DARK ENERGY (DE) AND EXPANDING UNIVERSE(EU) AN AUGMENTATION -DETRITION MODEL

[1]DR K N PRASANNA KUMAR, [2]PROF B S KIRANAGI AND [3]PROF C S BAGEWADI

ABSTRACT: *A system of dark energy-expanding universe is investigated. It is shown that the time independence of the contributions portrays another system by itself and constitutes the equilibrium solution of the original time independent system. With the methodology reinforced and revitalized with the explanations, we write the governing equations with the nomenclature for the systems in the foregoing. Further papers extensively draw inferences upon such concatenation process, ipso facto,fait accompli desideratum.*

INDEX TERMS: Dark Energy, Expanding Universe ,Empty Space, Repulsive Force

INTRODUCTION:

In physical cosmology, astronomy and celestial mechanics, dark energy is a hypothetical form of energy that permeates all of space and tends to accelerate the expansion of the universe. Dark energy is the most accepted hypothesis to explain observations since the 1990s that indicate that the universe is expanding at an accelerating rate. In the standard model of cosmology, dark energy currently accounts for 73% of the total mass-energy of the universe.
At the UK-Germany National Astronomy Meeting NAM2012, the Baryon Oscillation Spectroscopic Survey (BOSS) team has just announced the most accurate measurement yet of the distribution of galaxies between five and six billion years ago. This was the key 'pivot' moment at which the expansion of the universe stopped slowing down due to gravity and started to accelerate instead, due to a mysterious force dubbed "dark energy." The nature of this "dark energy" is one of the big mysteries in cosmology today, and scientists need precise measurements of the expansion history of the universe to unravel this mystery -- BOSS provides this kind of data. In a set of six joint papers presented March 30, the BOSS team, an international group of scientists with the participation of the Max Planck Institute of Extraterrestrial Physics in Garching, Germany, used these data together with previous measurements to place tight constraints on various cosmological models. At the UK-Germany National Astronomy Meeting NAM2012, the Baryon Oscillation Spectroscopic Survey (BOSS) team has just announced the most accurate measurement yet of the distribution of galaxies between five and six billion years ago. This was the key 'pivot' moment at which the expansion of the universe stopped slowing down due to gravity and started to accelerate instead, due to a mysterious force dubbed "dark energy." The nature of this "dark energy" is one of the big mysteries in cosmology today, and scientists need precise measurements of the expansion history of the universe to unravel this mystery -- BOSS provides this kind of data. In a set of six joint papers presented March 30, the BOSS team, an international group of scientists with the participation of the Max Planck Institute of Extraterrestrial Physics in Garching, Germany, used these data together with previous measurements to place tight constraints on various cosmological

Two proposed forms for dark energy are the cosmological constant, a constant energy density filling space homogeneously, and scalar fields such as quintessence or moduli, dynamic quantities whose energy density can vary in time and space. Contributions from scalar fields that are constant in space are usually also included in the cosmological constant. The cosmological constant is physically equivalent to vacuum energy. Scalar fields which do change in space can be difficult to distinguish from a cosmological constant because the change may be extremely slow.

High-precision measurements of the expansion of the universe are required to understand how the expansion rate changes over time. In general relativity, the evolution of the expansion rate is parameterized by the cosmological equation of state (the relationship between temperature, pressure, and combined matter, energy, and vacuum energy density for any region of space). Measuring the equation of state for dark energy is one of the biggest efforts in observational cosmology today. Adding the cosmological constant to cosmology's standard FLRW metric leads to the Lambda-CDM model, which has been referred to as the "standard model" of cosmology because of its precise agreement with observations. Dark energy has been used as a crucial ingredient in a recent attempt to formulate a cyclic model for the universe.

A 2011 survey of more than 200,000 galaxies appears to confirm the existence of dark energy, although the exact physics behind it remains unknown. The first suggestion for dark energy from observed data happened in 1992, when Hungarian astronomer György Paál and his collaborators published a paper. Previously, in 1990 Broadhurst et al had published the so called "pencil beam survey" about the irregularities in the galaxy distribution. Using this data Paál et al. found in some cosmological model the irregularities became more regular. In 1998, published observations of Type Ia supernovae ("one-A") by the High-z Supernova Search Team followed in 1999 by the Supernova Cosmology Project suggested that the expansion of the universe is accelerating. This work was awarded by the Nobel Prize in Physics in 2011.

Since then, these observations have been corroborated by several independent sources. Measurements of the cosmic microwave background, gravitational lensing, and the large scale structure of the cosmos as well as improved measurements of supernovae have been consistent with the Lambda-CDM model. Supernovae are useful for cosmology because they are excellent standard candles across cosmological distances. They allow the expansion history of the Universe to be measured by looking at the relationship between the distance to an object and its redshift, which gives how fast it is receding from us. The relationship is roughly linear, according to Hubble's law. It is relatively easy to measure redshift, but finding the distance to an object is more difficult. Usually, astronomers use standard candles: objects for which the intrinsic brightness, the absolute magnitude, is known. This allows the object's distance to be measured from its actual observed brightness, or apparent magnitude. Type Ia supernovae are the best-known standard candles across cosmological distances because of their extreme, and extremely consistent, brightness. Recent observations of supernovae are consistent with a universe made up 71.3% of dark energy and 27.4% of a combination of dark matter and baryonic matter.

COSMIC MICROWAVE BACKGROUND

Estimated distribution of dark matter and dark energy in the universe
The existence of dark energy, in whatever form, is needed to reconcile the measured geometry of space with the total amount of matter in the universe. Measurements of cosmic microwave background (CMB) anisotropies, most recently by the WMAP spacecraft, indicate that the universe is close to flat. For the shape of the universe to be flat, the mass/energy density of the universe must be equal to a certain critical density. The total amount of matter in the universe (including baryons and dark matter), as measured by the CMB, accounts for only about 30% of the critical density. This implies the existence of an additional form of energy to account for the remaining 70%.[15] The WMAP five-year analysis estimate a universe made up of 74% dark energy, 22% dark matter, and 4% ordinary matter.[17] More recently, the WMAP seven-year analysis gave an estimate of 72.8% dark energy, 22.7% dark matter and 4.6% ordinary matter.
Large-scale structure

The theory of large scale structure, which governs the formation of structures in the universe (stars, quasars, galaxies and galaxy clusters), also suggests that the density of matter in the universe is only 30% of the critical density.

The Wiggle Z galaxy survey from Australian Astronomical Observatory scanned 200,000 galaxies to determine their redshift. Then, by exploiting the fact that baryon acoustic oscillations have left voids regularly of ~150 Mpc diameter, surrounded by the galaxies, the voids were used as standard rulers to determine distances to galaxies as far as 2000 Mpc (redshift 0.6), which allowed astronomers to determine more accurately the speeds of the galaxies from their redshift and distance. The data confirmed cosmic acceleration up to half of the age of the universe (7 billion years), and constrain its inhomogeneity to 1 part in 10, This provides a confirmation to cosmic acceleration independent of supernovas.

LATE-TIME INTEGRATED SACHS-WOLFE EFFECT

Accelerated cosmic expansion causes gravitational potential wells and hills to flatten as photons pass through them, producing cold spots and hot spots on the CMB aligned with vast supervoids and superclusters. This so-called late-time Integrated Sachs-Wolfe effect (ISW) is a direct signal of dark energy in a flat universe. It was reported at high significance in 2008 by Ho et al. and Giannantonio et al.

NATURE

The nature of this dark energy is a matter of speculation. The evidence for dark energy is only indirect coming from distance measurements and their relation to redshift. It is thought to be very homogeneous, not very dense and is not known to interact through any of the fundamental forces other than gravity. Since it is not very dense—roughly 10−29 grams per cubic centimeter—it is unlikely to be detectable in laboratory experiments. Dark energy can only have such a profound effect on the universe, making up 74% of universal density, because it uniformly fills otherwise empty space. The two leading models are a cosmological constant and quintessence. Both models include the common characteristic that dark energy must have negative pressure.

NEGATIVE PRESSURE

Independently from its actual nature, dark energy would need to have a strong negative pressure (acting repulsively) in order to explain the observed acceleration in the expansion rate of the universe. According to General Relativity, the pressure within a substance contributes to its gravitational attraction for other things just as its mass density does. This happens because the physical quantity that causes matter to generate gravitational effects is the Stress-energy tensor, which contains both the energy (or matter) density of a substance and its pressure and viscosity.

In the Friedmann-Lemaître-Robertson-Walker metric, it can be shown that a strong constant negative pressure in the entire universe causes acceleration in universe expansion if the universe is already expanding or a deceleration in universe contraction if the universe is already contracting. This accelerating expansion effect is sometimes labeled "gravitational repulsion", which is a colorful but possibly confusing expression. In fact a negative pressure does not influence the gravitational interaction between masses—which remains attractive—but rather alters the overall evolution of the universe at the cosmological scale, typically resulting in the accelerating expansion of the universe despite the attraction among the masses present in the universe.

COSMOLOGICAL CONSTANT

The simplest explanation for dark energy is that it is simply the "cost of having space": that is, a volume of space has some intrinsic, fundamental energy. This is the cosmological constant, sometimes called Lambda (hence Lambda-CDM model) after the Greek letter Λ, the symbol used to mathematically represent this quantity. Since energy and mass are related by E = mc2, Einstein's theory of general relativity predicts that this energy will have a gravitational effect. It is sometimes called a vacuum energy because it is the energy density of empty vacuum. In fact, most theories of particle physics predict vacuum fluctuations that would give the vacuum this sort of energy. This is related to the Casimir Effect, in which there is a small suction into regions where virtual particles are geometrically inhibited from forming (e.g. between plates with tiny separation). The cosmological constant is estimated by cosmologists to be on the order of 10−29g/cm3, or about 10−120 in reduced Planck units. Particle physics predicts a natural value of 1 in reduced Planck units, leading to a large discrepancy.
The cosmological constant has negative pressure equal to its energy density and so causes the expansion of the universe to accelerate. The reason why a cosmological constant has negative pressure can be seen from classical thermodynamics; Energy must be lost from inside a container to do work on the container. A change in volume dV requires work done equal to a change of energy −P dV, where P is the pressure. But the amount of energy in a container full of vacuum actually increases when the volume increases (dV is positive), because the energy is equal to ρV, where ρ (rho) is the energy density of the cosmological constant. Therefore, P is negative and, in fact, P = −ρ.

A major outstanding problem is that most quantum field theories predict a huge cosmological constant from the energy of the quantum vacuum, more than 100 orders of magnitude too large. This would need to be cancelled almost, but not exactly, by an equally large term of the opposite sign. Some super symmetric theories require a cosmological constant that is exactly zero, which does not help. The present scientific consensus amounts to extrapolating the empirical evidence where it is relevant to predictions, and fine-tuning theories until a more elegant solution is found. Technically, this amounts to checking theories against macroscopic observations. Unfortunately, as the known error-margin in the constant predicts the fate of the universe more than its present state, many such "deeper" questions remain unknown.

Another problem arises with inclusion of the cosmological constant in the standard model: i.e., the appearance of solutions with regions of discontinuities (see classification of discontinuities for three examples) at low matter density, Discontinuity also affects the past sign of the pressure assigned to the cosmological constant, changing from the current negative pressure to attractive, as one looks back towards the early Universe. A systematic, model-independent evaluation of the supernovae data supporting inclusion of the cosmological constant in the standard model indicates these data suffer systematic error. The supernovae data are not overwhelming evidence for an accelerating Universe expansion which may be simply gliding, A numerical evaluation of WMAP and supernovae data for evidence that our local group exists in a local void with poor matter density compared to other locations, uncovered possible conflict in the analysis used to support the cosmological constant, A recent theoretical investigation found the cosmological time, dt, diverges for any finite interval, ds, associated with an observer approaching the cosmological horizon, representing a physical limit to observation. This is a key component required for a complete interpretation of astronomical observations, particularly pertaining to the nature of dark energy. The identification of dark energy as a cosmological constant does not appear to be consistent with the data. These findings should be considered shortcomings of the standard model, but only when a term for vacuum energy is included.

In spite of its problems, the cosmological constant is in many respects the most economical solution to the problem of cosmic acceleration. One number successfully explains a multitude of observations. Thus, the current standard model of cosmology, the Lambda-CDM model, includes the cosmological constant as an essential feature.

QUINTESSENCE

In quintessence models of dark energy, the observed acceleration of the scale factor is caused by the potential energy of a dynamical field, referred to as quintessence field. Quintessence differs from the cosmological constant in that it can vary in space and time. In order for it not to clump and form structure like matter, the field must be very light so that it has a large Compton wavelength.

No evidence of quintessence is yet available, but it has not been ruled out either. It generally predicts a slightly slower acceleration of the expansion of the universe than the cosmological constant. Some scientists think that the best evidence for quintessence would come from violations of Einstein's equivalence principle and variation of the fundamental constants in space or time. Scalar fields are predicted by the standard model and string theory, but an analogous problem to the cosmological constant problem (or the problem of constructing models of cosmic inflation) occurs: renormalization theory predicts that scalar fields should acquire large masses.

The cosmic coincidence problem asks why the cosmic acceleration began when it did. If cosmic acceleration began earlier in the universe, structures such as galaxies would never have had time to form and life, at least as we know it, would never have had a chance to exist. Proponents of the anthropic principle view this as support for their arguments. However, many models of quintessence have a so-called tracker behavior, which solves this problem. In these models, the quintessence field has a density which closely tracks (but is less than) the radiation density until matter-radiation equality, which triggers quintessence to start behaving as dark energy, eventually dominating the universe. This naturally sets the low energy scale of the dark energy.

In 2004, when scientists fit the evolution of dark energy with the cosmological data, they found that the equation of state had possibly crossed the cosmological constant boundary ($w=-1$) from above to below. A No-Go theorem has been proved that gives this scenario at least two degrees of freedom as required for dark energy models. This scenario is so-called Quintom scenario. Some special cases of quintessence are phantom energy, in which the energy density of quintessence actually increases with time, and k-essence (short for kinetic quintessence) which has a non-standard form of kinetic energy. They can have unusual properties: phantom energy, for example, can cause a Big Rip.

ALTERNATIVE IDEAS

Recent research by Christos Tsagas, a cosmologist at Aristotle University of Thessaloniki in Greece, has argued that it's likely that the accelerated expansion of the universe is an illusion caused by the relative motion of us to the rest of the universe. The peer reviewed journal entry cites data showing that the 2.5 billion ly wide region of space we are inside of is moving very quickly relative to everything around it. If his theory is confirmed, then dark energy would not exist (but the "dark flow" still might). His research can be found in the journal, Physical Review D.

Some theorists think that dark energy and cosmic acceleration are a failure of general relativity on very large scales, larger than superclusters. However most attempts at modifying general relativity have turned out to be either equivalent to theories of quintessence, or inconsistent with observations.

Alternative ideas for dark energy have come from string theory, brane cosmology and the holographic principle, but have not yet proved. as compellingly as quintessence and the cosmological constant.

STRING THEORY

String theories, popular with many particle physicists, make it possible, even desirable, to think that the observable universe is just one of 10500 universes in a grander multiverse, says [Leonard Susskind, a cosmologist at Stanford University in California]. The vacuum energy will have different values in different universes, and in many or most it might indeed be vast. But it must be small in ours because it is only in such a universe that observers such as ourselves can evolve.

Paul Steinhardt in the same article criticizes string theory's explanation of dark energy stating "...Anthropics and randomness don't explain anything... I am disappointed with what most theorists are willing to accept.
Yet another, "radically conservative" class of proposals aims to explain the observational data by a more refined use of established theories rather than through the introduction of dark energy, focusing, for example, on the gravitational effects of density in homogeneities, or on consequences of electroweak symmetry breaking in the early universe. If we are located in an emptier-than-average region of space, the observed cosmic expansion rate could be mistaken for a variation in time, or acceleration.

Another class of theories attempts to come up with an all-encompassing theory of both dark matter and dark energy as a single phenomenon that modifies the laws of gravity at various scales. An example of this type of theory is the theory of dark fluid. Another class of theories that unifies dark matter and dark energy are suggested to be covariant theories of modified gravities. These theories alter the dynamics of the space-time such that the modified dynamic stems what have been assigned to the presence of dark energy and dark matter.

Another set of proposals is based on the possibility of a double metric tensor for space-time. It has been argued that time reversed solutions in general relativity require such double metric for consistency, and that both dark matter and dark energy can be understood in terms of time reversed solutions of general relativity]Implications for the fate of the universe Cosmologists estimate that the acceleration began roughly 5 billion years ago. Before that, it is thought that the expansion was decelerating, due to the attractive influence of dark matter and baryons. The density of dark matter in an expanding universe decreases more quickly than dark energy, and eventually the dark energy dominates. Specifically, when the volume of the universe doubles, the density of dark matter is halved but the density of dark energy is nearly unchanged (it is exactly constant in the case of a cosmological constant).If the acceleration continues indefinitely, the ultimate result will be that galaxies outside the local super cluster will have a line-of-sight velocity that continually increases with time, eventually far exceeding the speed of light. This is not a violation of special relativity, because the notion of "velocity" used here is different from that of velocity in a local inertial frame of reference, which is still constrained to be less than the speed of light for any massive object. Because the Hubble parameter is decreasing with time, there can actually be cases where a galaxy that is receding from us faster than light does manage to emit a signal which reaches us eventually. However, because of the accelerating expansion, it is projected that most galaxies will eventually cross a type of cosmological event horizon where any light they emit past that point will never be able to reach us at any time in the infinite future, because the light never reaches a point where its "peculiar velocity" towards us exceeds the expansion velocity away from us. Assuming the dark energy is constant (a cosmological constant), the current distance to this cosmological event horizon is about 16 billion light years, meaning that a signal from an event happening at present would eventually be able to reach us in the future if the event was less than 16 billion light years away, but the signal would never reach us if the event was more than 16 billion light years away.

As galaxies approach the point of crossing this cosmological event horizon, the light from them will become more and more red shifted, to the point where the wavelength becomes too large to detect in practice and the galaxies appear to disappear completely. The Earth, the Milky Way and the Virgo super cluster, however, would remain virtually undisturbed while the rest of the universe recedes and disappears from view. In this scenario, the local super cluster would ultimately suffer heat death, just as was thought for the flat, matter-dominated universe, before measurements of cosmic acceleration.

There are some very speculative ideas about the future of the universe. One suggests that phantom energy causes divergent expansion, which would imply that the effective force of dark energy continues growing until it dominates all other forces in the universe. Under this scenario, dark energy would ultimately tear apart all gravitationally bound structures, including galaxies and solar systems, and eventually overcome the electrical and nuclear forces to tear apart atoms themselves, ending the universe in a "Big Rip". On the other hand, dark energy might dissipate with time, or even become attractive. Such uncertainties leave open the possibility that gravity might yet rule the day and lead to a universe that contracts in on itself in a "Big Crunch". Some scenarios, such as the cyclic model suggest this could be the case. It is also possible the universe may never have an end and continue in its present state forever. While these ideas are not supported by observations, they are not ruled out.
The cosmological constant was first proposed by Einstein as a mechanism to obtain a stable solution of the gravitational field equation that would lead to a static universe, effectively using dark energy to balance gravity. Not only was the mechanism an inelegant example of fine-tuning, it was soon realized that Einstein's static universe would actually be unstable because local inhomogeneities would ultimately lead to either the runaway expansion or contraction of the universe. The equilibrium is unstable: if the universe expands slightly, then the

expansion releases vacuum energy, which causes yet more expansion. Likewise, a universe which contracts slightly will continue contracting. These sorts of disturbances are inevitable, due to the uneven distribution of matter throughout the universe. More importantly, observations made by Edwin Hubble showed that the universe appears to be expanding and not static at all. Einstein famously referred to his failure to predict the idea of a dynamic universe, in contrast to a static universe, as his greatest blunder. Following this realization, the cosmological constant was largely ignored as a historical curiosity.

Alan Guth proposed in the 1970s that a negative pressure field, similar in concept to dark energy, could drive cosmic inflation in the very early universe. Inflation postulates that some repulsive force, qualitatively similar to dark energy, resulted in an enormous and exponential expansion of the universe slightly after the Big Bang. Such expansion is an essential feature of most current models of the Big Bang. However, inflation must have occurred at a much higher energy density than the dark energy we observe today and is thought to have completely ended when the universe was just a fraction of a second old. It is unclear what relation, if any, exists between dark energy and inflation. Even after inflationary models became accepted, the cosmological constant was thought to be irrelevant to the current universe.

The term "dark energy", echoing Fritz Zwicky's "dark matter" from the 1930s, was coined by Michael Turner in 1998. By that time, the missing mass problem of big bang nucleosynthesis and large scale structure was established, and some cosmologists had started to theorize that there was an additional component to our universe. The first direct evidence for dark energy came from supernova observations of accelerated expansion, in Riess et al. and later confirmed in Perlmutter et al. This resulted in the Lambda-CDM model, which as of 2006 is consistent with a series of increasingly rigorous cosmological observations, the latest being the 2005 Supernova Legacy Survey. First results from the SNLS reveal that the average behavior (i.e., equation of state) of dark energy behaves like Einstein's cosmological constant to a precision of 10%. Recent results from the Hubble Space Telescope Higher-Z Team indicate that dark energy has been present for at least 9 billion years and during the period preceding cosmic acceleration.

Although gravitation is extremely weak, it always wins over cosmological distances and therefore is the most important force for the understanding of the large-scale structure and evolution of the Universe.

UNIFICATION OF THE FORCES OF NATURE

Fundamental forces in our present Universe are intuitively seem distinct and have very different characteristics, but the current thinking in theoretical physics is that this was not always so. There is a rather strong belief (although it is yet to be confirmed experimentally) that in the very early Universe when temperatures were very high compared with today, the weak, electromagnetic, and strong forces were unified into a single force. Only when the temperature dropped did these forces separate from each other, with the strong force separating first and then at a still lower temperature the electromagnetic and weak forces separating to leave us with the 4 distinct forces that we see in our present Universe. The process of the forces separating from each other is called spontaneous symmetry breaking.

There is further speculation, which is even less firm than that above, that at even higher temperatures (the Planck Scale) all four forces were unified into a single force. Then, as the temperature dropped, gravitation separated first and then the other 3 forces separated as described above. The time and temperature scales for this proposed sequential loss of unification are illustrated in the following table.

Loss of Unity in the Forces of Nature			
Characterization	Forces Unified	Time Since Beginning	Temperature (GeV)*
All 4 forces unified	Gravity, Strong, Electromagnetic, Weak	~0	~infinite
Gravity separates (Planck Scale)	Strong, Electromagnetic, Weak	10-43 s	1019
Strong force separates (GUTs Scale)	Electromagnetic, Weak	10-35 s	1014
Split of weak and electromagnetic forces	None	10-11 s	100
Present Universe	None	1010 y	10-12

*Temperature Conversion: 1 GeV = 1.2 x 1013 K

Theories that postulate the unification of the strong, weak, and electromagnetic forces are called Grand Unified

Theories (often known by the acronym GUTs). Theories that add gravity to the mix and try to unify all four fundamental forces into a single force are called Super unified Theories. The theory that describes the unified electromagnetic and weak interactions is called the Standard Electroweak Theory, or sometimes just the Standard Model.

Grand Unified and Super unified Theories remain theoretical speculations that are as yet unproven, but there is strong experimental evidence for the unification of the electromagnetic and weak interactions in the Standard Electroweak Theory. Furthermore, although GUTs are not proven experimentally, there is strong circumstantial evidence to suggest that a theory at least like a Grand Unified Theory is required to make sense of the Universe.
The universe is expanding at the same speed Einstein predicated driven by dark energy says a new study, from University of Portsmouth and Max Planck Institute of Theoretical Physics. They have reported that the period of five to six billion years ago has been examined and made measurements of extraordinary accuracy within 1.7 percent. The findings ,support Einstein*s general Theory Of Relativity which predicts how fast galaxies separated by large distances should be moving towards one another and at what rate the structure of the universe must be growing. He findings are in consonance and conformality with the notions ,doctrines ,axiomatic predications and postulational alcovishness of the concordance model of the universe that bloomed from the big bang some 13.7 billion years ago the papers have reported
Team member Dr.Rita Tojeiro is reported to have opined that the results are the best intergalactic measurement of of distances ever made by the cosmologists, means that the cosmologists are closer to the understanding of why the universe's expansion is accelerating. One of the exceptional qualities of Einstein's Theory of relativity is that it is testable, in no uncertain terms. These results reported by the team stated in the foregoing are consistent with the notion that constant vacuum energy –empty space creating a repulsive force-is driving the acceleration of the universe. These are far more profound statements that describe the physics of our universe at the most fundamental level. Critically, the results found no evidence that the dark energy is the figment of imagination or the product of puerile prognostication of human beings, and that possibly makes the stringent test for Einstein's Theory rigorously passed.

EXPANDING UNIVERSE

For thousands of years, astronomers wrestled with basic questions about the size and age of the universe. Does the universe go on forever, or does it have an edge somewhere? Has it always existed, or did it come to being some time in the past? In 1929, Edwin Hubble, an astronomer at Caltech, made a critical discovery that soon led to scientific answers for these questions: he discovered that the universe is expanding.The ancient Greeks recognized that it was difficult to imagine what an infinite universe might look like. But they also wondered that if the universe were finite, and you stuck out your hand at the edge, where would your hand go? The Greeks' two problems with the universe represented a paradox - the universe had to be either finite or infinite, and both alternatives presented problems.After the rise of modern astronomy, another paradox began to puzzle astronomers. In the early 1800s, German astronomer Heinrich Olbers argued that the universe must be finite. If the Universe were infinite and contained stars throughout, Olbers said, then if you looked in any particular direction, your line-of-sight would eventually fall on the surface of a star. Although the apparent size of a star in the sky becomes smaller as the distance to the star increases, the brightness of this smaller surface remains a constant. Therefore, if the Universe were infinite, the whole surface of the night sky should be as bright as a star. Obviously, there are dark areas in the sky, so the universe must be finite.But, when Isaac Newton discovered the law of gravity, he realized that gravity is always attractive. Every object in the universe attracts every other object. If the universe truly were finite, the attractive forces of all the objects in the universe should have caused the entire universe to collapse on itself. This clearly had not happened, and so astronomers were presented with a paradox.
When Einstein developed his theory of gravity in the General Theory of Relativity, he thought he ran into the same problem that Newton did: his equations said that the universe should be either expanding or collapsing, yet he assumed that the universe was static. His original solution contained a constant term, called the cosmological constant, which cancelled the effects of gravity on very large scales, and led to a static universe. After Hubble discovered that the universe was expanding, Einstein called the cosmological constant his "greatest blunder."
At around the same time, larger telescopes were being built that were able to accurately measure the spectra, or the intensity of light as a function of wavelength, of faint objects. Using these new data, astronomers tried to understand the plethora of faint, nebulous objects they were observing. Between 1912 and 1922, astronomer Vesto Slipher at the Lowell Observatory in Arizona discovered that the spectra of light from many of these objects was systematically shifted to longer wavelengths, or redshifted. A short time later, other astronomers showed that these nebulous objects were distant galaxies.

Meanwhile, other physicists and mathematicians working on Einstein's theory of gravity discovered the equations had some solutions that described an expanding universe. In these solutions, the light coming from distant objects would be redshifted as it traveled through the expanding universe. The redshift would increase with increasing distance to the object.In 1929 Edwin Hubble, working at the Carnegie Observatories in Pasadena, California, measured the redshifts of a number of distant galaxies. He also measured their relative distances by measuring the apparent brightness of a class of variable stars called Cepheids in each galaxy. When he plotted redshift against relative distance, he found that the redshift of distant galaxies increased as a linear function of their distance. The only explanation for this observation is that the universe was expanding.Once scientists understood that the universe was expanding, they immediately realized that it would have been smaller in the past. At some point in the past, the entire universe would have been a single point. This point, later called the big bang, was the beginning of the universe as we understand it today.The expanding universe is finite in both time and space. The reason that the universe did not collapse, as Newton's and Einstein's equations said it might, is that it had been expanding from the moment of its creation. The universe is in a constant state of change. The expanding universe, a new idea based on modern physics, laid to rest the paradoxes that troubled astronomers from ancient times until the early 20th Century.

Properties of the Expanding Universe

The equations of the expanding universe have three possible solutions, each of which predicts a different eventual fate for the universe as a whole. Which fate will ultimately befall the universe can be determined by measuring how fast the universe expands relative to how much matter the universe contains.The three possible types of expanding universes are called open, flat, and closed universes. If the universe were open, it would expand forever. If the universe were flat, it would also expand forever, but the expansion rate would slow to zero after an infinite amount of time. If the universe were closed, it would eventually stop expanding and recollapse on itself, possibly leading to another big bang. In all three cases, the expansion slows, and the force that causes the slowing is gravity.

A simple analogy to understand these three types of universes is to consider a spaceship launched from the surface of the Earth. If the spaceship does not have enough speed to escape the Earth's gravity, it will eventually fall back to Earth. This is analogous with a closed universe that recollapses. If the spaceship is given enough speed so that it has just enough energy to escape, then at an infinite distance away from the Earth, it will come to a stop (this is the flat universe). And lastly, if the ship is launched with more than enough energy to escape, it will always have some speed, even when it is an infinite distance away (the open universe).

The Fate of the Universe

For the last eighty years, astronomers have been making increasingly accurate measurements of two important cosmological parameters: Ho - the rate at which the universe expands - and w - the average density of matter in the universe. Knowledge of both of these parameters will tell which of the three models describes the universe we live in, and thus the ultimate fate of our universe. The Sloan Digital Sky Survey, with its large systematic measurement of the galaxy density in the Universe, should enable astronomers to precisely measure the density parameter w.

The Heavy Elements

Astronomers are not only interested in the fate of the universe; they are also interested in understanding its present physical state. One question they try to answer is why the universe is primarily composed of hydrogen and helium, and what is responsible for the relatively small concentration of the heavier elements.

With the rise of nuclear physics in the 1930s and 40s, scientists started to try to explain the abundances of heavier elements by assuming they were synthesized out of primordial hydrogen in the early universe. In the late 1940s, American physicists George Gamow, Robert Herman, and Ralph Alpher realized that long ago, the universe was much hotter and denser. They made calculations to show whether nuclear reactions that took place at those higher temperatures could have created the heavy elements. Unfortunately, with the exception of helium, they found that it was impossible to form heavier elements in any appreciable quantity. Today, we understand that heavy elements were synthesized either in the cores of stars or during supernovae, when a large dying star implodes.

Gamow, Herman, and Alpher did realize, though, that if the universe were hotter and denser in the past, radiation should still be left over from the early universe. This radiation would have a well-defined spectrum (called a blackbody spectrum) that depends on its temperature. As the universe expanded, the spectrum of this light would have been redshifted to longer wavelengths, and the temperature associated with the spectrum would have decreased by a factor of over one thousand as the universe cooled.

The Cosmic Microwave Background Radiation

In 1963, Arno Penzias and Robert Wilson, two scientists in Holmdale, New Jersey, were working on a satellite designed to measure microwaves. When they tested the satellite's antenna, they found mysterious microwaves coming equally from all directions. At first, they thought something was wrong with the antenna. But after checking and rechecking, they realized that they had discovered something real. What they discovered was the radiation predicted years earlier by Gamow, Herman, and Alpher. The radiation that Penzias and Wilson discovered, called the Cosmic Microwave Background Radiation, convinced most astronomers that the Big Bang theory was correct.

After Penzias and Wilson found the Cosmic Microwave Background Radiation, astrophysicists began to study whether they could use its properties to study what the universe was like long ago. According to Big Bang theory, the radiation contained information on how matter was distributed over ten billion years ago, when the universe was only 500,000 years old.

At that time, stars and galaxies had not yet formed. The Universe consisted of a hot soup of electrons and atomic nuclei. These particles constantly collided with the photons that made up the background radiation, which then had a temperature of over 3000 C.

Soon after, the Universe expanded enough, and thus the background radiation cooled enough, so that the electrons could combine with the nuclei to form atoms. Because atoms were electrically neutral, the photons of the background radiation no longer collided with them.

When the first atoms formed, the universe had slight variations in density, which grew into the density variations we see today - galaxies and clusters. These density variations should have led to slight variations in the temperature of the background radiation, and these variations should still be detectable today. Scientists realized that they had an exciting possibility: by measuring the temperature variations of the Cosmic Microwave Background Radiation over different regions of the sky, they would have a direct measurement of the density variations in the early universe, over 10 billion years ago.

Density Variations in the Early Universe

In 1990, a satellite called the Cosmic Microwave Background Explorer (COBE) measured background radiation temperatures over the whole sky. COBE found variations that amounted to only about 5 parts in 100,000, but revealed the density fluctuations in the early universe.

The initial density variations would be the seeds of structure that would grow over time to become the galaxies, clusters of galaxies, and superclusters of galaxies observed today by the Sloan Digital Sky Survey. With the Sloan data, along with data from COBE, astronomers will be able to reconstruct the evolution of structure in the universe over the last 10 to 15 billion years. With this information, we will have a deep understanding of the history of the universe, which will be an almost unbelievable scientific and intellectual achievement.

But measuring the evolution of the density variations in the universe still does not answer the most important question: why does the universe contain these differences in density in the first place? To answer this question, astronomers and astrophysicists must understand the nature of the density variations and construct theories of the origin of the universe that predict how these variations should occur.

Our galaxy and other galaxies must contain large amounts of dark matter ,whose presence is amply testified by the fact that gravitational attraction on the orbits of stars and galaxies. This, together with any other matter would increase the density of the universe up to the critical value needed to halt the expansion (Hawking page 49 A Brief History Of Time)

ASSUMPTIONS:

1) Expanding universe is classified into three categories; Category 1 representative of the universe in the first interval vis-à-vis category 1 of dark energy. Category 2 (second interval) comprising of expanding universe corresponding to category 2 of dark energy regimentation. Category 3 constituting expanding universe belong to higher age than that of category 1 and category 2.This is concomitant to category 3 of dark energy classification. In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the dark matter stratification would be in the fitness of things. For category 3. “Over and above” nomenclature could be used. Similarly, a “less than” scale for category 1 can be used.

a) The speed of growth of expanding universe under category 1 is proportional to the speed of growth of universe under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between consumption due to cellular respiration under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable that would render the systemic equations unsolvable. **Note that the increase in the dissipation coefficient bears ample testimony, and infallible observatory to the fact that the dark energy is utilized by the expansionary universe**

b) The dissipation in all the three categories is attributable to the following two phenomenon :
   1) Aging phenomenon: The aging process leads to transference of the that part of the expanding universe to the next category, no sooner than the age of the dark energy crosses the boundary of demarcation
   2) Depletion phenomenon: Natural calamities leading to destruction of dark energy purported to have been produced by empty space dissipates the growth speed by an equivalent extent.

NOTATION :

G_{13} : Part of The Expanding Universe corresponding to Category 1 of dark energy

G_{14} :Part of the Expanding universe corresponding to category 2 of dark energy

G_{15} : Part of The Expanding Universe corresponding to Category 3 of dark energy

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}$: Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

(a) The growth speed in category 1 is the sum of a accentuation term $(a_{13})^{(1)}G_{14}$ and a dissipation term $-(a'_{13})^{(1)}G_{13}$, the amount of dissipation taken to be proportional to the concomitant category of dark energy

(b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(1)}G_{13}$ and $-(a'_{14})^{(1)}G_{14}$ the inflow from the category 1 ,

(c) The growth speed in category 3 is equivalent to $(a_{15})^{(1)}G_{14}$ and $-(a'_{15})^{(1)}G_{15}$ dissipation, or the slowing down of the pace of the expanding universe.

GOVERNING EQUATIONS:
The differential equations governing the above system can be written in the following form

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}$$ 1

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}$$ 2

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15}$$ 3

$$(a_i)^{(1)} > 0 \quad , \quad i = 13,14,15$$ 4

$$(a'_i)^{(1)} > 0 \quad , \quad i = 13,14,15$$ 5

$$(a_{14})^{(1)} < (a'_{13})^{(1)}$$ 6

$$(a_{15})^{(1)} < (a'_{14})^{(1)}$$ 7

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = dt$$ 8

$$\frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = dt$$ 9

Or we write a single equation as

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = \frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = \frac{dG_{15}}{(a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15}} = dt$$ 10

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α ,β ,γ all positive we can write equation (10) as

$$\frac{\alpha dG_{13}}{\alpha\left((a_{13})^{(1)}G_{14}-(a_{13}^{'})^{(1)}G_{13}\right)}=\frac{\beta dG_{14}}{\beta\left((a_{14})^{(1)}G_{13}-(a_{14}^{'})^{(1)}G_{14}\right)}=\frac{\gamma dG_{15}}{\gamma\left((a_{15})^{(1)}G_{14}-(a_{15}^{'})^{(1)}G_{15}\right)}=dt \quad 11$$

$\alpha_i G_i+\beta_i G_i+\gamma_i G_i = C_i e_i^{\lambda_i t}$ Where $i=13{,}14{,}15$ and C_{13}, C_{14}, C_{15} are arbitrary constant coefficients.

STABILITY ANALYSIS :

Supposing $G_i(0)=G_i^0(0)>0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. If $(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)}-(a_{13})^{(1)}(a_{14})^{(1)}>0$ all the components of the solution, ie all the three parts of the expanding universe tend to zero, and the solution is stable with respect to the initial data.

2. If $(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)}-(a_{13})^{(1)}(a_{14})^{(1)}<0$ and $\left(\lambda_{14}+(a_{13}^{'})^{(1)}\right)G_{13}^0-(a_{13})^{(1)}G_{14}^0\neq 0,(\lambda_{14}<0)$, the first two components of the solution tend to infinity as $t\to\infty$, and $G_{15}\to 0$, ie. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tend to zero

3. If $(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)}-(a_{13})^{(1)}(a_{14})^{(1)}<0$ and $\left(\lambda_{14}+(a_{13}^{'})^{(1)}\right)G_{13}^0-(a_{13})^{(1)}G_{14}^0=0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of system of expanding universe converges to equilibrium.
2.The approach to equilibrium is a steady one , and there exists progressively diminishing oscillations around the equilibrium point
3.Conditions 1 and 2 are independent of the size and direction of initial disturbance
4.The actual shape of the time path of expanding universe is determined by efficiency parameter , the strength of the response of the portfolio in question, and the initial disturbance
5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold.
6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question.
7. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

DARK ENERGY PORTFOLIO:

ASSUMPTIONS:
Dark energy is classified into three categories analogous to the stratification that was resorted to in expanding universe **with an eye on the empty spaces which produce repulsive forces**., whose equations are concatenated at the end in the annexure. Process of transmission from one category to another is based on such phenomenon that takes place on the expanding universe.

NOTATION :

T_{13} : Dark energy Balance standing in the category 1

T_{14} : Dark energy Balance standing in the category 2
T_{15} : Dark energy Balance standing in the category 3

$(b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$: Accentuation coefficients

$(b'_{13})^{(1)}, (b'_{14})^{(1)}, \left(b'_{15}\right)^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

Under the above assumptions, we derive the following :

a) The growth speed in category 1 is the sum of two parts:
b) A term $+(b_{13})^{(1)}T_{14}$ proportional to the balance of the expanding universe in the category2.A term $-(b'_{13})^{(1)}T_{13}$ representing the quantum of balance dissipated from category 1 .This comprises of expanded universe which have grown old, qualified to be classified under category 2 and category 2 is the sum of two parts:
   1. A term $+(b_{14})^{(1)}T_{13}$ constitutive of the amount of inflow from the category 1
   2. A term $-(b'_{14})^{(1)}T_{14}$ the dissipation factor arising due to aging of expanding universe than the one that is expanding.
   The growth speed under category 3 is attributable to inflow from category 2 Any stalling, deceleration, of the expansion of the universe could also be taken in to consideration.

GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the Dark Energy Portfolio:

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13} \qquad 12$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14} \qquad 13$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left(b'_{15}\right)^{(1)}T_{15} \qquad 14$$

$$(b_i)^{(1)} > 0 \quad , \quad i = 13,14,15 \qquad 15$$

$$\left(b'_i\right)^{(1)} > 0 \quad , \quad i = 13,14,15 \qquad 16$$

$$(b_{14})^{(1)} < (b'_{13})^{(1)} \qquad 17$$

$$(b_{15})^{(1)} < (b'_{14})^{(1)} \qquad 18$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha'_iT_i + \beta'_iT_i + \gamma'_iT_i = C'_ie_i{}^{\lambda'_it}$, $i = 13,14,15$ where $C'_{13}, C'_{14}, C'_{15}$ are arbitrary constant coefficients and $\alpha'_{13}, \alpha'_{14}, \alpha'_{15}, \gamma'_{13}, \gamma'_{14}, \gamma'_{15}$ corresponding multipliers to the characteristic roots of the system

EXPANDING UNIVERSE AND DARK ENERGY THE DUAL SYSTEM PROBLEM

We will denote

1) By $T_i(t), i = 13,14,15$, the three parts of the DRAK energy system analogously to the G_i of the expanding universe portfolio

2) By $\left(a''_i\right)^{(1)}(T_{14}, t)$ $(T_{14} \geq 0, t \geq 0)$,the contribution of the *dark* energy to the dissipation coefficient of the expanding universe
3) By $\left(-b''_i\right)^{(1)}(G_{13}, G_{14}, G_{15}, t) = -\left(b''_i\right)^{(1)}(G, t)$, the contribution of the expanding universe to the dissipation coefficient of the dark energy

GOVERNING EQUATIONS

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13}$ 19

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14}$ 20

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)} + \left(a''_{15}\right)^{(1)}(T_{14},t)\right]G_{15}$ 21

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13}$ 22

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14}$ 23

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)} - \left(b''_{15}\right)^{(1)}(G,t)\right]T_{15}$ 24

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor attributable to EXPANDING UNIVERSE

$-(b''_{13})^{(1)}(G,t) =$ First detrition factor contributed by EXPANDING UNIVERSE
Where we suppose

(A) $(a_i)^{(1)}, \left(a'_i\right)^{(1)}, \left(a''_i\right)^{(1)}, (b_i)^{(1)}, \left(b'_i\right)^{(1)}, \left(b''_i\right)^{(1)} > 0, \quad i,j = 13,14,15$

(B) The functions $\left(a''_i\right)^{(1)}, \left(b''_i\right)^{(1)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$\left(a''_i\right)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$ 25

$\left(b''_i\right)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$ 26

(C) $lim_{T_2\to\infty}\left(a''_i\right)^{(1)}(T_{14},t) = (p_i)^{(1)}$ 27

$\lim_{G\to\infty}\left(b''_i\right)^{(1)}(G,t) = (r_i)^{(1)}$ 28

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}$ are positive constants
and $i = 13,14,15$

They satisfy Lipschitz condition:

29

$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T'_{14}|e^{-(\hat{M}_{13})^{(1)}t}$

30

$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,T)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14},t)$ and$(a''_i)^{(1)}(T_{14},t)$. (T'_{14},t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a''_i)^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ 31

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and \ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + \ (\hat{P}_{13})^{(1)} \ (\hat{k}_{13})^{(1)}] < 1$$ 32

$$\frac{1}{(\hat{M}_{13})^{(1)}} [\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)} \ (\hat{k}_{13})^{(1)}] < 1$$ 33

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0) \, , T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{13} \right)^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \ (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i \, , \, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \, , \, T_i(0) = T_i^0 \, , \, G_i^0 \leq (\hat{P}_{13})^{(1)} \, , T_i^0 \leq (\hat{Q}_{13})^{(1)},$$ 34

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$$ 35

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$$ 36

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)} \left(T_{14}\left(s_{(13)}\right), s_{(13)} \right) \right) G_{13}\left(s_{(13)}\right) \right] ds_{(13)}$$ 37

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)} \left(T_{14}\left(s_{(13)}\right), s_{(13)} \right) \right) G_{14}\left(s_{(13)}\right) \right] ds_{(13)}$$ 38

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)} \left(T_{14}\left(s_{(13)}\right), s_{(13)} \right) \right) G_{15}\left(s_{(13)}\right) \right] ds_{(13)}$$ 39

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b_{13}')^{(1)} - \ (b_{13}'')^{(1)} \left(G\left(s_{(13)}\right), s_{(13)} \right) \right) T_{13}\left(s_{(13)}\right) \right] ds_{(13)}$$ 40

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}\left(s_{(13)}\right) - \left((b_{14}')^{(1)} - \ (b_{14}'')^{(1)} \left(G\left(s_{(13)}\right), s_{(13)} \right) \right) T_{14}\left(s_{(13)}\right) \right] ds_{(13)}$$ 41

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b_{15}')^{(1)} - \ (b_{15}'')^{(1)} \left(G\left(s_{(13)}\right), s_{(13)} \right) \right) T_{15}\left(s_{(13)}\right) \right] ds_{(13)}$$ 42

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] \, ds_{(13)} =$$

$$\big(1 + (a_{13})^{(1)}t\big)G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)}t} - 1\right)$$ 43

From which it follows that

$$(G_{13}(t) - G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right)e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$ 44

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose
$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)}$$ 45

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)}$$ 46

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$ 47

Indeed if we denote

<u>Definition of</u> $\tilde{G}, \tilde{T}$:

$$\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$$ 48

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)} - G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)} +$$
$$\int_0^t\{(a_{13}^{'})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$
$$(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$
$$G_{13}^{(2)}|(a_{13}^{''})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a_{13}^{''})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$ 49

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on 25,26,27,28 and 29 it follows

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t} \leq$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a_{13}^{'})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$ 50

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows

<u>Remark 1:</u> The fact that we supposed $(a_{13}^{''})^{(1)}$ and $(b_{13}^{''})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 51

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.
Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results 52

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)}G_{14}$ and by integrating 53

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 54

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14}\ \to \infty$.

Definition of $(m)^{(1)}$ and ε_1 : 55

Indeed let t_1 be so that for $t\ > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14}\ \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \ \frac{1}{2}$ it results

$T_{14}\ \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right),\quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42 56

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}\,,(\sigma_2)^{(1)}\,,(\tau_1)^{(1)}\,,(\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}\,,(\sigma_2)^{(1)}\,,(\tau_1)^{(1)}\,,(\tau_2)^{(1)}$ four constants satisfying

$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$ 57

58

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$:

59

(b) By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ 60

and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ and 61

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 62

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the

roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ 63

and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ 64

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:-

(c) If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by 65

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$ 66

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}\ , if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$ 67

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(\ m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$ 68

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$ 69

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\ , if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$ 70

$(\ \mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined by 59 and 61 respectively 71

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$ 72

where $(p_i)^{(1)}$ is defined by equation 25

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$ 73

$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}\left((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)}\right)}\left[e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} - e^{-(S_2)^{(1)}t}\ \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq\right.$ 74

$$\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a_{15}')^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a_{15}')^{(1)}t}\right]+\ G_{15}^0e^{-(a_{15}')^{(1)}t})$$

75

$$\boxed{T_{13}^0e^{(R_1)^{(1)}t}\leq T_{13}(t)\leq T_{13}^0e^{((R_1)^{(1)}+(r_{13})^{(1)})t}}$$

76

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^0e^{(R_1)^{(1)}t}\leq T_{13}(t)\leq\frac{1}{(\mu_2)^{(1)}}T_{13}^0e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b_{15}')^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b_{15}')^{(1)}t}\right]+T_{15}^0e^{-(b_{15}')^{(1)}t}\leq T_{15}(t)\leq$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^0e^{-(R_2)^{(1)}t}$$ 77

<u>Definition of</u> $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:-

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a_{13}')^{(1)}$

78

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b_{13}')^{(1)}$

79

$(R_2)^{(1)}=(b_{15}')^{(1)}-(r_{15})^{(1)}$

<u>Proof</u> : From 19,20,21,22,23,24 we obtain

$$\frac{d\nu^{(1)}}{dt}=(a_{13})^{(1)}-\left((a_{13}')^{(1)}-(a_{14}')^{(1)}+(a_{13}'')^{(1)}(T_{14},t)\right)-(a_{14}'')^{(1)}(T_{14},t)\nu^{(1)}-(a_{14})^{(1)}\nu^{(1)}$$ 80

<u>Definition of</u> $\nu^{(1)}$:- $\boxed{\nu^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_2)^{(1)}\nu^{(1)}-(a_{13})^{(1)}\right)\leq\frac{d\nu^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_1)^{(1)}\nu^{(1)}-(a_{13})^{(1)}\right)$$ 81

From which one obtains

<u>Definition of</u> $(\bar{\nu}_1)^{(1)},(\nu_0)^{(1)}$:-

(a) For $0<\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}<(\nu_1)^{(1)}<(\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t)\geq\frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\nu_1)^{(1)}$

In the same manner , we get

$$\nu^{(1)}(t)\leq\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$ 82

From which we deduce $(\nu_0)^{(1)}\leq\nu^{(1)}(t)\leq(\bar{\nu}_1)^{(1)}$

83

(b) If $0<(\nu_1)^{(1)}<(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}<(\bar{\nu}_1)^{(1)}$ we find like in the previous case, 84

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

(c) If $0<(\nu_1)^{(1)}\le(\bar{\nu}_1)^{(1)}\le \boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)} \quad 85$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:- 86

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t)=\frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t)=\frac{T_{13}(t)}{T_{14}(t)}} \quad 87$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)}=(a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)}=(\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)}=(\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)}=(\nu_1)^{(1)}$ then $\nu^{(1)}(t)=(\nu_0)^{(1)}$ and as a consequence $G_{13}(t)=(\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case .

Analogously if $(b_{13}'')^{(1)}=(b_{14}'')^{(1)}, then\ (\tau_1)^{(1)}=(\tau_2)^{(1)}$ and then $(u_1)^{(1)}=(\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)}=(u_1)^{(1)}$ then $T_{13}(t)=(u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Theorem 3: If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions (with the notations 25,26,27,28) 88

$$(a_{13}')^{(1)}(a_{14}')^{(1)}-(a_{13})^{(1)}(a_{14})^{(1)}<0$$

$$(a_{13}')^{(1)}(a_{14}')^{(1)}-(a_{13})^{(1)}(a_{14})^{(1)}+(a_{13})^{(1)}(p_{13})^{(1)}+(a_{14}')^{(1)}(p_{14})^{(1)}+(p_{13})^{(1)}(p_{14})^{(1)}>0$$

$$(b_{13}')^{(1)}(b_{14}')^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}>0 ,$$

$$(b_{13}')^{(1)}(b_{14}')^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-(b_{13}')^{(1)}(r_{14})^{(1)}-(b_{14}')^{(1)}(r_{14})^{(1)}+(r_{13})^{(1)}(r_{14})^{(1)}<0$$

$with$ $(p_{13})^{(1)},(r_{14})^{(1)}$ as defined by equation 25 are satisfied , then the system

$$(a_{13})^{(1)}G_{14}-\left[(a_{13}')^{(1)}+(a_{13}'')^{(1)}(T_{14})\right]G_{13}=0 \quad 89$$

$$(a_{14})^{(1)}G_{13}-\left[(a_{14}')^{(1)}+(a_{14}'')^{(1)}(T_{14})\right]G_{14}=0 \quad 90$$

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 91

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 92

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 93

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 94

has a unique positive solution , which is an equilibrium solution for the system (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) =$
$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) +$
$(a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$ 95

Definition and uniqueness of T^*_{14} :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$ 96

(b) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$
$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$ 97

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0\ , \varphi(\infty)\ < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$, $G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$ 98

$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]}$, $T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$ 99

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}\ and\ (b''_i)^{(1)}$ Belong to $C^{(1)}(\ \mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

ISSN 2250-3153

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$ 100

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(\, G^* \,) = s_{ij}$$ 101

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14}$$ 102

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14}$$ 103

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14}$$ 104

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right)$$ 105

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right)$$ 106

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right)$$ 107

The characteristic equation of this system is

$$\begin{aligned}
&\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)\\
&\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]\\
&\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)\\
&+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)\\
&\quad\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)\\
&\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&\quad\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)\\
&+\left(\left((\lambda)^{(1)}\right)^2 + \left(\,(a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}\\
&+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\\
&\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0
\end{aligned}$$ 108

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

GOVERNING EQUATIONS

EXPANDING UNIVERSE

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a_{13}')^{(1)}G_{13}$$ 1a

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a_{14}')^{(1)}G_{14}$$ 2a

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left(a_{15}'\right)^{(1)}G_{15}$$ 3a

DARK ENERGY

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b_{13}')^{(1)}T_{13}$$ 4a

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b_{14}')^{(1)}T_{14}$$ 5a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left(b'_{15}\right)^{(1)} T_{15}$ 6a

EMPTY SPACE:

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - (a'_{16})^{(2)} G_{16}$ 7a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - (a'_{17})^{(2)} G_{17}$ 8a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - (a'_{18})^{(2)} G_{18}$ 9a

REPULSIVE FORCE

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - (b'_{16})^{(2)} T_{16}$ 10a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - (b'_{17})^{(2)} T_{17}$ 11a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - (b'_{18})^{(2)} T_{18}$ 12a

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS

$\left(-b''_i\right)^{(1)}(G_{13}, G_{14}, G_{15}, t) = -\left(b''_i\right)^{(1)}(G, t)$, $i = 13,14,15$ the contribution to the dissipation coefficient of the dark energy portfolio.

ACCELERATION OF THE EXPANDING UNIVERSE:

$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)}\right] G_{13}$ 13a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)}\right] G_{14}$ 14a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[\left(a'_{15}\right)^{(1)} \boxed{+\left(a''_{15}\right)^{(1)}(T_{14}, t)}\right] G_{15}$ 15a

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+\left(a''_{15}\right)^{(1)}(T_{14}, t)}$ are first augmentation coefficients for category 1, 2 and 3 of the categories of expanding universe.

DARK ENERGY PORTFOLIO:

$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)}\right] T_{13}$ 16a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)}\right] T_{14}$ 17a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[\left(b'_{15}\right)^{(1)} \boxed{-\left(b''_{15}\right)^{(1)}(G, t)}\right] T_{15}$ 18a

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-\left(b''_{15}\right)^{(1)}(G, t)}$ are first detrition coefficients for category 1, 2 and 3 of dark energy portfolio

EMPTY SPACE LEADS TO THE PRODUCTION OF REPULSIVE FORCEGOVERNING EQUATIONS:
REPULSIVE FORCE:

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)}\right] G_{16}$ 19a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)}\right] G_{17}$ 20a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)}\right] G_{18}$ 21a

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are first augmentation coefficients for category 1, 2 and 3 due to EMPTY SPACE PRODUCING RF

EMPTY SPACE:

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)}\right] T_{16}$ 22a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)}\right] T_{17}$ 23a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}^{'})^{(2)}\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}\right]T_{18}$ 24a

Where $\boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 of EMPTY SPACE.

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

EXPANDING UNIVERSE AND DARK ENERGY; AND REPULSIVE FORCE AND EMPTY SPACE COMBINATION:

DARK ENERGY:

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)}\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}\boxed{-(a_{13}^{''})^{(1,1)}(T_{14},t)}\right]G_{16}$ 25a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)}\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}\boxed{-(a_{14}^{''})^{(1,1)}(T_{14},t)}\right]G_{17}$ 26a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)}\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}\boxed{-\left(a_{15}^{''}\right)^{(1,1)}(T_{14},t)}\right]G_{18}$ 27a

Where $\boxed{+(a_{16}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{-(a_{13}^{''})^{(1,1)}(T_{14},t)}$, $\boxed{-(a_{14}^{''})^{(1,1)}(T_{14},t)}$, $\boxed{-\left(a_{15}^{''}\right)^{(1,1)}(T_{14},t)}$ are second detrition coefficients for category 1, 2 and 3

REPULSIVE FORCE:

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b_{13}^{'})^{(1)}\boxed{-(b_{13}^{''})^{(1)}(G,t)}\boxed{+(b_{16}^{''})^{(2,2)}(G_{19},t)}\right]T_{13}$ 28a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b_{14}^{'})^{(1)}\boxed{-(b_{14}^{''})^{(1)}(G,t)}\boxed{+(b_{17}^{''})^{(2,2)}(G_{19},t)}\right]T_{14}$ 29a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b_{15}^{'}\right)^{(1)}\boxed{-\left(b_{15}^{''}\right)^{(1)}(G,t)}\boxed{+(b_{18}^{''})^{(2,2)}(G_{19},t)}\right]T_{15}$ 30a

Where $\boxed{-(b_{13}^{''})^{(1)}(G,t)}$, $\boxed{-(b_{14}^{''})^{(1)}(G,t)}$, $\boxed{-\left(b_{15}^{''}\right)^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 and

$\boxed{+(b_{16}^{''})^{(2,2)}(G_{19},t)}$, $\boxed{+(b_{17}^{''})^{(2,2)}(G_{19},t)}$, $\boxed{+(b_{18}^{''})^{(2,2)}(G_{19},t)}$ are second augmentation coefficients for category 1, 2 and 3

EXPANDING UNIVERSE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a_{13}^{'})^{(1)}\boxed{+(a_{13}^{''})^{(1)}(T_{14},t)}\right]G_{13}$ 31a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a_{14}^{'})^{(1)}\boxed{+(a_{14}^{''})^{(1)}(T_{14},t)}\right]G_{14}$ 32a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a_{15}^{'}\right)^{(1)}\boxed{+\left(a_{15}^{''}\right)^{(1)}(T_{14},t)}\right]G_{15}$ 33a

Where $\boxed{+(a_{13}^{''})^{(1)}(T_{14},t)}$, $\boxed{+(a_{14}^{''})^{(1)}(T_{14},t)}$, $\boxed{+\left(a_{15}^{''}\right)^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to EMPTY SPACE THAT PRODUCES REPULSIVE FORCE

EMPTY SPACE THAT PRODUCES REPULSIVE FORCE

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b_{16}^{'})^{(2)}\boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}\right]T_{16}$ 34a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b_{17}^{'})^{(2)}\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}\right]T_{17}$ 35a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}^{'})^{(2)}\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}\right]T_{18}$ 36a

Where $\boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}$, $\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}$, $\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

GOVERNING EQUATIONS OF
THE DARK ENERGY AND REPULSIVE FORCE:

DARK ENERGY:

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b_{16}^{'})^{(2)}\boxed{-(b_{16}^{''})^{(2)}(G_{19},t)}\boxed{-(b_{13}^{''})^{(1,1)}(G,t)}\right]T_{16}$ 37a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b_{17}^{'})^{(2)}\boxed{-(b_{17}^{''})^{(2)}(G_{19},t)}\boxed{-(b_{14}^{''})^{(1,1)}(G,t)}\right]T_{17}$ 38a

$\frac{dT_6}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}^{'})^{(2)}\boxed{-(b_{18}^{''})^{(2)}(G_{19},t)}\boxed{-\left(b_{15}^{''}\right)^{(1,1)}(G,t)}\right]T_{18}$ 39a

ISSN 2250-3153

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 $\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-\left(b''_{15}\right)^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3

REPULSIVE FORCE:

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\right]G_{13} \quad 40a$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\right]G_{14} \quad 41a$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)}\boxed{+\left(a''_{15}\right)^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}\right]G_{15} \quad 42a$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{\left(a''_{15}\right)^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to EXPANDING UNIVERSE

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3 due to EMPTY SPACE THAT PRODUCES REPULSIVE FORCE

EMPTY SPACE

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\right]G_{16} \quad 43a$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\right]G_{17} \quad 44a$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\right]G_{18} \quad 45a$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to REPULSIVE FORCE CONTRIBUTION

EXPANDING UNIVERSE

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\right]T_{13} \quad 46a$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\right]T_{14} \quad 47a$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)}\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}\right]T_{15} \quad 48a$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to EXPANDING UNIVERSE ATTRIBUTABLE TO DARK ENERGY

GOVERNING EQUATIONS OF THE SYSTEM

EMPTY SPACES THAT PRODUCE REPULSIVE ENERGY:

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\right]G_{16} \quad 49a$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\right]G_{17} \quad 50a$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+\left(a''_{15}\right)^{(1,1,1)}(T_{14},t)}\right]G_{18} \quad 51a$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to REPULSIVE FORCE THAT INFACT IS PRODUCED BY THE EMPTY SPACE

And $\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}$, $\boxed{+\left(a''_{15}\right)^{(1,1,1)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3 due to EXPANDING UNIVERSE DECLERATED BY DARK ENERGY

EXPANDING UNIVERSE THAT DISSIPATES DARK ENERGY FOR THE ACCELERATION OF EXPANSION:

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\right]T_{13} \quad 52a$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\right]T_{14} \quad 53a$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)}\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\right]T_{15} \quad 54a$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to EXPANDING UNIVERSE THAT DISSIPATES THE DARK ENERGY.

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are second detrition coefficient for category 1, 2 and 3 due to EMPTY SPACES THAT PRODUCE REPULSIVE ENERGY

DARK ENERGY THAT IS RESPONSIBLE FOR THE ACCELERATION OF THE EXPANSION OF THE UNIVERSE:

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\right]G_{13}$ 55a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\right]G_{14}$ 56a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\right]G_{15}$ 57a

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to EXPANDING UNIVERSE THAT DISSIPATES THE DARK ENERGY

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3 due to REPULSIVE FORCE THAT IS PRODUCED BY EMPTY SPACES.

REPULSIVE FORCE THAT IS PRODUCED BY EMPTY SPACES:

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1)}(G,t)}\right]T_{16}$ 58a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1)}(G,t)}\right]T_{17}$ 59a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1)}(G,t)}\right]T_{18}$ 60a

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 due to EXPANDING UNIVERSE THAT UTILIZES DARK ENERGY

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$ are second detrition coefficients for category 1,2 and 3 due to EXPANDING UNIVERSE THAT DISSIPATES THE DARK ENERGY IN THE PROCESS OF THE ACCELERATION OF THE EXPANSION PROCESS

Acknowledgments:

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, We regret with great deal of compunction, contrition, and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

REFERENCES

1. Dr K N Prasanna Kumar, Prof B S Kiranagi, Prof C S Bagewadi - MEASUREMENT DISTURBS EXPLANATION OF QUANTUM MECHANICAL STATES-A HIDDEN VARIABLE THEORY - published at: ***"International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition".***
2. DR K N PRASANNA KUMAR, PROF B S KIRANAGI and PROF C S BAGEWADI -CLASSIC 2 FLAVOUR COLOR SUPERCONDUCTIVITY AND ORDINARY NUCLEAR MATTER-A NEW PARADIGM STATEMENT - published at: ***"International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012 Edition***
3. A HAIMOVICI: "On the growth of a two species ecological system divided on age groups". Tensor, Vol 37 (1982),Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday
4. FRTJOF CAPRA: "The web of life" Flamingo, Harper Collins See "Dissipative structures" pages 172-188
5. HEYLIGHEN F. (2001): "The Science of Self-organization and Adaptivity", in L. D. Kiel, (ed) . Knowledge Management, Organizational Intelligence and Learning, and Complexity, in: The Encyclopedia of Life Support Systems ((EOLSS), (Eolss Publishers, Oxford) [http://www.eolss.net
6. MATSUI, T, H. Masunaga, S. M. Kreidenweis, R. A. Pielke Sr., W.-K. Tao, M. Chin, and Y. J Kaufman (2006), "Satellite-based assessment of marine low cloud variability associated with aerosol, atmospheric stability, and the diurnal cycle", J. Geophys. Res., 111, D17204, doi:10.1029/2005JD006097
7. STEVENS, B, G. Feingold, W.R. Cotton and R.L. Walko, "Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus" J. Atmos. Sci., 53, 980-1006
8. FEINGOLD, G, Koren, I; Wang, HL; Xue, HW; Brewer, WA (2010), "Precipitation-generated oscillations in open cellular cloud fields" *Nature*, *466* (7308) 849-852, doi: 10.1038/nature09314, Published 12-Aug 2010

9. R WOOD "The rate of loss of cloud droplets by coalescence in warm clouds" J.Geophys. Res., 111, doi: 10.1029/2006JD007553, 2006

10. H. RUND, "The Differential Geometry of Finsler Spaces", Grund. Math. Wiss. Springer-Verlag, Berlin, 1959

11. A. Dold, "Lectures on Algebraic Topology", 1972, Springer-Verlag

12. S LEVIN "Some Mathematical questions in Biology vii ,Lectures on Mathematics in life sciences, vol 8" The American Mathematical society, Providence , Rhode island 1976

13. KNP KUMAR etal.,ozone-hydrocarbon problem a model;published in the commemorative volume of 21st century challenges in) mathematics,university of mysore

14. KNPKUMAR; etal;terrestial; organsims and oxygen consumption:presented at international conference at bangalkore university under the ageis of iang jong mathematical society

15. Multiple ozone support function; accepted for publication in sahyadri mathematical society journal

16. KNPKUMAR mathematical models in political science dlitt thesis(degree awarded)

17. .L P. J. E. Peebles and Bharat Ratra (2003). "The cosmological constant and dark energy". Reviews of Modern Physics 75 (2): 559–606. arXiv:astro-ph/0207347. Bibcode 2003RvMP...75..559P.doi:10.1103 /Rev ModPhys.75.559 .

18. Bibcode 1992Ap&SS.191..107P. doi:10.1007/BF00644200.

19. Broadhurst, T. J.; Ellis, R. S.; Koo, D. C.; Szalay, A. S. (1990). "Large-scale distribution of galaxies at the Galactic poles". Nature343: 726–728. Bibcode 1990Natur.343..726B.doi:10.1038/343726a0.

20. . Holba, Ágnes; Horváth, I.; Lukács, B.; Paál, G. (1994). "Once more on quasar periodicities". Astrophysics and Space Science222: 65. Bibcode 1994Ap&SS.222...65H.doi:10.1007/BF00627083.

21. Adam G. Riess et al. (Supernova Search Team) (1998). "Observational evidence from supernovae for an accelerating universe and a cosmological constant". Astronomical J. 116 (3): 1009–38. arXiv:astro-ph/9805201. Bibcode1998AJ....116.1009R. doi:10.1086/300499.

22. . S. Perlmutter et al. (The Supernova Cosmology Project) (1999). "Measurements of Omega and Lambda from 42 high redshift supernovae". Astrophysical J. 517 (2): 565–86. arXiv:astro-ph/9812133. Bibcode 1999ApJ...517..565P.doi:10.1086/307221.

23. The first paper, using observed data, which claimed a positive Lambda term was G. Paal et al. (1992). "Inflation and compactification from galaxy redshifts?". ApSS 191: 107–24.Bibcode 1992Ap&SS.191..107P. doi:10.1007/BF00644200.

24. Kowalski, Marek; Rubin, David (October 27, 2008). "Improved Cosmological Constraints from New, Old and Combined Supernova Datasets". The Astrophysical Journal (Chicago, Illinois:University of Chicago Press) 686 (2): 749–778. arXiv:0804.4142.Bibcode 2008ApJ...686..749K. doi:10.1086/589937.. They find a best fit value of the dark energy density, of 0.713+0.027–0.029(stat)+0.036–0.039(sys), of the total matter density, , of 0.274+0.016–0.016(stat)+0.013–0.012(sys) with an equation of state parameter w of −0.969+0.059–0.063(stat)+0.063–0.066(sys).

25. Shirley Ho; Hirata; Nikhil Padmanabhan; Uros Seljak; Neta Bahcall (2008). "Correlation of CMB with large-scale structure: I. ISW Tomography and Cosmological Implications". Phys. Rev. D 78(4). arXiv:0801.0642. Bibcode 2008PhRvD..78d3519H.doi: 10.1103/PhysRevD.78.043519.

26. Tommaso Giannantonio; Ryan Scranton; Crittenden; Nichol; Boughn; Myers; Richards (2008). "Combined analysis of the integrated Sachs-Wolfe effect and cosmological implications".Phys. Rev. D 77 (12). arXiv:0801.4380. Bibcode2008PhRvD..77l3520G. doi:10.1103/PhysRevD.77.123520.

27. R. Durrer (2011). What do we really know about Dark Energy?.arXiv:1103.5331. Bibcode 2011 arXiv1103.5331D.

28. A.M. Öztas and M.L. Smith (2006). "Elliptical Solutions to the Standard Cosmology Model with Realistic Values of Matter Density". International Journal of Theoretical Physics 45 (5): 925–936. Bibcode 2006IJTP...45..896O. doi:10.1007/s10773-006-9082-7.

29. Stephon Alexander, Tirthabir Biswas, Alessio Notari, and Deepak Vaid (2008). "Local Void vs Dark Energy: Confrontation with WMAP and Type Ia Supernovae". Journal of Cosmology and Astroparticle Physics 2009 (09): 025–025. arXiv:0712.0370. Bibcode2009JCAP...09..025A. doi:10.1088/1475-7516/2009/09/025.

30. F. Melia and M. Abdelqadar (2009). "The Cosmological Spacetime". International Journal of Modern Physics D 18 (12): 1889–1901. Bibcode 2009IJMPD..18.1889M.doi:10.1142/S0218271809015746.

31. . Tsagas, Christos G. (2011). "Peculiar motions, accelerated expansion, and the cosmological axis". Physical Review D 84: 063503. Bibcode 2011PhRvD..84f3503T.doi:10.1103/PhysRevD.84.063503.

32. Hogan, Jenny (2007). "Unseen Universe: Welcome to the dark side". Nature 448 (7151): 240–245. Bibcode 2007Natur.448. .240H. doi:10.1038/448240a.PMID 17637630.

33. Wiltshire, David L. (2007). "Exact Solution to the Averaging Problem in Cosmology". Phys. Rev. Lett. 99 (25): 251101. Bibcode2007PhRvL..99y1101W. doi:10.1103/PhysRevLett.99.251101.PMID 18233512.

34. . Teppo Mattsson (2007). "Dark energy as a mirage". Gen. Rel. Grav. 42 (3): 567–599. arXiv:0711.4264. Bibcode2010GReGr..42..567M. doi:10.1007/s10714-009-0873-z.

35. Clifton, Timothy; Pedro Ferreira (April 2009). "Does Dark Energy Really Exist?". Scientific American 300 (4): 48–55.doi:10.1038/scientificamerican0409-48. PMID 19363920. Retrieved April 30, 2009.

36. Exirifard, Q. (2010). "Phenomenological covariant approach to gravity". General Relativity and Gravitation 43: 93–106. Bibcode2011GReGr..43...93E. doi:10.1007/s10714-010-1073-6.

37. Hossenfelder, S. (2008). "A Bi-Metric Theory with Exchange Symmetry". Physical Review D 78 (4): 044015. Bibcode2008PhRvD..78d4015H. doi:10.1103/PhysRevD.78.044015.

38. . Henry-Couannier, F. (2005). International Journal of Modern Physics A 20 (11): 2341. arXiv:gr-qc/0410055. Bibcode2005IJMPA..20.2341H. doi:10.1142/S0217751X05024602.

39. . Ripalda, Jose M. (1999). "Time reversal and negative energies in general relativity". Eprint arXiv: 6012. arXiv:gr-qc/9906012.Bibcode 1999gr.qc.....6012R.

40. . a b Lineweaver, Charles; Tamara M. Davis (2005)."Misconceptions about the Big Bang". Scientific American. Retrieved 2008-11-06.
. Loeb, Abraham (2002). "The Long-Term Future of Extragalactic Astronomy". Physical Review D 65 (4). arXiv:/0107568 astro-ph /0107568. Bibcode 2002PhRvD..65d7301L.doi:10.1103/PhysRevD.65.047301.

41. Krauss, Lawrence M.; Robert J. Scherrer (2007). "The Return of a Static Universe and the End of Cosmology". General Relativity and Gravitation 39 (10): 1545–1550. arXiv:0704.0221. Bibcode2007GReGr..39.1545K. doi:10.1007/s10714-007-0472-9.

42. Pierre Astier et al. (Supernova Legacy Survey) (2006). "The Supernova legacy survey: Measurement of omega(m), omega(lambda) and W from the first year data set". Astronomy and Astrophysics 447: 31–48. arXiv:astro-ph/0510447. Bibcode2006A&A...447...31A. doi:10.1051/0004-6361:20054185.

43. Beck, C; MacKey, M (2005). "Could dark energy be measured in the lab?". Physics Letters B 605 (3–4): 295–300. arXiv:astro-ph/0406504. Bibcode 2005PhLB..605..295B.doi:10.1016/j.physletb.2004.11.060. Retrieved 2011-01-06.

First Author: [1]**Mr. K. N.Prasanna Kumar** has three doctorates one each in Mathematics, Economics, Political Science. Thesis was based on Mathematical Modeling. He was recently awarded D.litt., for his work on 'Mathematical Models in Political Science'--- Department of studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India **Corresponding Author:drknpkumar@gmail.com**

Second Author: [2]**Prof. B.S Kiranagi** is the Former Chairman of the Department of Studies in Mathematics, Manasa Gangotri and present Professor Emeritus of UGC in the Department. Professor Kiranagi has guided over 25 students and he has received many encomiums and laurels for his contribution to Co homology Groups and Mathematical Sciences. Known for his prolific writing, and one of the senior most Professors of the country, he has over 150 publications to his credit. A prolific writer and a prodigious thinker, he has to his credit several books on Lie Groups, Co Homology Groups, and other mathematical application topics, and excellent publication history.-- UGC Emeritus Professor (Department of studies in Mathematics), Manasagangotri, University of Mysore, Karnataka, India

Third Author: [3]**Prof. C.S. Bagewadi** is the present Chairman of Department of Mathematics and Department of Studies in Computer Science and has guided over 25 students. He has published articles in both national and international journals. Professor Bagewadi specializes in Differential Geometry and its wide-ranging ramifications. He has to his credit more than 159 research papers. Several Books on Differential Geometry, Differential Equations are coauthored by him--- Chairman, Department of studies in Mathematics and Computer science, Jnanasahyadri Kuvempu university, Shankarghatta, Shimoga district, Karnataka, India