Controller Performance Evaluation for Concentration Control of Isothermal Continuous Stirred Tank Reactor

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Abstract: This paper presents a comparative analysis of performance of PID controller and hybrid fuzzy controller for concentration control of isothermal continuous stirred tank reactor, which is used to carry out chemical reactions in an industry. Isothermal continuous stirred tank reactor is one of the types of reactor which operates at a constant temperature. The authors developed a mathematical model of the isothermal CSTR and implemented PID controller and PD-fuzzy controller to control the product concentration of the reactor irrespective of the disturbances and delays. Time domain analysis of the controller is performed to study the performance of different controllers and it is observed that PD-fuzzy controller performs better than the conventional PID controller to control the product concentration of isothermal CSTR.

Keywords: CSTR, PID, PD-Fuzzy

I. Introduction

PID controllers are most widely used controllers in past two decades. There are many tuning methods available and most of the PID controller tuning uses frequency response methods for example Zeigler-Nichols rule, symmetric optimum rule, Cohen-Coon tuning, internal model control, ITAE tuning rules etc. These tuning rules provide a simple way to calculate the parameters of PID controllers. But in most of the cases, it doesn't provide satisfactory closed loop performance.

The widespread use of chemical reactors has led to design of different control mechanism to control different parameters of the reactor. The control mechanism can be a conventional control or an intelligent control. This paper considers an isothermal CSTR and models the system to obtain the state space and transfer function model of the system. The primary objective of the control mechanism developed for the isothermal CSTR is that the product concentration should be controlled irrespective of the different disturbances and delays. To obtain this control mechanism, conventional and intelligent controller is developed and a comparative study of controller is performed.

II. Nomenclature

CSTR	Continuous Stirred Tank Reactor
A	Cyclopentadine
B	Cyclopentenol
C	Cylcopentanediol
D	Dicyclopentadiene
k_1	Rate constant for $A \to B$ $\left(\min^{-1}\right)$
k_2	Rate constant for $B \to C$ $\left(\min^{-1}\right)$
k_3	Rate constant for $2A \to D$ $\left(mol/l - \min\right)$
r_A	Molar rate of formation of A
r_B	Molar rate of formation of B
r_C	Molar rate of formation of A
r_D	Molar rate of formation of A
C_A	Concentration of A
C_B	Concentration of B
C_{As}	Steady state concentration of A
C_{Bs}	Steady state concentration of B

III. Chemical Reactor

Chemical reactors are the most important unit of a chemical plant used for unit operations. Basically a chemical reactor is a device in which chemical reaction takes place. Chemical reactors can be classified according to different properties

1. Reaction phase
2. Operating modes

According to the reaction phase chemical reactor can be classified as

1. Homogeneous reactor
2. Heterogeneous reactor

According to the operating modes chemical reactors can be classified as

1. Continuous stirred tank reactor
2. Batch stirred tank reactor
3. Semi batch
4. Tubular reactor

While designing a chemical reactor following factor has to be considered, (i) Overall size of reactor, (ii) Products emerging from reactor, (iii) Temperature inside the reactor (iv) Pressure inside the reactor (v) Rate of reaction (vi) Activity and mode of catalyst (vii) Stability and controllability of reactor

IV. Isothermal Reactor & Modelling

Isothermal CSTR is a type of CSTR which is operating at a constant temperature. The volume is also assumed to be constant. The reaction scheme consists of the following irreversible reactions. The feed stream contains only component A. The isothermal CSTR has following reaction scheme which is called Van de Vusse reaction.

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
$$2A \xrightarrow{k_3} D$$

For the above reaction the values of rate constant are

$$k_1 = 50h^{-1} = 0.83\min^{-1}$$
$$k_2 = 100h^{-1} = 1.66\min^{-1}$$
$$k_3 = 10moll^{-1}h^{-1} = 0.166moll^{-1}\min^{-1}$$

Steady state feed concentration is $C_{Afs} = 10gmoll^{-1}$

Overall material balance is given as $\frac{d(V\rho)}{dt} = F_i\rho - F\rho$ (1)

So, $F = F_i$ (2)

Component material balance can be shown as

$$\frac{d(VC_A)}{dt} = F(C_{Af} - C_A) - Vk_1C_A - Vk_3C_A^2 \quad (3)$$

Simplifying eq(3) we obtain eq(4)

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - k_1C_A - k_3C_A^2 \quad (4)$$
$$\frac{dC_B}{dt} = -\frac{F}{V}C_B + k_1C_A - k_2C_B \quad (5)$$
$$\frac{dC_C}{dt} = -\frac{F}{V}C_C + k_2C_B \quad (6)$$
$$\frac{dC_D}{dt} = -\frac{F}{V}C_D + \frac{1}{2}k_3C_A^2 \quad (7)$$

The molar rate of formation for each component (per unit volume) is

$$r_A = -k_1C_A - k_3C_A^2 \quad (8)$$
$$r_B = k_1C_A - k_2C_B \quad (9)$$
$$r_C = k_2C_B \quad (10)$$
$$r_D = \frac{1}{2}k_3C_A^2 \quad (11)$$

Solving eq(4) and eq(5)

$$-k_3C_{As}^2 + \left(-k_1 - \frac{F_s}{V}\right)C_{As} + \frac{F_s}{V}C_{Afs} = 0 \quad (12)$$

Steady state concentration of A and B is defined as

$$C_{As} = \frac{-\left(k_1 + \frac{F_s}{V}\right)}{2k_3} + \frac{\sqrt{\left(k_1 + \frac{F_s}{V}\right)^2 + 4k_3\frac{F_s}{V}C_{Afs}}}{2k_3} \quad (13)$$

$$C_{Bs} = \frac{k_1C_{As}}{\frac{F_s}{V} + k_2} \quad (14)$$

The linear state space model is represented as

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

The state variable is represented as $x = \begin{bmatrix} C_A & -C_{As} \\ C_B & -C_{Bs} \end{bmatrix}$

The output variable is represented as $y = \begin{bmatrix} C_A & -C_{As} \\ C_B & -C_{Bs} \end{bmatrix}$

The input variable is represented as $u = \begin{bmatrix} \frac{F}{V} & \frac{-F_s}{V} \end{bmatrix}$

Two dynamic functional equation is represented as

$$\frac{dC_A}{dt} = f_1\left(C_A, C_B, \frac{F}{V}\right) = \frac{F}{V}(C_{Af} - C_A) - k_1C_A - k_3C_A^2$$
$$\frac{dC_B}{dt} = f_2\left(C_A, C_B, \frac{F}{V}\right) = -\frac{F}{V}C_B + k_1C_A - k_2C_B$$

The elements of state space A matrix is found by

$$A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{x_s, u_s}$$

The elements of state space B matrix is found by

$$B_{ij} = \left.\frac{\partial f_i}{\partial u_j}\right|_{x_s, u_s}$$

The state space model is represented as

$$A = \begin{bmatrix} -\frac{F_s}{V} - k_1 - 2k_3C_{As} & 0 \\ k_1 & \frac{F_s}{V} - k_2 \end{bmatrix}$$

$$B = \begin{bmatrix} C_{Afs} - C_{As} & \frac{F_s}{V} \\ -C_{Bs} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Based on steady state operating point $C_{As} = 3gmoll^{-1}$,

$C_{Bs} = 1.117gmoll^{-1}$, $\frac{F_s}{V} = 0.5714\min^{-1}$

$$A = \begin{bmatrix} -2.4 & 0 \\ 0.83 & -2.23 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 0.57 \\ -1.117 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Converting the state space model to transfer function

$$G(s) = C\left(sI - A\right)^{-1} B$$

$$g_p(s) = \frac{-1.117s + 3.1472}{s^2 + 4.6429s + 5.3821} \qquad (15)$$

$$g_p(s) = \frac{-1.117s + 3.1472e^{-0.5s}}{s^2 + 4.6429s + 5.3821} \qquad (16)$$

$$g_d(s) = \frac{0.4762}{s^2 + 4.6429s + 5.3821} \qquad (17)$$

Eq(15) represents the process transfer function, eq(16) represents the process transfer function with delay and eq(17) represents the disturbance transfer function.

V. Conventional Control Of Reactor

Control of isothermal CSTR has generated a lot of research interest and a large number of literatures can be found in this area. Some of the research findings are discussed in this section.

Jose Alvarez-Ramirez et.al presents proportional-integral (PI) control of continuously stirred tank reactors (CSTR). The main ingredient in the formulation is the use of a novel PI control configuration derived from modeling error compensation ideas. The main theoretical contribution is a novel stability analysis of a wide class of CSTR. It is shown that the performance of an inverse dynamics feedback control can be recovered by classical PI control. This performance recovery includes the region of attraction and transient response [6].

Nina F. Thornhill et.al presents the simulation of CSTR. In this article, volumetric and heat balance equations are presented along with algebraic equations derived from experimental data for calibration of sensors and actuators and unknown quantities such heat transfer through the heating coils. Many of these relationships have nonlinearities, and hard constraints such as the tank being full are also captured. A valuable feature is that the model uses measured, not simulated, noise and disturbances and therefore provides a realistic platform for data-driven identification and fault detection [14].

S M Giriraj Kumar et.al has proposed Genetic Algorithm to improve the performance of bioreactor [16].

J Prakash et.al has presented a design criterion for nonlinear PID controller and non linear model predictive controller for a CSTR system which exhibits dynamic nonlinearity [19].

R Suja Mani Malar et.al has propsed the use of Artificial Neural Network to model and control the CSTR [20, 21].

In this research paper, the primary control objective is to control the product concentration of isothermal CSTR by varying the rate of dilution of the feed flow. The schematic diagram of the feedback control loop of isothermal CSTR is shown in figure 1.

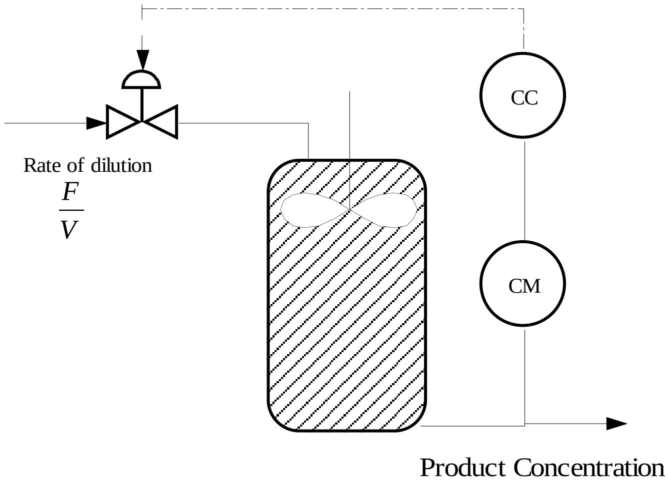


Figure 1: Feedback control mechanism for concentration control of isothermal CSTR

Here CM represents the measurement of concentration and CC represents the concentration controller. Figure 2 shows the block diagram approach of feedback control scheme.

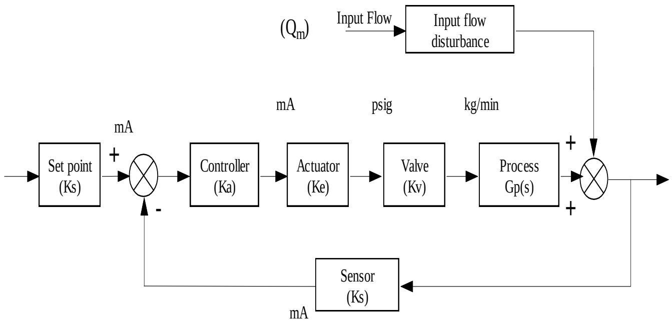


Figure 2: Block diagram based feedback control approach for concentration control of isothermal CSTR

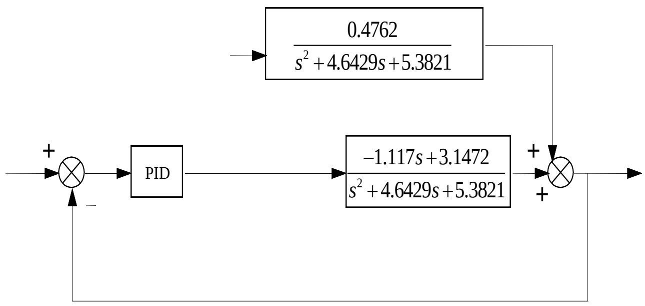


Figure 3: Transfer function based feedback control approach for concentration control of isothermal CSTR

Figure 3 shows the transfer function model of the feedback control scheme for concentration control of isothermal CSTR. The transfer function for process and the disturbance is derived in section IV. Ideal PID controller in continuous time is given as

$$u(t) = K_c\left(e(t) + \frac{1}{\tau_i}\int_0^t e(t)dt + \tau_d \frac{de(t)}{dt}\right)$$

The PID controller is tuned using Zeigler-Nichols criteria of tuning and the unit step response of feedback control is shown in figure 4. The values of proportional gain, integral gain and derivative gain of PID controller are 0.2, 0.95 and 0.23 respectively.

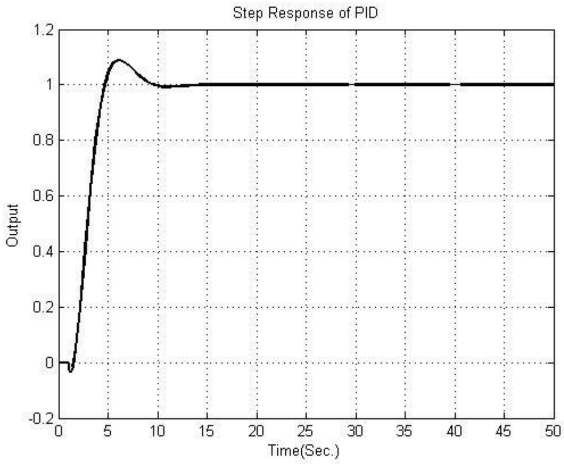


Figure 4: Unit step response of PID controller for concentration control

Figure 4 represents the unit step response of PID controller Figure 5 shows the unit step response of feedback control scheme with a disturbance. Due to this disturbance, the peak overshoot increases.

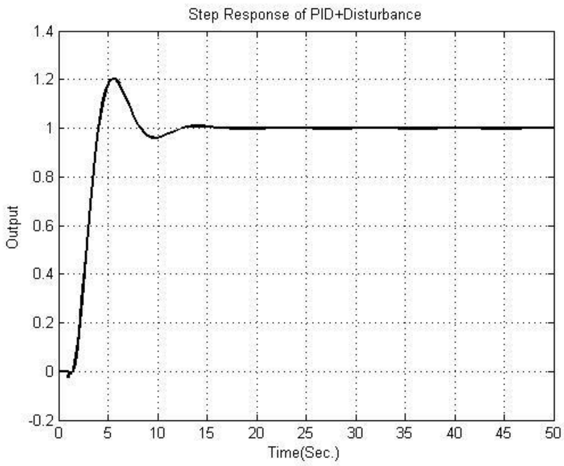


Figure 5: Unit step response of PID controller with disturbances

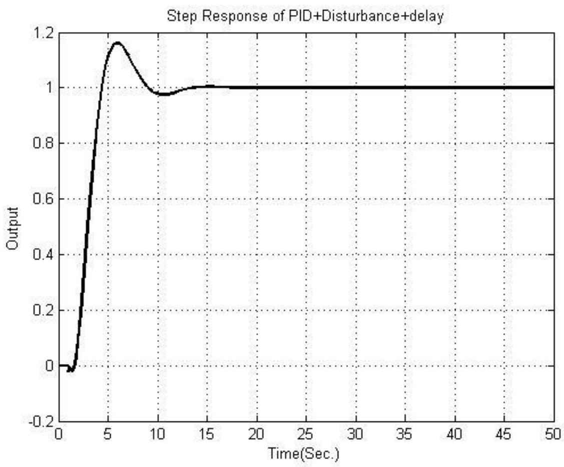


Figure 6: Unit step response of PID controller with delay and disturbances

VI. Fuzzy Control Of Isothermal Reactor

Fuzzy logic is a form of logic that is the extension of boolean logic, which incorporates partial values of truth. Instead of sentences being "completely true" or "completely false," they are assigned a value that represents their degree of truth. In fuzzy systems, values are indicated by a number (called a truth value) in the range from 0 to 1, where 0.0 represents absolute false and 1.0 represents absolute truth.

The fuzzy logic controller provides an algorithm, which converts the expert knowledge into an automatic control strategy. Fuzzy logic is capable of handling approximate information in a systematic way and therefore it is suited for controlling non linear systems and is used for modeling complex systems, where an inexact model exists or systems where ambiguity or vagueness is common. The fuzzy control systems are rule-based systems in which a set of fuzzy rules represent a control decision mechanism for adjusting the effects of certain system stimuli. With an effective rule base, the fuzzy control systems can replace a skilled human operator. The rule base reflects the human expert knowledge, expressed as linguistic variables, while the membership functions represent expert interpretation of those variables.

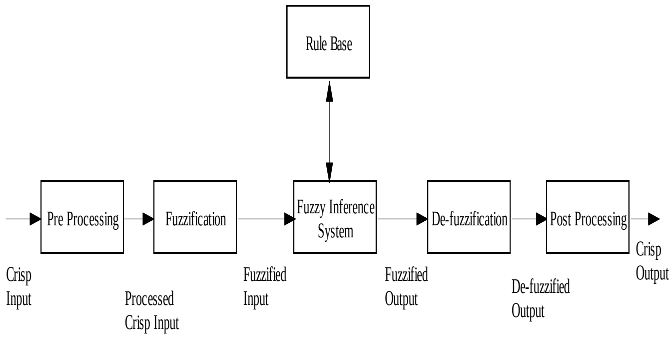


Figure 7: Block diagram of fuzzy inference system

Hybrid PD-Fuzzy Controller

Although it is possible to design a fuzzy logic type of PID controller by a simple modification of the conventional ones, via inserting some meaningful fuzzy logic IF- THEN rules into the control system, these approaches in general complicate the overall design and do not come up with new fuzzy PID controllers that capture the essential characteristics and nature of the conventional PID controllers. Besides, they generally do not have analytic formulas to use for control specification and stability analysis. The fuzzy PD, PI, and PI+D controllers to be introduced below are natural extensions of their conventional versions, which preserve the linear structures of the PID controllers, with simple and conventional analytical formulas as the final results of the design. Thus, they can directly replace the conventional PID controllers in any operating control systems (plants, processes).

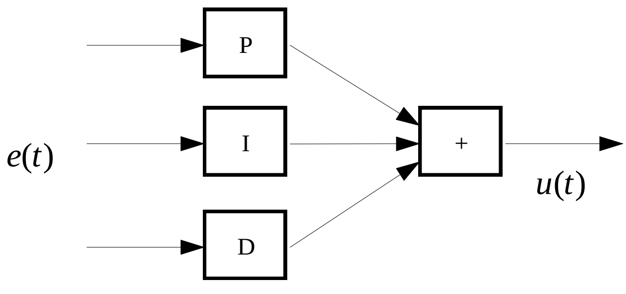


Figure 8: Parallel form of PID controller

The conventional design of PID controller was some what modified and a new hybrid fuzzy PID controller was designed. Instead of summation effect a mamdani based fuzzy inference system is implemented. The inputs to the mamdani based fuzzy inference system are error and change in error.

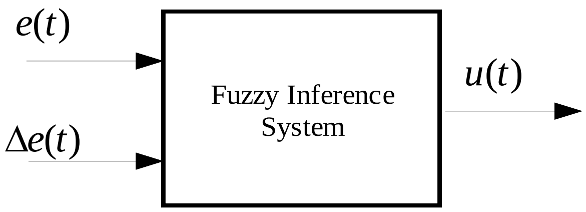


Figure 9: Fuzzy inference system

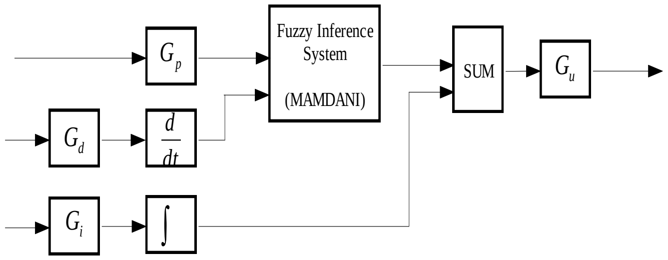


Figure 10: Architecture of proposed hybrid fuzzy controller

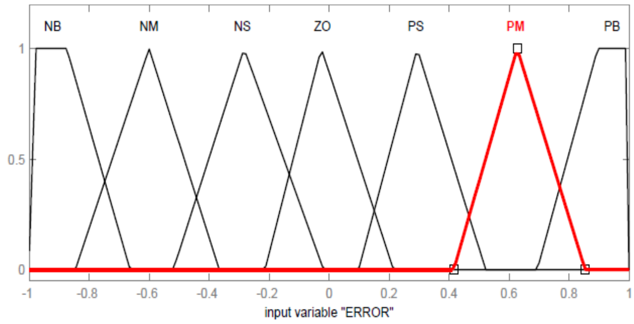
Figure 9 shows the fuzzy inference system developed for hybrid fuzzy controller. Figure 10 shows the structure of hybrid fuzzy logic controller, which keeps the general architecture of PID controller as shown in figure 8 with some slight modifications. A mamdani based fuzzy inference system is implemented in between proportional and derivative term. The integral term is then added to the output of fuzzy inference system.

G_p, G_d and G_i are scaling factors for the input where as G_u is the scaling factor for the output. In this design the input and output scaling factors are determined by trial and error methods and are taken very small. The linguistic variables used in the membership functions are described in table 1.

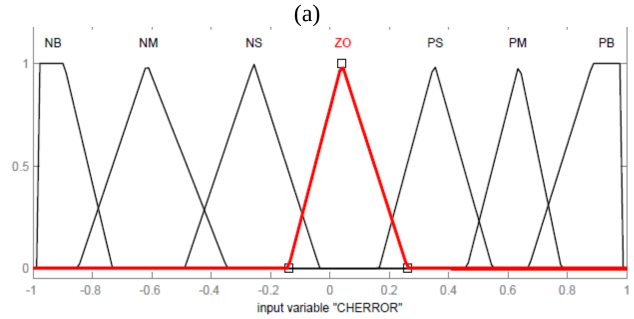
Table 1: Linguistic variable for fuzzy logic

Error e(t)		Change in error Δe(t)		Controller output u(t)	
NB	Negative Big	NB	Negative Big	NB	Negative Big
NM	Negative Medium	NM	Negative Medium	NM	Negative Medium
NS	Negative Small	NS	Negative Small	NS	Negative Small
ZO	Zero	ZO	Zero	ZO	Zero
PS	Positive Small	PS	Positive Small	PS	Positive Small
PM	Positive Medium	PM	Positive Medium	PM	Positive Medium
PB	Positive Big	PB	Positive Big	PB	Positive Big

Figure 11 (a), (b) and (c) shows the inputs and output membership functions for mamdani based fuzzy inference system.



(a)



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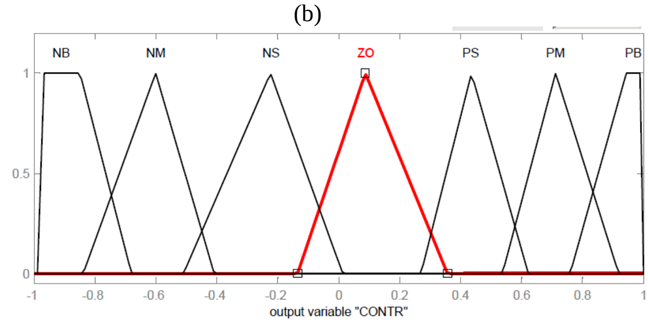

(c)

Figure 11 (a), (b) and (c): Membership function for error, change in error and output

Table 2: Fuzzy rule base

		e(t)						
u(t)		**NB**	**NM**	**NS**	**ZO**	**PS**	**PM**	**PB**
Δe(t)	**NB**	NB	NB	NB	NB	NM	NS	ZO
	NM	NB	NB	NB	NM	NS	ZO	PS
	NS	NB	NB	NM	NS	NS	PS	PS
	ZO	NB	NM	NS	ZO	ZO	PM	PM
	PS	NM	NS	ZO	PS	PS	PB	PB
	PM	NS	ZO	PS	PM	PM	PB	PB
	PB	ZO	PS	PM	PB	PB	PB	PB

Table 2 shows the fuzzy rule base for mamdani fuzzy inference system.

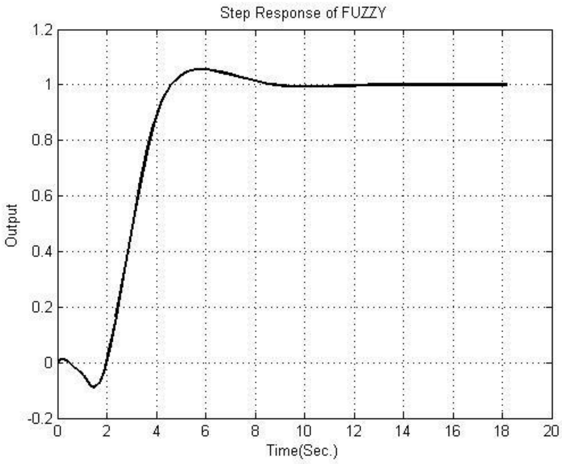

Figure 12: Unit step response of hybrid fuzzy controller for concentration control of isothermal CSTR

Figure 12 shows the unit step response of hybrid fuzzy controller for concentration control of isothermal CSTR. It is evident from the response that the overshoot is less as compared to the PID controller, which shows the efficiency of fuzzy based controllers.

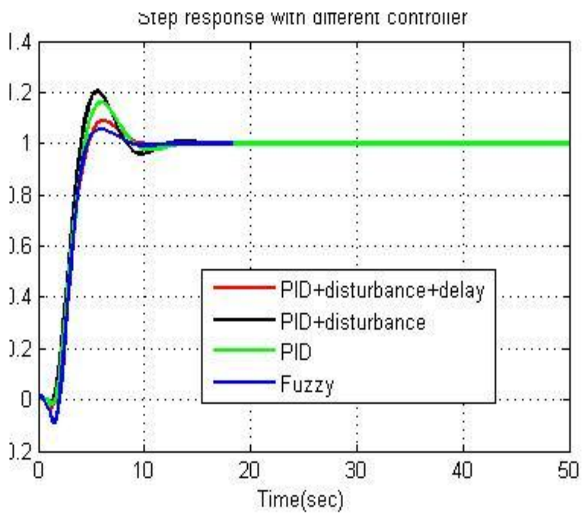

Figure 13: Comparative response of unit step response of different controller

Figure 13 shows comparative response of controller.

VII. Results & Discussion

This section evaluates the controller perfromance on the basis of tranisent response and error criteria. Table 3 shows the comparative tranisent response of conventional PID controller and fuzzy controller.

Table 3: Transient response

Parameters/ type	Peak overshoot (%)	Rise time (Sec)	Delay time (Sec)	Sett time (Sec)	Peak time (Sec)
PID	9.1251	4.679	3.0459	7.4062	6.1049
PID with disturbances	20.3425	4.365	2.956	7.0852	5.6760
PID with disturbances and delay	16.0946	4.379	2.982	7.5983	6.0825
Fuzzy Controller	5.5092	4.580	3.115	6.5306	6.0402

Table 4: Performance criteria of controllers

	IAE	ISE	ITAE
PID	2.279	1.579	6.226
PID with disturbance	2.49	1.552	8.2
PID with disturbance with delay	2.528	1.661	7.974
Fuzzy controller	2.342	1.872	5.702

Table 4 shows the error criteria for different controllers. From transient response analysis in table 3 and error analysis in table 4 it is evident that the hybrid fuzzy controller is best for concentration control.

VIII. Conclusion

This paper presents a comparative study of performance of different conventional and fuzzy based controllers. The aim of the proposed controller is to regulate the product concentration of isothermal CSTR. After time response analysis it is observed that hybrid fuzzy controller provides a satisfactory control performance.

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