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P-FUZZY SUBALGEBRA AND ITS PROPERTIES

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Abstract- We give definition for fuzzy subalgebras for partially ordered algebra, Fuzzy subgroup and fuzzy coset. Then we study their connection of P – fuzzy subalgebras with fuzzy subgroup and fuzzy coset.

Index Terms- P – fuzzy subalgebras, fuzzy subgroup and fuzzy coset.

I. INTRODUCTION

In 1965, Lotfi. Zadeh introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. A fuzzy set A is defined as a map from A to the real unit interval
I = [0, 1]. The set of all fuzzy sets on A is usually denoted by I^A. It is also known that under the natural ordering I^A is a complete lattice. Early Greeks, such as Pythagoras and Euclid, relied on geometry to express all of their logical proofs. It was about 250CE when the Greek Diophantus began using Greek letters as numbers and other mathematical symbols. Finally, algebra began to take on its modern symbolic look when viete used letter for variables around 1600. Then in 1637, Descartes wrote la geometrie. Before these mathematicians had thought of polynomials specially, so X_3 actually represented a cube. His written algebra is the first to look almost exactly like ours.

The definitions given to the concept of P – fuzzy subalgebra, fuzzy subgroup and fuzzy coset also involve different operations. Now we unify and generalize these definitions. The theorems proved also highly generalize the existing ones.

II. PRELIMINARIES

Definition 2.1 If $(P,\leq)$ is a partially ordered set and X is a nonempty set, then any mapping
$A : X\rightarrow P$ is a partially ordered fuzzy set on X.

Definition 2.2 A P – fuzzy set $\mu \in P^A$ is called a P – fuzzy algebra or fuzzy subalgebra on the algebra A, if

- For any n – ary ($n\geq1$) operation $f \in F$
$\mu(f(x_1\ldots\ldots\ldots..x_n)) \geq \mu(x_1)*\ldots\ldots\ldots\ldots\ldots..*\mu(x_n)$ for all $x_1\ldots\ldots\ldots..x_n \in A$

- For any constant (nullary operation) C
$\mu(c) \geq \mu(x)$ for all $x \in A$.

Definition 2.3 Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group if

- $\mu (xy) \geq \min \{\mu(x) , \mu(y)\}$ for every $x,y \in G$ and
- $\mu (x^{-1}) = \mu (x)$ for every $x \in G$.

Definition 2.4 Let μ be a fuzzy subgroup of a group g. For any $a \in G$, a μ defined by
$(a\mu)x = \mu(a^{-1}x)$ for every $x \in G$ is called the fuzzy coset of the group G determined by a and μ.

Result: 2.5
A fuzzy subset μ of a group g is a fuzzy subgroup G iff $\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$ for every $x,y \in G$.

III. RESULTS

Theorem:3.1 P – fuzzy subalgebra is a fuzzy subgroup.

Proof:
Consider a P – fuzzy subalgebra on the algebra A then there eists a P –fuzzy set
$\mu \in P^A$ then for
(i) n – ary operation
$\mu(f(x_1\ldots\ldots\ldots..x_n)) \geq \mu(x_1)*\ldots\ldots\ldots\ldots\ldots..*\mu(x_n)$ for all $x_1\ldots\ldots\ldots..x_n \in A$-(1)
from (1) let $x_2 = y$
$\mu(f(xy)) \geq \mu(x)*\mu(y)$-(2)
since $\mu(x*y) \geq \min \{\mu(x) , \mu(y)\}$-(3)
from (2) and (3)
$\mu(x)*\mu(y) = \mu(x*y) \geq \min \{\mu(x) , \mu(y)\}$-(4)
Let $\mu: X\rightarrow[0,1]$
Here μ is a function defined that X is a set which map it each and every element between 0 and 1.
Also the set of all elements of X^{-1} maps the values between 0 and 1.
Therefore $\mu: X^{-1} \rightarrow[0,1]$.
This implies $\mu(x^{-1}) = \mu(x)$-(5)
(ii) nullary operation (for any constant)
If there exist any constant μ(c) then
$\mu(c) * \mu(x) = \mu(c * x) \geq \min \{\mu(c) , \mu(x)\}= \mu(x)$-(6)
$\mu(c) \geq \mu(x)$, for all $x \in A$.

Since all the element of μ(x) lies between 0 and 1. Also μ(c) is any constant and it is greater than or equal to 1. Using the results (4), (5) and (6), it is clear that P – fuzzy subalgebra is a subgroup.

Example:

Let P = {1,-1, i,-i} be the P- fuzzy set in P- fuzzy algebra.

Define

μ: P→[0,1] by

$$\mu(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.5 & \text{if } x = -1 \\ 0 & \text{if } x = i, -i, 1/i, -1/i \end{cases}$$

Clearly μ is a fuzzy subgroup of the P. Also P is a group under operation ∗.

Theorem:3.2 A P – fuzzy subalgebra of a group G is a fuzzy subgroup of the group G iff

$\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$ for every x,y Є G.

Proof:

Real part:

Consider a P – fuzzy subalgebra of a group G is a fuzzy subgroup on the algebra A then we have to prove that

$\mu(xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$

proof:

For P – Fuzzy subalgebra, there exist a P –fuzzy set μ Є P^A, then

(i) For n – ary operation

$\mu(f(x_1 \ldots\ldots\ldots x_n)) \geq \mu(x_1)* \ldots\ldots\ldots\ldots\ldots *\mu(x_n)$ for all $x_1 \ldots\ldots\ldots x_n$ Є A

Let x_1, x_2 Є P where P Є G

Consider $x_1 = x$ and $x_2 = y^{-1}$

Since it is a fuzzy subgroup using theorem (1)

Therefore $\mu(f(x_1\, x_2)) = \mu(f(x\, y^{-1}))$

$\geq \mu(x) * \mu(y^{-1})$

$\mu(x) * \mu(y^{-1}) = \mu(x * y^{-1})$

since fuzzy algebra satisfies fuzzy relation

so $\mu(x * y^{-1}) \geq \mu(x) \wedge \mu(y^{-1})$ using equation (5)

$\geq \mu(x) \wedge \mu(y)$

$\geq \min\{\mu(x), \mu(y)\}$

(ii) for nullary operation

$\mu(c) \geq \mu(x)$ for all x Є A.

using theorem (1), it is a subgroup

therefore there exist an identity element,let it be μ(e)

we know that $\mu(c) \geq \mu(x)$

$\mu(c) = \mu(c) * \mu(e^{-1}) \geq \mu(c) * \mu(e)$ using equation (5)

$\geq \mu(c) \wedge \mu(e)$

$\geq \min\{\mu(c), \mu(e)\}= \mu(e)$-(7)

C is any constant it may be greater than or equal to identity element.

Also

$\mu(x) = \mu(x) * \mu(e^{-1}) \geq \mu(x) * \mu(e)$ using equation (5)

$\geq \mu(x) \wedge \mu(e)$

$\geq \min \{\mu(x), \mu(e)\}= \mu(x)$-(8)

All the elements of x are less than or equal to identity element.

Converse part:

If $\mu (xy^{-1}) \geq \min \{\mu(x) , \mu(y) \}$ for every x, y Є P and P Є G then a fuzzy partially ordered subset μ of a group is a fuzzy subgroup of the group G.

Proof:

$\mu (xy^{-1}) \geq \min \{ \mu(x) , \mu(y^{-1}) \}$

$\geq \min \{ \mu(x) , \mu(y) \}$ using (5)

$\geq \mu(x) \wedge \mu(y)$

$\mu (xy^{-1}) = \mu(x * y^{-1})$ using fuzzy algebra relation.

$\mu(x * y^{-1}) = \mu(x) * \mu(y^{-1}) = \mu(x) * \mu(y)$

$\leq \mu(f(x\, y))$ (n- ary operation)

This implies $\mu(f(x\, y)) \geq \mu(x) * \mu(y)$-(9)

Let μ(c) be any constant and μ(e) be an identity element, then

Assume that

$\mu(c) = \mu(x)$

$\mu(c) * \mu(e^{-1}) = \mu(x) * \mu(e^{-1})$

using (5)

$\mu(c) * \mu(e) = \mu(x) * \mu(e)$

$\mu(c) * \mu(e) \geq \min \{ \mu(c) , \mu(e) \} = \mu(e)$

$\mu(x) * \mu(e) \geq \min \{ \mu(x) , \mu(e) \} = \mu(x)$

C is any constant it may be greater than or equal to identity element and x.

Therefore, $\mu(c) \geq \mu(x)$-(10)

Using (9) and (10)

If $\mu (xy^{-1}) \geq \min \{ \mu(x) , \mu(y) \}$then P- fuzzy subalgebra is a fuzzy subgroup.

Theorem:3.3 P – fuzzy subalgebra is a fuzzy coset of the group.

Proof:

Let μ be a fuzzy subgroup of a group G from fuzzy algebra A.

To prove that: For any a Є G, a μ is defined by $(a\mu)x = \mu(a^{-1}x)$ for every x Є G, also x Є P.

Consider P – fuzzy subalgebra to prove fuzzy coset, assume that n = 1

Then we use n – ary operation

$\mu(f(x)) \geq \mu(x)$

Let a Є P and P Є G

$\Rightarrow a\, \mu(f(x)) \geq a\mu(x)$

$\Rightarrow (a\mu)x = (\mu a)x$

Since P – fuzzy algebra satisfy commutative law

$\Rightarrow (\mu a)x = \mu(ax)$ using (5)

$= \mu(a^{-1}x)$

Since P fuzzy subalgebra is afuzzy subgroup

Therefore,

$(a\mu)x = \mu(a^{-1}x)$ for every x Є G

IV. CONCLUSION

This paper concludes the connection of P – fuzzy subalgebra with fuzzy subalgebra and fuzzy coset of a group.

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