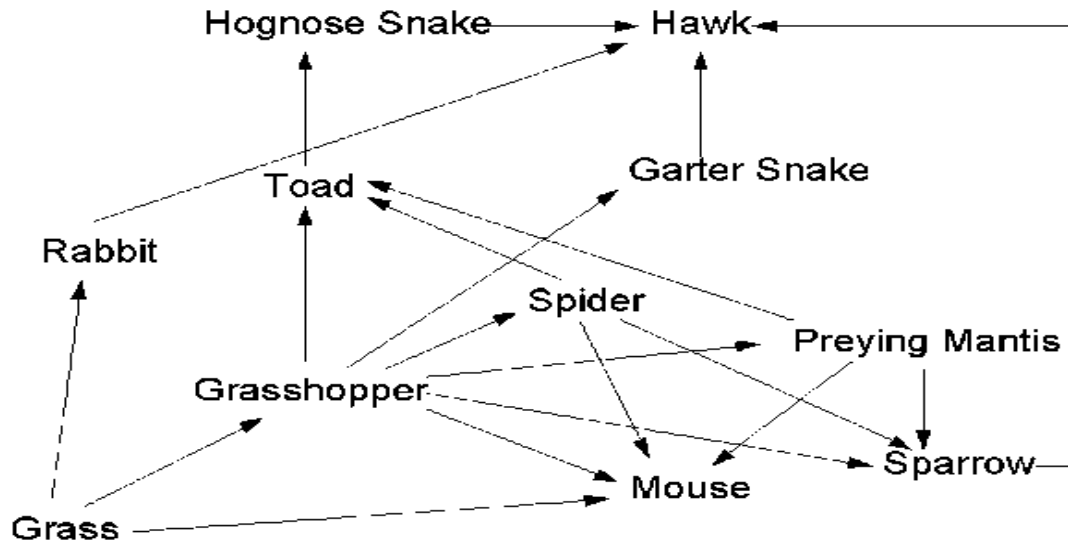
A food chain is the path of food from a given final consumer back to a producer. For instance, a typical food chain in a field ecosystem might be:

grass ---> grasshopper --> mouse ---> snake ---> hawk

Note that even though I said the food chain is the path of food from a given final consumer back to a producer we typically list a food chain from producer on the left (or at the bottom) to final consumer on the right (or at the top). Note to international readers: In Hebrew or Aramaic, or other languages which are read right-to-left, is it customary to list the food chains in the reverse order? By the way, you should be able to look at the food chain above and identify the autotrophs and heterotrophs, and classify each as a herbivore, carnivore, etc. You should also be able to determine that the hawk is a quaternary consumer.

The real world, of course, is more complicated than a simple food chain. While many organisms do specialize in their diets (anteaters come to mind as a specialist), other organisms do not. Hawks don't

limit their diets to snakes, snakes eat things other than mice, mice eat grass as well as grasshoppers, and so on. A more realistic depiction of who eats whom is called a food web; an example is shown below:



It is when we have a picture of a food web in front of us that the definition of food chain makes more sense. We can now see that a food web consists of interlocking food chains, and that the only way to untangle the chains is to trace *back* along a given food chain to its source.

The food webs you see here are *grazing food chains* since at their base are producers which the herbivores then graze on. While grazing food chains are important, in nature they are outnumbered by *detritus-based food chains*. In detritus-based food chains, decomposers are at the base of the food chain, and sustain the carnivores which feed on them. In terms of the weight (or biomass) of animals in many ecosystems, more of their body mass can be traced back to detritus than to living producers.

Pyramids

The concept of biomass is important. It is a general principle that the further removed a trophic level is from its source (detritus or producer), the less biomass it will contain (biomass here would refer to the combined weight of all the organisms in the trophic level). This reduction in biomass occurs for several reasons:

- Not everything in the lower levels gets eaten
- Not everything that is eaten is digested
- Energy is always being lost as heat

It is important to remember that the decrease in number is best detected in terms of biomass. Numbers of organisms are unreliable in this case because of the great variation in the biomass of *individual* organisms. For instance, squirrels feed on acorns. The oak trees in a forest will always outnumber the squirrels in terms of combined weight, but there may actually be more squirrels than oak trees. Remember that an individual oak tree is huge, weighing thousands of kilograms, while an individual squirrel weighs perhaps 1 kilogram at best. There are few exceptions to the pyramid of biomass scheme. One occurs in aquatic systems where the algae may be both outnumbered and outweighed by the organisms that feed on the algae. The algae can support the greater biomass of the next trophic level only because they can reproduce as fast as they are eaten. In this way, they are never completely consumed. It is interesting to note that this exception to the rule of the pyramid of biomass also is a partial exception to at least 2 of the 3 reasons for the pyramid of biomass given above. While not all the algae are consumed, a greater proportion of them are, and while not completely digestible, algae are far more nutritious overall than the average woody plant is (most organisms cannot digest wood and extract energy from it).

A generalization exists among ecologists that on average, about 10% of the energy available in one trophic level will be passed on to the next; this is primarily due to the 3 reasons given above. Therefore, it is also reasonable to assume that in terms of biomass, each trophic level will weigh only about 10% of the level below it, and 10x as much as the level above it. It also seems, however, that every time I go to measure, test, or model this assumption I run into an inconsistency, so take this generalization with a big

grain of salt. Still, it comes in useful In terms of human diet and feeding the world's population, consider this. If we all ate corn, there would be enough food for 10 xs as many of us as compared to a world where we all eat beef (or chicken, fish, pork, etc.). Another way of looking at it is this. Every time you eat meat, you are taking food out of the mouths of 9 other people, who could be fed with the plant material that was fed to the animal you are eating. Of course, it's not quite that simple, but you get the general idea.

Biological Magnification

Biological magnification is the tendency of pollutants to become concentrated in successive trophic levels. Often, this is to the detriment of the organisms in which these materials concentrate, since the pollutants are often toxic.

Biomagnifications occurs when organisms at the bottom of the food chain concentrate the material above its concentration in the surrounding soil or water. Producers, as we saw earlier, take in inorganic nutrients from their surroundings. Since a lack of these nutrients can limit the growth of the producer, producers will go to great lengths to obtain the nutrients. They will spend considerable energy to pump them into their bodies. They will even take up more than they need immediately and store it, since they can't be "sure" of when the nutrient will be available again (of course, plants don't think about such things, but, as it turns out, those plants, which, for whatever reason, tended to concentrate inorganic nutrients have done better over the years). The problem comes up when a pollutant, such as DDT or mercury, is present in the environment. Chemically, these pollutants resemble essential inorganic nutrients and are brought into the producer's body and stored "by mistake". This is the first step in biomagnifications; the pollutant is at a higher concentration inside the producer than it is in the environment.

The second stage of biomagnifications occurs when the producer is eaten. Remember from our discussion of a pyramid of biomass that relatively little energy is available from one trophic level to the next. This means that a consumer (of any level) has to consume a lot of biomass from the lower trophic level. If that biomass contains the pollutant, the pollutant will be taken up in large quantities by the consumer. Pollutants that biomagnify have another characteristic. Not only are they taken up by the producers, but they are absorbed and stored in the bodies of the consumers. This often occurs with pollutants soluble in fat such as DDT or PCB's. These materials are digested from the producer and move into the fat of the consumer. If the consumer is caught and eaten, its fat is digested and the pollutant moves to the fat of the new consumer. In this way, the pollutant builds up in the fatty tissues of the consumers. Water-soluble pollutants usually cannot biomagnify in this way because they would dissolve in the bodily fluids of the consumer. Since every organism loses water to the environment, as the water is lost the pollutant would leave as well. Alas, fat simply does not leave the body.

The "best" example of biomagnifications comes from DDT. This long-lived pesticide (insecticide) has improved human health in many countries by killing insects such as mosquitoes that spread disease. On the other hand, DDT is effective in part because it does not break down in the environment. It is picked up by organisms in the environment and incorporated into fat. Even here, it does no real damage in many organisms (including humans). In others, however, DDT is deadly or may have more insidious, long-term effects. In birds, for instance, DDT interferes with the deposition of calcium in the shells of the bird's eggs. The eggs laid are very soft and easily broken; birds so afflicted are rarely able to raise young and this causes a decline in their numbers. This was so apparent in the early 1960's that it led the scientist Rachel Carson to postulate a "silent spring" without the sound of bird calls. Her book "*Silent Spring*" led to the banning of DDT, the search for pesticides that would not biomagnify, and the birth of the "modern" environmental movement in the 1960's. Birds such as the bald eagle have made comebacks in response to the banning of DDT in the US. Ironically, many of the pesticides which replaced DDT are more dangerous to humans, and, without DDT, disease (primarily in the tropics) claims more human lives.

Summary:

In order for a pollutant to biomagnify, the following conditions must be met:

The pollutant must be long-lived.

The pollutant must be concentrated by the producers.

The pollutant must be fat-soluble.

Human vs. Natural Food Chains

Human civilization is dependent on agriculture. Only with agriculture can a few people feed the rest of the population; the part of the population freed from raising food can then go on to do all the things we associate with civilization. Agriculture means manipulating the environment to favor plant species that we can eat. In essence, humans manipulate competition, allowing favored species (crops) to thrive and thwarting species which might otherwise crowd them out (weeds). In essence, with agriculture we are creating a very simple ecosystem. At most, it has only three levels - producers (crops), primary

consumers (livestock, humans) and secondary consumers (humans). This means that little energy is lost between trophic levels, since there are fewer trophic levels present.

This is good for humans, but what type of "ecosystem" have we created? Agricultural ecosystems have several problems. First, we create *monocultures*, or fields with only one crop. This is simplest for planting, weeding, and harvesting, but it also packs many similar plants into a small area, creating a situation ideal for disease and insect pests. In natural ecosystems, plants of one species are often scattered. Insects, which often specialize on feeding on a particular plant species, have a hard time finding the scattered plants. Without food, the insect populations are kept in check. In a field of corn however, even the most inept insect can find a new host plant with a jump in any direction. Likewise, disease is more easily spread if the plants are in close proximity. It takes lots of chemicals (pesticides) to keep a monoculture going.

Another problem with human agriculture is that we rely on relatively few plants for food. If the corn and rice crops failed worldwide in the same year, we would be hard-pressed to feed everyone (not that we're doing a great job of it now). Natural ecosystems usually have alternate sources of food available if one fails.

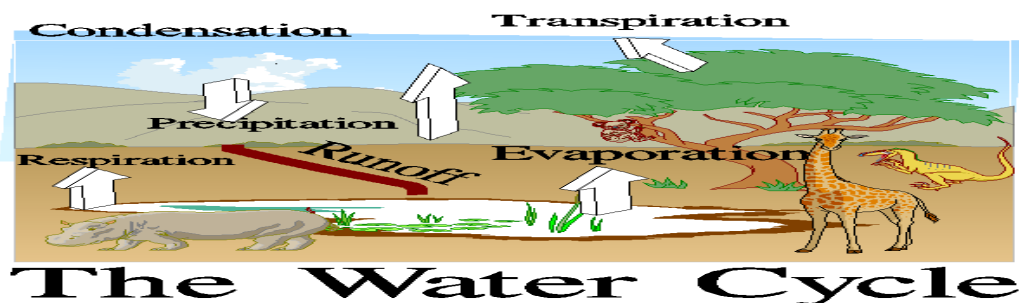
A final problem associated with agro ecosystems is the problem of inorganic nutrient recycling. In a natural ecosystem, when a plant dies it falls to the ground and rots, and its inorganic nutrients are returned to the soil from which they were taken. In human agriculture, however, we harvest the crop, truck it away, and flush it down the toilet to be run off in the rivers to the ocean. Aside from the water pollution problems this causes, it should be obvious to you that the nutrients are not returned to the fields. They have to be replaced with chemical fertilizers, and that means mining, transportation, electricity, etc. Also, the chemical fertilizers tend to run off the fields (along with soil disrupted by cultivation) and further pollute the water.

Some solutions are at hand, but they bring on new problems, too. No-till farming uses herbicides to kill plants in a field; the crop is then planted through the dead plants without plowing up the soil. This reduces soil and fertilizer erosion, but the herbicides themselves may damage ecosystems. In many areas, sewage sludge is returned to fields to act as a fertilizer. This reduces the need for chemical fertilizers, but still requires a lot of energy to haul the sludge around. Further, if one is not careful, things such as household chemicals and heavy metals may contaminate the sewage sludge and biomagnify in the crops which we would then eat.

Biogeochemical Cycles.

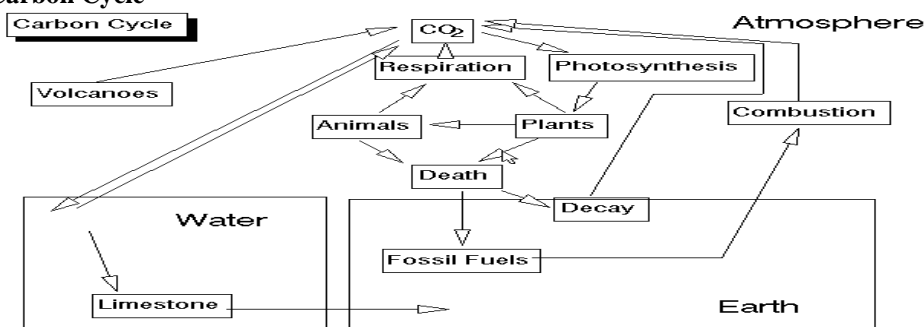
We have already seen that while energy does not cycle through an ecosystem, chemicals do. The inorganic nutrients cycle through more than the organisms, however, they also enter into the atmosphere, the oceans, and even rocks. Since these *chemicals* cycle through both the *biological* and the *geological* world, we call the overall cycle's biogeochemical cycles. Each chemical has its own unique cycle, but all of the cycles do have some things in common. *Reservoirs* are those parts of the cycle where the chemical is held in large quantities for long periods of time. In *exchange pools*, on the other hand, the chemical is held for only a short time. The length of time a chemical is held in an exchange pool or a reservoir is termed its *residence* time. The oceans are a reservoir for water, while a cloud is an exchange pool. Water may reside in an ocean for thousands of years, but in a cloud for a few days at best. The biotic community includes all living organisms. This community may serve as an exchange pool (although for some chemicals like carbon, bound in a sequoia for a thousand years, it may seem more like a reservoir), and also serve to move chemicals from one stage of the cycle to another. For instance, the trees of the tropical rain forest bring water up from the forest floor to be evaporated into the atmosphere. Likewise, coral endosymbionts take carbon from the water and turn it into limestone rock. The energy for most of the transportation of chemicals from one place to another is provided either by the sun or by the heat released from the mantle and core of the Earth.

While all inorganic nutrients cycle, we will focus on only 4 of the most important cycles - water, carbon (and oxygen), nitrogen, and phosphorous. Water Cycle:



Key Features: In the water cycle, energy is supplied by the sun, which drives evaporation whether it be from ocean surfaces or from treetops. The sun also provides the energy which drives the weather systems which move the water vapor (clouds) from one place to another (otherwise, it would only rain over the oceans). Precipitation occurs when water condenses from a gaseous state in the atmosphere and falls to earth. Evaporation is the reverse process in which liquid water becomes gaseous. Once water condenses, gravity takes over and the water is pulled to the ground. Gravity continues to operate, either pulling the water underground (groundwater) or across the surface (runoff). In either event, gravity continues to pull water lower and lower until it reaches the oceans (in most cases; the Great Salt Lake, Dead Sea, Caspian Sea, and other such depressions may also serve as the lowest basin into which water can be drawn). Frozen water may be trapped in cooler regions of the Earth (the poles, glaciers on mountaintops, etc.) as snow or ice, and may remain as such for very long periods of time. Lakes, ponds, and wetlands form where water is temporarily trapped. The oceans are salty because any weathering of minerals that occurs as the water runs to the ocean will add to the mineral content of the water, but water cannot leave the oceans except by evaporation, and evaporation leaves the minerals behind. Thus, rainfall and snowfall are comprised of relatively clean water, with the exception of pollutants (such as acids) picked up as the water falls through the atmosphere. Organisms play an important role in the water cycle. As you know, most organisms contain a significant amount of water (up to 90% of their body weight). This water is not held for any length of time and moves out of the organism rather quickly in most cases. Animals and plants lose water through evaporation from the body surfaces, and through evaporation from the gas exchange structures (such as lungs). In plants, water is drawn in at the roots and moves to the gas exchange organs, the leaves, where it evaporates quickly. This special case is called transpiration because it is responsible for so much of the water that enters the atmosphere. In both plants and animals, the breakdown of carbohydrates (sugars) to produce energy (respiration) produces both carbon dioxide and water as waste products. Photosynthesis reverses this reaction, and water and carbon dioxide are combined to form carbohydrates. Now you understand the relevance of the term carbohydrate; it refers to the combination of carbon and water in the sugars we call carbohydrates.

Carbon Cycle



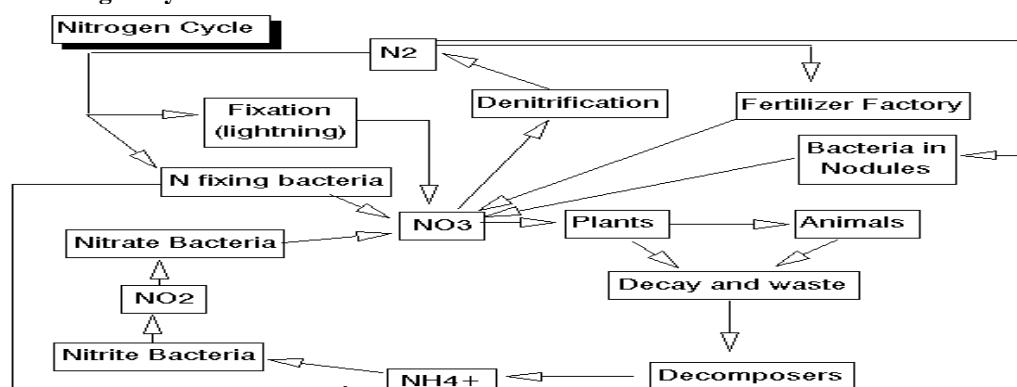
Once you understand the water cycle, the carbon cycle is relatively simple. From a biological perspective, the key events here are the complementary reactions of respiration and photosynthesis. Respiration takes carbohydrates and oxygen and combines them to produce carbon dioxide, water, and energy. Photosynthesis takes carbon dioxide and water and produces carbohydrates and oxygen. The outputs of respiration are the inputs of photosynthesis, and the outputs of photosynthesis are the inputs of respiration. The reactions are also complementary in the way they deal with energy. Photosynthesis takes energy from the sun and stores it in the carbon-carbon bonds of carbohydrates; respiration releases that energy. Both plants and animals carry on respiration, but only plants (and other producers) can carry on photosynthesis. The chief reservoirs for carbon dioxide are in the oceans and in rock. Carbon dioxide

dissolves readily in water. Once there, it may precipitate (fall out of solution) as a solid rock known as calcium carbonate (limestone). Corals and algae encourage this reaction and build up limestone reefs in the process. On land and in the water, plants take up carbon dioxide and convert it into carbohydrates through photosynthesis. This carbon in the plants now has 3 possible fates. It can be liberated to the atmosphere by the plant through respiration; it can be eaten by an animal, or it can be present in the plant when the plant dies. Animals obtain all their carbon in their food, and, thus, all carbon in biological systems ultimately comes from plants (autotrophs). In the animal, the carbon also has the same 3 possible fates. Carbon from plants or animals that is released to the atmosphere through respiration will either be taken up by a plant in photosynthesis or dissolved in the oceans. When an animal or a plant dies, 2 things can happen to the carbon in it. It can either be respired by decomposers (and released to the atmosphere), or it can be buried intact and ultimately form coal, oil, or natural gas (fossil fuels). The fossil fuels can be mined and burned in the future; releasing carbon dioxide to the atmosphere. Otherwise, the carbon in limestone or other sediments can only be released to the atmosphere when they are sub ducted and brought to volcanoes, or when they are pushed to the surface and slowly weathered away. Humans have a great impact on the carbon cycle because when we burn fossil fuels we release excess carbon dioxide into the atmosphere. This means that more carbon dioxide goes into the oceans, and more is present in the atmosphere. The latter condition causes global warming, because the carbon dioxide in the atmosphere allows more energy to reach the Earth from the sun than it allows to escape from the Earth into space.

The Oxygen Cycle:

If you look back at the carbon cycle, you will see that we have also described the oxygen cycle, since these atoms often are combined. Oxygen is present in the carbon dioxide, in the carbohydrates, in water, and as a molecule of two oxygen atoms. Oxygen is released to the atmosphere by autotrophs during photosynthesis and taken up by both autotrophs and heterotrophs during respiration. In fact, all of the oxygen in the atmosphere is *biogenic*; that is, it was released from water through photosynthesis by autotrophs. It took about 2 billion years for autotrophs (mostly cyanobacteria) to raise the oxygen content of the atmosphere to the 21% that it is today; this opened the door for complex organisms such as multicellular animals, which need a lot of oxygen.

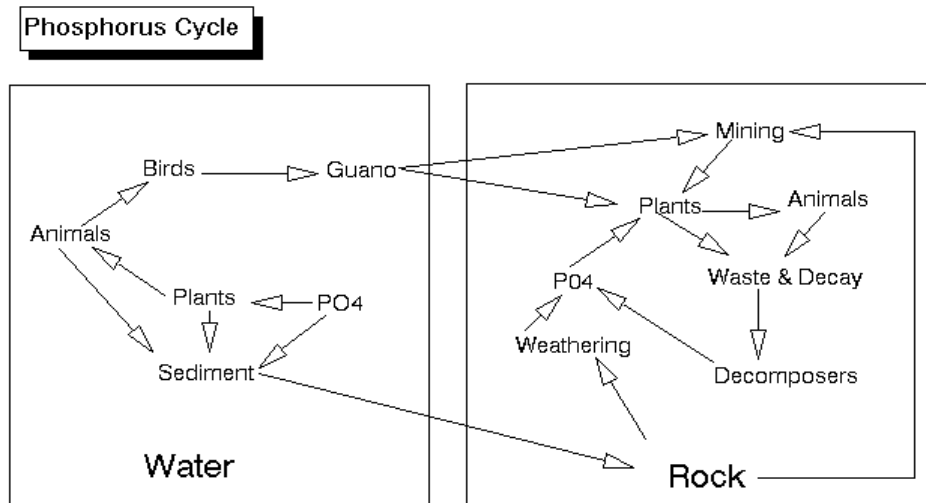
The Nitrogen Cycle:



The nitrogen cycle is one of the most difficult of the cycles to learn, simply because there are so many important forms of nitrogen, and because organisms are responsible for each of the inter conversions. Remember that nitrogen is critically important in forming the amino portions of the amino acids which in turn form the proteins of your body. Proteins make up skin and muscle, among other important structural portions of your body, and all enzymes are proteins. Since enzymes carry out almost all of the chemical reactions in your body, it's easy to see how important nitrogen is. The chief reservoir of nitrogen is the atmosphere, which is about 78% nitrogen. It is here we reach one of the limits of the hypertext language currently (1995-1996) most in vogue on the WWW. This version does not allow for superscripts or subscripts, so I will have to stick to the longer chemical names. Nitrogen gas in the atmosphere is composed of two nitrogen atoms bound to each other. It is a pretty non-reactive gas; it takes a lot of energy to get nitrogen gas to break up and combine with other things, such as carbon or oxygen. Nitrogen gas can be taken from the atmosphere (fixed) in two basic ways. First, lightning provides enough energy to "burn" the nitrogen and fix it in the form of nitrate, which is a nitrogen with three oxygens attached. This process is duplicated in fertilizer factories to produce nitrogen fertilizers. Biology 101 students will also recall the experiments of Stanley Miller, who used electrical discharges to show how nitrogen in the Earth's early atmosphere might have combined to form amino acids. The other form of nitrogen fixation is by nitrogen fixing bacteria, who use special enzymes instead of the extreme amount of energy found in lightning to fix nitrogen. These nitrogen-fixing bacteria come in

three forms: some are free-living in the soil; some form symbiotic, mutualistic associations with the roots of bean plants and other legumes (rhizobial bacteria); and the third form of nitrogen-fixing bacteria are the photosynthetic cyanobacteria (blue-green algae) which are found most commonly in water. All of these fix nitrogen, either in the form of nitrate or in the form of ammonia (nitrogen with 3 hydrogen attached). Most plants can take up nitrate and convert it to amino acids. Animals acquire all of their amino acids when they eat plants (or other animals). When plants or animals die (or release waste) the nitrogen is returned to the soil. The usual form of nitrogen returned to the soil in animal wastes or in the output of the decomposers, is ammonia. Ammonia is rather toxic, but, fortunately there are nitrite bacteria in the soil and in the water which take up ammonia and convert it to nitrite, which is nitrogen with two oxygens. Nitrite is also somewhat toxic, but another type of bacteria, nitrate bacteria, take nitrite and convert it to nitrate, which can be taken up by plants to continue the cycle. We now have a cycle set up in the soil (or water), but what returns nitrogen to the air? It turns out that there are denitrifying bacteria which take the nitrate and combine the nitrogen back into nitrogen gas.

The nitrogen cycle has some important practical considerations, as anyone who has ever set up a saltwater fish tank has found out. It takes several weeks to set up such a tank, because you must have sufficient numbers of nitrite and nitrate bacteria present to detoxify the ammonia produced by the fish and decomposers in the tank. Otherwise, the ammonia levels in the tank will build up and kill the fish. This is usually not a problem in freshwater tanks for two reasons. One, the pH in a freshwater tank is at a different level than in a saltwater tank. At the pH of a freshwater tank, ammonia is not as toxic. Second, there are more multicellular plant forms that can grow in freshwater, and these plants remove the ammonia from the water very efficiently. It is hard to get enough plants growing in a saltwater tank to detoxify the water in the same way



The phosphorous cycle is the simplest of the cycles that we will examine (I like to save the simplest for the last sometimes; it's like a cool-down period after a long run). For our purposes, phosphorous has only one form, phosphate, which is a phosphorous atom with 4 oxygen atoms. This heavy molecule never makes its way into the atmosphere, it is always part of an organism, dissolved in water, or in the form of rock. When rock with phosphate is exposed to water (especially water with a little acid in it), the rock is weathered out and goes into solution. Autotrophs take this phosphorous up and use it in a variety of ways. It is an important constituent of cell membranes, DNA, RNA, and, of course ATP, which, after all, stands for adenosine triphosphate. Heterotrophs (animals) obtain their phosphorous from the plants they eat, although one type of heterotroph, the fungi, excel at taking up phosphorous and may form mutualistic symbiotic relationships with plant roots. These relationships are called *mycorrhizae*; the plant gets phosphate from the fungus and gives the fungus sugars in return. Animals, by the way, may also use phosphorous as a component of bones, teeth and shells. When animals or plants die (or when animals defecate), the phosphate may be returned to the soil or water by the decomposers. There, it can be taken up by another plant and used again. This cycle will occur over and over until at last the phosphorous is lost at the bottom of the deepest parts of the ocean, where it becomes part of the sedimentary rocks forming there. Ultimately, this phosphorous will be released if the rock is brought to the surface and weathered. Two types of animals play a unique role in the phosphorous cycle. Humans often mine rock rich in phosphorous. For instance, in Florida, which was once sea floor, there are extensive phosphate mines. The phosphate is then used as fertilizer. This mining of phosphate and use of the phosphate as fertilizer greatly accelerates the phosphorous cycle and may cause local overabundance

of phosphorous, particularly in coastal regions, at the mouths of rivers, and anyplace where there is a lot of sewage released into the water (the phosphate placed on crops finds its way into our stomachs and from there to our toilets). Local abundance of phosphate can cause overgrowth of algae in the water; the algae can use up all the oxygen in the water and kill other aquatic life. This is called eutrophication. The other animals that play a unique role in the phosphorous cycle are marine birds. These birds take phosphorous containing fish out of the ocean and return to land, where they defecate. Their guano contains high levels of phosphorous and in this way marine birds return phosphorous from the ocean to the land. The guano is often mined and may form the basis of the economy in some areas!

PRIGOGINE;'S STRUCTURES:

Prigogine and coworkers proposed a Lowville space extension of quantum mechanics aimed to solving the arrow of time problem of thermodynamics and the measurement problem of quantum mechanics. He also co-authored several books with Isabelle Stengers', including *End of Certainty* and *La Nouvelle Alliance* (*The New Alliance*).

The End of Certainty

In his 1997 book, *The End of Certainty*, Prigogine contends that determinism is no longer a viable scientific belief. "The more we know about our universe, the more difficult it becomes to believe in determinism." This is a major departure from the approach of Newton, Einstein and Schrödinger, all of whom expressed their theories in terms of deterministic equations. According to Prigogine, determinism loses its explanatory power in the face of irreversibility and instability.

Prigogine traces the dispute over determinism back to Darwin, whose attempt to explain individual variability according to evolving populations inspired Ludwig Boltzmann to explain the behavior of gases in terms of populations of particles rather than individual particles. This led to the field of statistical mechanics and the realization that gases undergo irreversible processes. In deterministic physics, all processes are time-reversible, meaning that they can proceed backward as well as forward through time. As Prigogine explains, determinism is fundamentally a denial of the arrow of time. With no arrow of time, there is no longer a privileged moment known as the "present," which follows a determined "past" and precedes an undetermined "future." All of time is simply given, with the future as determined or undetermined as the past. With irreversibility, the arrow of time is reintroduced to physics. Prigogine notes numerous examples of irreversibility, including diffusion, radioactive decay, solar radiation, weather and the emergence and evolution of life. Like weather systems, organisms are unstable systems existing far from thermodynamic equilibrium. Instability resists standard deterministic explanation. Instead, due to sensitivity to initial conditions, unstable systems can only be explained statistically, that is, in terms of probability.

Prigogine asserts that Newtonian physics has now been "extended" three times with the use of the wave function in quantum mechanics, then with the introduction of space-time in general relativity and finally with the recognition of indeterminism in the study of unstable systems.

In his celebrated paper Adolf Haimovici (1), studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of a system of oxygen consumption by terrestrial organisms. Notations are changed towards the end of obtaining higher number of equations in the holistic study of the global climate models. Quintessentially, Haimovician diurnal dynamics are used to draw interesting inferences, from the simple fact that terrestrial organisms consume oxygen due to cellular respiration, Decomposer Organisms (DO) disintegrate Dead Organic Matter (DOM and Green Plants (GP) use nutrients.

NOTATION FOR THE HOLISTIC SYSTEM TERRESTRIAL ORGANISMS (TO)-OXYGEN CONSUMPTION(OC);DEAD ORGANIC MATTER(DOM)-DECOMPOSER ORGANISM (DO); GREEN PLANTS(GP)-NUTRIENTS(NR)

G_{13} : Quantum of oxygen consumption due to cellular respiration in category 1 of terrestrial organism

G_{14} : Quantum of oxygen consumption due to cellular respiration in category 2 of terrestrial organism

G_{15} : Quantum of oxygen consumption due to cellular respiration in category 3 of terrestrial organism

T_{13} : Balance standing in the category 1 of terrestrial organism

T_{14} : Balance standing in the category 2 of terrestrial organism

T_{15} : Balance standing in the category 3 of terrestrial organism

G_{16} : Quantum of dead organic matter in category 1

G_{17} : Quantum of dead organic matter in category 2

G_{18} : Quantum of dead organic matter in category 3

T_{16} : Balance of decomposer organisms (DO) standing in the category 1

T_{17} : Balance of decomposer organisms standing in the category 2 vis-à-vis that of terrestrial organism

T_{18} : Balance of decomposer organisms standing in the category 3 vis-à-vis of terrestrial organism

G_{20} : Quantum of NR (Nutrients) vis-à-vis -Dead Organic Matter (DOM)- vis-à-vis oxygen consumption (OC)

G_{21} : Quantum of Nutrients(NR) vis-à-vis -DOM vis-à-vis oxygen consumption (OC) due to cellular respiration in category 2 of terrestrial organism

G_{22} : Quantum of Nutrients(NR-) Dead Organic Matter(DOM) vis-à-vis - oxygen consumption (OC) due to cellular respiration in category 3 of terrestrial organism(TO)

T_{20} : Balance standing in the category 1 of Green plants(GP) vis-à-vis terrestrial organism

T_{21} : Balance standing in the category 2 of Green Plants (GP)that corresponds to the concomitant category of terrestrial organism(TO)

T_{22} : Balance standing in the category 3 of Green Plants(GP) vis-à-vis terrestrial organism(TO) concomitant with the stratification of oxygen consumption(OC)

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$ are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$ are dissipation coefficients

TERRESTRIAL ORGANISM –OXYGEN CONSUMPTION SYSTEM

GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor attributable to cellular respiration of terrestrial organism, to the dissipation of oxygen consumption

$-(b''_{13})^{(1)}(G, t) =$ First detrition factor contributed by oxygen consumption to the dissipation of terrestrial organisms

DEAD ORGANIC MATTER-DECOMPOSER ORGANISMS-GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18}$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18}$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor attributable to Dead Organic Matter (DOM) due to Decomposer Organisms (DO). Dead Organic Matter is vis-à-vis Oxygen Consumption in the first category of classification done erstwhile

$-(b''_{16})^{(2)}((G_{19}), t) =$ First detrition factor contributed by disintegration of Dead Organic Matter (DOM) Decomposer Organisms (DO), This Decomposer Organism is vis-à-vis Terrestrial Organism Classification that dissipated Oxygen Consumption.

GREEN PLANTS VIS-À-VIS NUTRIENTS

GOVERNING EQUATIONS

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22}$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22}$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor attributable to Green Plants exhausting Nutrients. Green Plants correspond to the classification of Oxygen Consumption in the first schedule and Dead Organic Matter (DOM) in the Second schedule.

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detrition factor contributed by Green Plants (GP) utilizing Nutrients (NR). Nutrients correspond to the Terrestrial Organism in the first constitution group and Decomposer Organisms (DO) that disintegrate in the second scroll. Oxygen consumption to the dissipation of

terrestrial organisms

GOVERNING EQUATIONS OF THE SYSTEM TERRESTRIAL ORGANISMS(TO)-OXYGEN CONSUMPTION(OC);DEAD ORGANIC MATTER(DOM)-DECOMPOSER ORGANISM (DO); GREEN PLANTS(GP)-NUTRIENTS(NR)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)} \right] G_{15}$$

Where $\boxed{(a'_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a'_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a'_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b''_{13})^{(1,1)}(G, t)} \boxed{-(b''_{13})^{(1,1,1)}(G, t)} \right] T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b''_{14})^{(1,1)}(G, t)} \boxed{-(b''_{14})^{(1,1,1)}(G, t)} \right] T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \boxed{-(b''_{15})^{(1,1)}(G, t)} \boxed{-(b''_{15})^{(1,1,1)}(G, t)} \right] T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$ are third detritions coefficients for category 1,2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)} \right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3)}(G_{23}, t)}$ are second detritions coefficients for category 1, 2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} \right] G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} \right] G_{22}$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)} \right] G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)} \right] G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)} \right] G_{18}$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are first augmentation coefficients for category 1, 2 and 3

And $\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b''_{13})^{(1,1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)} \right] T_{20}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b''_{14})^{(1,1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)} \right] T_{21}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \boxed{-(b''_{15})^{(1,1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)} \right] T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1)}(G, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19}, t)}$ are third detrition coefficients for category 1, 2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21}, t)} \boxed{+(a''_{16})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} \right] G_{20}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} \right] G_{21}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} \boxed{+(a''_{17})^{(2,2)}(T_{17}, t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} \right] G_{22}$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\boxed{(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1,1)}(G, t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}} \right] T_{16}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\boxed{(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1,1)}(G, t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}} \right] T_{17}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\boxed{(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)} \boxed{-(b''_{15})^{(1,1,1)}(G, t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}} \right] T_{18}$$

where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G, t)}$ are second detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23}, t)}$ are third detrition coefficients for category 1,2 and 3

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\boxed{(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)} \boxed{+(a''_{20})^{(3,3)}(T_{21}, t)} \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}} \right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\boxed{(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)} \boxed{+(a''_{21})^{(3,3)}(T_{21}, t)} \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}} \right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\boxed{(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)} \boxed{+(a''_{22})^{(3,3)}(T_{21}, t)} \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}} \right] G_{15}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{(a''_{15})^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3)}(T_{21}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\boxed{(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{13})^{(1,1,1,1)}(G, t)}} \right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\boxed{(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{14})^{(1,1,1,1)}(G, t)}} \right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\boxed{(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)} \boxed{-(b''_{15})^{(1,1,1,1)}(G, t)}} \right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1)}(G, t)$ are third detritions coefficients for category 1, 2 and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0$,

$i, j = 13, 14, 15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$

$\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$ are positive constants and $i = 13, 14, 15$

They satisfy Lipschitz condition:

$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\hat{M}_{13})^{(1)}t}$

$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} ||G - G'| e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with the constants $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$,

satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$

(F) $(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16, 17, 18$

(G) The functions $(a''_i)^{(2)}, (b''_i)^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$:

$$(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$$

$$(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b'_i)^{(2)} \leq (\hat{B}_{16})^{(2)}$$

(H) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)}$

$$\lim_{G \rightarrow \infty} (b''_i)^{(2)}(G_{19}, t) = (r_i)^{(2)}$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T'_{17}, t) - (a''_i)^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T'_{17}| e^{-(\hat{M}_{16})^{(2)}t}$$

$$|(b''_i)^{(2)}((G_{19})', t) - (b''_i)^{(2)}((G_{19}), T_{19})| < (\hat{k}_{16})^{(2)} \|(G_{19}) - (G_{19})'\| e^{-(\hat{M}_{16})^{(2)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(2)}(T'_{17}, t)$ and $(a''_i)^{(2)}(T_{17}, t)$. (T'_{17}, t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a''_i)^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a''_i)^{(2)}(T_{17}, t)$, the first augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

(J) **Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18$,

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a'_i)^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b'_i)^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1$$

(K) $(a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0,$

$$i, j = 20, 21, 22$$

(L) The functions $(a''_i)^{(3)}, (b''_i)^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$(M) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$ are positive constants and $i = 20, 21, 22$

They satisfy Lipschitz condition:

$$|(a_i'')^{(3)}(T'_{21}, t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T'_{21} - T_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}((G_{23})', t) - (b_i'')^{(3)}(G_{23}, T)| < (\hat{k}_{20})^{(3)} \|G_{23} - (G_{23})'\| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T'_{21}, t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T'_{21}, t) and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient would be absolutely continuous.

(N) **Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

(O) There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the **inequalities (N)**

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Theorem 1: if the conditions (A)-(O) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad T_i(0) = T_i^0 > 0$$

Definition of $G_i(0), T_i(0)$ $i=16, 17, 18$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad T_i(0) = T_i^0 > 0$$

Definition of $G_i(0), T_i(0)$ for $i=20, 21, 22$

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t}$$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t [(a_{13})^{(1)} G_{14}(s_{(13)}) - ((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})] G_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t [(a_{14})^{(1)} G_{13}(s_{(13)}) - ((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}))] G_{14}(s_{(13)})] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t [(a_{15})^{(1)} G_{14}(s_{(13)}) - ((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}))] G_{15}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t [(b_{13})^{(1)} T_{14}(s_{(13)}) - ((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{13}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t [(b_{14})^{(1)} T_{13}(s_{(13)}) - ((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{14}(s_{(13)})] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t [(b_{15})^{(1)} T_{14}(s_{(13)}) - ((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}))] T_{15}(s_{(13)})] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t [(a_{16})^{(2)} G_{17}(s_{(16)}) - ((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})] G_{16}(s_{(16)})] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t [(a_{17})^{(2)} G_{16}(s_{(16)}) - ((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}))] G_{17}(s_{(16)})] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t [(a_{18})^{(2)} G_{17}(s_{(16)}) - ((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}))] G_{18}(s_{(16)})] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t [(b_{16})^{(2)} T_{17}(s_{(16)}) - ((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}))] T_{16}(s_{(16)})] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t [(b_{17})^{(2)} T_{16}(s_{(16)}) - ((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}))] T_{17}(s_{(16)})] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying the system into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}s_{(13)}} \right) \right] ds_{(13)} = \\ (1 + (a_{13})^{(1)}t)G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)}t} - 1 \right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying the system into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}s_{(16)}} \right) \right] ds_{(16)} = \\ (1 + (a_{16})^{(2)}t)G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)}t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0)e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[((\hat{P}_{16})^{(2)} + G_{17}^0)e^{-\frac{((\hat{P}_{16})^{(2)} + G_{17}^0)}{G_{17}^0}} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying the system into itself. Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}s(20)} \right) \right] dS_{(20)} = \\ (1 + (a_{20})^{(3)}t)G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)}t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^0)e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[((\hat{P}_{20})^{(3)} + G_{21}^0)e^{-\frac{((\hat{P}_{20})^{(3)} + G_{21}^0)}{G_{21}^0}} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0 \right) e^{-\frac{((\hat{P}_{13})^{(1)} + G_j^0)}{G_j^0}} \right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0 \right) e^{-\frac{((\hat{Q}_{13})^{(1)} + T_j^0)}{T_j^0}} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying the system into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote

Definition of \tilde{G}, \tilde{T} : $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\hat{M}_{13})^{(1)}s(13)} e^{(\hat{M}_{13})^{(1)}s(13)} dS_{(13)} +$$

$$\int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\hat{M}_{13})^{(1)}s(13)} e^{-(\hat{M}_{13})^{(1)}s(13)} +$$

$$(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) | G_{13}^{(1)} - G_{13}^{(2)} | e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} + G_{13}^{(2)} | (a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)}(T_{14}^{(2)}, s_{(13)}) | e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)} t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}} ((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}))$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ and $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$, $i = 13, 14, 15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})) ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)} t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{13})^{(1)})_1, ((\widehat{M}_{13})^{(1)})_2$ and $((\widehat{M}_{13})^{(1)})_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < (\widehat{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b'_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = \log \frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b'_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of governing equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose

$(\widehat{P}_{16})^{(2)}$ and $(\widehat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[(\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{16})^{(2)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{16})^{(2)}$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} \left[((\widehat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{16})^{(2)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{16})^{(2)} \right] \leq (\widehat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying the system into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote

Definition of $\widetilde{G}_{19}, \widetilde{T}_{19}$: $(\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$\begin{aligned} |\widetilde{G}_{16}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} ds_{(16)} + \\ &\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} + \\ &(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} + \\ &G_{16}^{(2)} | (a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)}) | e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)} \end{aligned}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left| (G_{19})^{(1)} - (G_{19})^{(2)} \right| e^{-(\widehat{M}_{16})^{(2)}t} \leq \frac{1}{(\widehat{M}_{16})^{(2)}} \left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)} (\widehat{k}_{16})^{(2)} \right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)}); (G_{19})^{(2)}, (T_{19})^{(2)} \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$, $i = 16, 17, 18$ depend only on T_{17} and respectively on (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a'_i)^{(2)} - (a''_i)^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1$, $((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$:

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < ((\widehat{M}_{16})^{(2)})_1$ it follows $\frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)}G_{17}$ and by integrating

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 5: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$ then $T_{17} \rightarrow \infty$.

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b''_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2} \right)$, $t = \log \frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded.

The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b''_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of governing equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} , \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[(\widehat{P}_{20})^{(3)} + ((\widehat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{20})^{(3)}$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} \left[((\widehat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)} \right] \leq (\widehat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$d\left((G_{23})^{(1)}, (T_{23})^{(1)}, (G_{23})^{(2)}, (T_{23})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{20})^{(3)}t} \right\}$$

Indeed if we denote

$$\underline{\text{Definition of}} \widetilde{G}_{23}, \widetilde{T}_{23} : (\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}(G_{23}, T_{23})$$

It results

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_{20}^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{-(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\bar{M}_{20})^{(3)}s_{(20)}} e^{(\bar{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(\bar{M}_{20})^{(3)}t} &\leq \\ &\frac{1}{(\bar{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)} \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$ and $(\widehat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(3)} - (a''_i)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \} ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i^{(3)})t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2$ and $((\widehat{M}_{20})^{(3)})_3$:

Remark 3: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$$G_{20} < ((\widehat{M}_{20})^{(3)}) \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 5: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(3)})((G_{23})(t), t) = (b'_{21})^{(3)}$ then $T_{21} \rightarrow \infty$.

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), t = \log \frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t \rightarrow \infty} ((b_{22}'')^{(3)})((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions of resultant equations of the governing equations in the forging:

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

(b) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations

$$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0 \text{ and } (b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

Then the solution satisfies the inequalities

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

$$\text{Where } (S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solution equations for Terrestrial Organism(TO) and Oxygen Consumption(OC)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

(d) $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) - (b''_{17})^{(2)}(G_{19}, t) \leq -(\tau_1)^{(2)}$$

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:-

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)},$$

and
$$(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)}$$

and analogously

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and
$$(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}$$

Then the solution satisfies the inequalities

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined in the foregoing

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:-

$$\text{Where } (S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \text{ and } (R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solution equations for the concatenated set of equations of Dead Organic Matter (DOM) and Decomposer Organisms(DO):

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

(b) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined in the foregoing

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}}$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Proof : From solution we obtain

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

$$\text{Definition of } v^{(1)} \text{ :- } \boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$$

It follows

$$- \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(a) For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner, we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (\bar{v}_1)^{(1)}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (v_0)^{(1)}$$

we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing in the system we get easily the result stated in the theorem.

Particular case :

If $(a_{13}''^{(1)}) = (a_{14}''^{(1)})$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ this also defines $(v_0)^{(1)}$ for

the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$. This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From the resultant equations of the governing equations we obtain (System: Dead Organic Matter(DOM) and Decomposer Organisms(DO))

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:-
$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows

$$- \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

(d) For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_0)^{(2)})t]}} , \quad (C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} , \quad (\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$

(e) If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case,

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((v_1)^{(2)} - (v_2)^{(2)})t]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (\bar{v}_1)^{(2)}$$

(f) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)})t]}} \leq (v_0)^{(2)}$$

we have

Definition of $v^{(2)}(t)$:-

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result we get easily the result stated in the theorem for the concatenated system of Terrestrial Organisms (TO)-Oxygen Consumption (OC)-Dead Organic Matter (DOM)-Decomposer Organisms (DO).

Particular case :

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

Proof : From solution equations we obtain:

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a'_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$- \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_0)^{(3)})t]}} , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get

$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case,

$$\begin{aligned}
 (v_1)^{(3)} &\leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}} \leq v^{(3)}(t) \leq \\
 &\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (\bar{v}_1)^{(3)}
 \end{aligned}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

$$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (v_0)^{(3)}$$

And so with the notation, we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result in the solution equations of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR), we obtain the result stated in the Theorem.

Particular case :System Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

We can prove the following

THEOREM 3: If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t , and the conditions for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR), Satisfies the following:

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied, then the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

If $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ are independent on t , and the conditions

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied, then the system

If $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ are independent on t , and the conditions for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ are satisfied, then the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0$$

has a unique positive solution, which is an equilibrium solution

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0$$

has a unique positive solution, which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0$$

has a unique positive solution , which is an equilibrium solution

Proof: (a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

(b)Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(c) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

Definition and uniqueness of T_{14}^* for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR)

, After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* for the: the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{19})$ are being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value , we obtain for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

Definition and uniqueness of T_{21}^* for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T)$ are being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

By the same argument, the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

(b) admit solutions G_{13}, G_{14} if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \\ [(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR), admit solutions G_{16}, G_{17} if

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \\ [(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{17}^* such that $\varphi((G_{19})^*) = 0$ for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

By the same argument the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR), admit solutions G_{20}, G_{21} if

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - \\ [(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

Finally we obtain the unique solution of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]} \\ T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

the system being Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

Obviously, these values represent an equilibrium solution of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

Finally we obtain the unique solution for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

Finally we obtain the unique solution for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR)

ASYMPTOTIC STABILITY ANALYSIS(for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),)

THEOREM 4: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(1)}$ and $(b'_i)^{(1)}$ belong to $C^{(1)}(\mathbb{R}_+)$ then the above system Terrestrial Organism (TO)-Oxygen Consumption (OC)-Dead Organic matter (DOM)-Decomposer Organisms (DO)-Green Plants(GP)-Nutrients(NR), is asymptotically stable.

Proof:

Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i , T_i = T_i^* + T_i$$

$$\frac{\partial (a'_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} , \frac{\partial (b'_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account the governing equations and the resultant equations of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^*T_{14}$$

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^*T_{14}$$

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^*T_{14}$$

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*G_j$$

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*G_j$$

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*G_j$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),. The following is the second condition to be satisfied.

Definition of G_i, T_i :- for $i=16,17,18$

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij}$$

taking into account governing equations for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),and neglecting the powers of 2, following equations are satisfied for the holistic system. These equations are in addition to the equations mentioned in TO_OC case.

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17}$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17}$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17}$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)(j)})T_{16}^*G_j$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)(j)})T_{17}^*G_j$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)(j)})T_{18}^*G_j$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable, for the global system under consideration namely the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),

Denote

Definition of G_i, T_i :- for $i=20,21,22$,

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations for the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR),,after neglect ion of the powers of 2,

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21}$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21}$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21}$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^*G_j$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^*G_j$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^*G_j$$

The characteristic equation of the system Terrestrial Organism(TO)-Oxygen Consumption(OC)-Dead Organic matter(DOM)-Decomposer Organisms(DO)-Green Plants(GP)-Nutrients(NR), is given by the following:

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ & \left[((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\ & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\ & + \left(((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\ & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\ & \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & \left(((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & + \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\ & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) ((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*) \\ & \left. \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \right\} \end{aligned}$$

+

$$\begin{aligned} & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\ & \left[((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \\ & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\ & + \left(((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\ & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \end{aligned}$$

$$\begin{aligned} & \left((\lambda^{(2)})^2 + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda^{(2)}) \\ & \left((\lambda^{(2)})^2 + (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} \right) (\lambda^{(2)}) \\ & + \left((\lambda^{(2)})^2 + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda^{(2)}) (q_{18})^{(2)} G_{18} \\ & + \left((\lambda^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) (a_{18})^{(2)} (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \\ & \left. \left((\lambda^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}) s_{(17),(18)} T_{17}^* + (b_{17})^{(2)} s_{(16),(18)} T_{17}^* \right) \right\} \end{aligned}$$

+

$$\begin{aligned} & \left((\lambda^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}) \left\{ (\lambda^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}) \right. \right. \\ & \left. \left[\left((\lambda^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \right] \right. \\ & \left. \left((\lambda^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \right) \right. \\ & \left. + \left((\lambda^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}) (q_{20})^{(3)} G_{20}^* + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^* \right) \right. \\ & \left. \left((\lambda^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \right) \right. \\ & \left. \left((\lambda^{(3)})^2 + (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda^{(3)}) \right. \\ & \left. \left((\lambda^{(3)})^2 + (b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda^{(3)}) \right. \\ & \left. + \left((\lambda^{(3)})^2 + (a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda^{(3)}) (q_{22})^{(3)} G_{22} \right. \\ & \left. + \left((\lambda^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \right. \\ & \left. \left. \left((\lambda^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{21}^* \right) \right\} = 0 \right. \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, We regret with great deal of compunction, contrition, and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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