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Fuzzy Sub - Trigroup – Trilevel Properties

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Abstract- In this paper, some characteristic description and a kind of representation of the fuzzy sub-trigroup with respect to operation addition are Introduced.Fuzzy sub-trigroup,union of fuzzy trigroups trilevel subset of a fuzzy sub-trigroup are defined. Properties of fuzzy sub-trigroup are proved with examples. some results of trilevel subset of fuzzy sub-trigroups are also proved.

Index Terms- Trilevel subset, Trigroup, Fuzzy sub-trigroup

I. Introduction

The concepts of fuzzy sets was introduced by Zadeh.Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers.The original concept of fuzzy sets was introduced as an extension of cribs (usual) sets, by enlarging the truth value set of "grade of members"from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld.

It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.W.B.Vasantha kandasamy introduced fuzzy sub-bigroup with respect to '+' and '.'and illustrate it with example. W.B.Vasantha kandasamy was the first one to introduce the notion of bigroups in the year 1994.Several mathematicians have followed them in investigating the fuzzy group theory. We now recall the previous and preliminary definitions, and results that are required in our discussion.

1. Preliminaries:

Definition: 1.1

Let X be a nonempty set.A fuzzy set μ of the set X is a function

$$\mu : X \to [0,1].$$

Definition: 1.2

The most commonly used range of membership functions is the unit interval [0,1].In this case,each membership function maps elements of a given universal set x,which is always a crisp set,into real numbers in [0,1]

Two distinct notations are most commonly employed in the literature to denote membership functions.

The membership functions of a fuzzy set A is denoted by μ_A

$$\tilde{A}:x \to [0,1].$$

Definition: 1.3

Let A and B be two fuzzy sets for all $\alpha \in [0,1]$

Min of A&B=$A_\alpha(\wedge)B_\alpha$

$$= [a_1^{(\alpha)},a_2^{(\alpha)}]\wedge[b_1^{(\alpha)}, b_2^{(\alpha)}]$$

$$= [a_1^{(\alpha)}\wedge b_1^{(\alpha)},a_2^{(\alpha)}\wedge b_2^{(\alpha)}]$$

Max of A&B= $A_\alpha(\vee)B_\alpha$

$$= [a_1^{(\alpha)},a_2^{(\alpha)}] \vee [b_1^{(\alpha)},b_2^{(\alpha)}]$$

$$= [a_1^{(\alpha)} \vee b_1^{(\alpha)},a_2^{(\alpha)} \vee b_2^{(\alpha)}]$$

Definition: 1.4

Let G be a group.A fuzzy subset μ of a group G is called a fuzzy subgroup of the G if i. $\mu(xy) \geq \min\{ \mu(x) , \mu(y)\}$ for every $x,y \in G$

ii. $\mu(x^{-1}) = \mu(x)$ for every $x \in G$

Definition: 1.5

Let μ be a fuzzy subset of G.For $t\in [0,1]$ the set $\mu_t = \{ x \in G / \mu(x) \geq t\}$ is called a level subset of the fuzzy subset μ.

Definition: 1.6

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2, then the fuzzy union of the fuzzy sets μ_1 and μ_2 is defined as a function.

$\mu_1\cup\mu_2: X_1\cup X_2 \to [0,1]$ given by

$$(\mu_1\cup\mu_2)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x)) & \text{if } x\in X_1\cap X_2 \\ \mu_1(x) & \text{if } x\in X_1 \text{ and } x\notin X_2 \\ \mu_2(x) & \text{if } x\in X_2 \text{ and } x\notin X_1 \end{cases}$$

Definition: 1.7

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2, μ_3 be a fuzzy subset of a set X_3then the fuzzy union of the fuzzy sets μ_1 and μ_2, μ_3 is defined as a function.

$\mu_1\cup\mu_2\cup\mu_3: X_1\cup X_2\cup X_3 \to [0,1]$ given by

$$(\mu_1\cup\mu_2 \cup \mu_3)(x) = \begin{cases} \max(\mu_1(x),\mu_2(x), \mu_3(x)) \\ \quad \text{if } x\in X_1\cap X_2\cap X_3 \\ \mu_1(x) \text{ if } x\in X_1 \,\&x \notin X_2, X_3 \\ \mu_2(x) \text{ if } x\in X_2 \,\&x \notin X_1, X_3 \\ \mu_3(x) \text{ if } x\in X_3 \& \, x \notin X_1, X_2 \end{cases}$$

II. Results

- Let $G = (G_1\cup G_2\cup G_3,+)$ be a trigroup.Then $\mu : G \to [0,1]$ is said to be a **fuzzy sub-trigroup** of the trigroup G if there exists three fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 such that
  (i) $(\mu_1,+)$ is a fuzzy subgroup of $(G_1,+)$
  (ii) $(\mu_2,+)$ is a fuzzy subgroup of $(G_2,+)$
  (iii) $(\mu_3,+)$ is a fuzzy subgroup of $(G_3,+)$
- $\mu = (\mu_1\cup\mu_2 \cup \mu_3)$.
  Let $(G = G_1\cup G_2 G_3,+)$ be a trigroup and

$\mu = (\mu_1\cup\mu_2 \cup \mu_3)$ be a fuzzy sub-trigroup of the trigroup G.The **tri level subset** of the fuzzy sub-trigroup μ of the trigroup G is defined as $(G_\mu)^t = (G_{1\mu t})^t \cup (G_{2\mu t})^t \cup (G_{3\mu t})^t$

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for every $t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ where e_1, e_2, e_3 are denotes the identity element of the group $(G_1, +)$, $(G_2, +)$, $(G_3, +)$ respectively.

- A fuzzy subset μ of a group G is said to be a union of three fuzzy sub-groups of the group G if there exists three fuzzy subgroups μ_1, μ_2 and μ_3 of μ ($\mu_1 = \mu$, $\mu_2 = \mu$ and $\mu_3 = \mu$) such that

$\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.Here by the term fuzzy subgroup λ of μ we mean that λ is a fuzzy subgroup of the group G and $\lambda \subseteq \mu$ (where μ is also a fuzzy subgroup of G).

- The condition

$t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ is essential for the trilevel subset to be a sub-trigroup for if

$t \notin [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ then the trilevel subset need not in general be a sub-trigroup of the trigroup G.

2. Properties of fuzzy sub –trigroup.

Example: 2.1

Let $x_1 = \{1,2,3,4,5\}$ and $X_2 = \{2,4,6,8,10\}, X_3 = \{1,3,5,7,9\}$ be three sets

Define $\mu_1: x_1 \to [0,1]$ by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 1,2 \\ 0.6 & \text{if } x = 3 \\ 0.2 & \text{if } x = 4,5 \end{cases}$$

Define $\mu_2: x_2 \to [0,1]$ by

$$\mu_2(x) = \begin{cases} 1 & \text{if } x = 2,4 \\ 0.6 & \text{if } x = 6 \\ 0.2 & \text{if } x = 8,10 \end{cases}$$

And Define $\mu_3: x_3 \to [0,1]$ by

$$\mu_3(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.6 & \text{if } x = 3,5 \\ 0.2 & \text{if } x = 7,9 \end{cases}$$

Hence

$$(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \begin{cases} 1 & \text{if } x = 1,2,4 \\ 0.6 & \text{if } x = 3,5,6 \\ 0.2 & \text{if } x = 7,8,9,10 \end{cases}$$

Example: 2.2

Consider the trigroup

$$G = \{\pm i, \pm 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\ldots\ldots..\}$$

Under the operation '+' where

$G_1 = \{0, \pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\ldots\ldots\ldots\}$

$G_2 = \{\pm i, 0\}, G_3 = \{0, \pm 2\}$.

Define $\mu: G \to [0,1]$ by

$$\mu(x) = \begin{cases} 1/3 & \text{if } x = i,-i \\ 1 & \text{if } x \in \{0, \pm 2, \pm 4, \ldots.\} \\ 1/2 & \text{if } x \in \{\pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\} \end{cases}$$

We can find

Define $\mu_1: G_1 \to [0,1]$ by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x \in \{0, \pm 2, \pm 4, \ldots\ldots.\} \\ 1/2 & \text{if } x \in \{\pm 1, \pm 3, \pm 5, \ldots\ldots\ldots\} \end{cases}$$

Define $\mu_2: G_2 \to [0,1]$ by

$$\mu_2(x) = \begin{cases} 1/3 & \text{if } x = \pm i \\ 1 & \text{if } x = 0 \end{cases}$$

Define $\mu_3: G_3 \to [0,1]$ by

$$\mu_3(x) = \begin{cases} 1 & \text{if } x = 0 \\ 1/2 & \text{if } x = \pm 2 \end{cases}$$

Hence there exists two fuzzy subgroups $\mu_1$of G_1, μ_2 of G_2 and μ_3 of G_3 such that $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Theorem: 2.1

Every t-level subset of a fuzzy sub-trigroup μ of a trigroup G need not in general be a sub-trigroup of the trigroup G

Proof:-

We can prove this theorem by an example

Take G = {-2,-1,0,1,2} to a bigroup under the operation '+'

Where $G_1 = \{-2,0,2\}$, $G_2 = \{-1,0,1\}$, $G_3 = \{0\}$ are groups w.r.to '+' .

Define $\mu: G \to [0,1]$ by

$$\mu(x) = \begin{cases} 1 & \text{if } x = -2,2 \\ 1/2 & \text{if } x = -1,1 \\ 1/4 & \text{if } x = 0 \end{cases}$$

Then clearly $(\mu,+)$ is a fuzzy sub-trigroup of the trigroup (G,+).

Now consider the level subset $G_\mu^{1/2}$ of the fuzzy sub-trigroup μ.

$G_\mu^{1/2} = \{-2,-1,1,2\}$ is not a sub – trigroup of the trigroup (G,+).

Hence the t level subset

G_μ^t (of t =1/2) of the fuzzy sub – trigroup μ is not a sub – trigroup (G,+).

Theorem: 2.2

Every trilevel subset of a fuzzy sub – trigroup μ of a trigroup G with respect to the usual addition is a sub-trigroup of trigroup G.

Proof :-

Let $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$ be the fuzzy subgroup of a trigroup $(G = G_1 \cup G_2 \cup G_3, +)$.

Consider the trilevel subset G_μ^t of the fuzzy sub – trigroup μ for every

$t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$

Where e_1, e_2 and e_3 denote the identity elements of the groups G_1, G_2 and G_3 respectively.

Then $G_\mu^t = G_{3\mu 3}^t \cup G_{2\mu 2}^t \cup G_{1\mu 1}^t$

Where $G_{1\mu 1}^t$, $G_{2\mu 2}^t$, and $G_{3\mu 3}^t$ are subgroups of G_1, G_2 and G_3 respectively.

Since $G_{1\mu 1}^t$ is a t-level subset of the group G_1, $G_{2\mu 2}^t$ is a t-level subset of the group G_2 and $G_{3\mu 3}^t$ is a t-level subset of the group G_3.

Hence the sub-trigroup G_μ^t is a sub-trigroup of the trigroup G.

Example: 2;3

G = {0, ±1, ±i} is a trigroup with repect to the addition.

Clearly $G_1 = \{0\}, G_2 = \{0, \pm 1\}$,

$G_3 = \{0, \pm i\}$ are groups with respect to addition.

Define $\mu: G \rightarrow [0,1]$ by

$$\mu(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0.5 & \text{if } x = \pm 1 \\ 0.3 & \text{if } x = \pm i \end{cases}$$

Therefore μ is a fuzzy sub – trigroup of the trigroup G as there exists three fuzzy subgroups $\mu_1: G_1 \rightarrow [0,1]$, $\mu_2: G_2 \rightarrow [0,1]$ and $\mu_3: G_3 \rightarrow [0,1]$ such that $\mu = \mu_1 U \mu_2 U \mu_3$

Where

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 0 \end{cases}$$

$$\begin{cases} 0.8 & \text{if } x = 0 \end{cases}$$

$$\mu_2(x) = \quad 0.5 \quad \text{if } x = \pm 1$$

$$\mu_3(x) = \begin{cases} 0.9 & \text{if } x = 0 \\ 0.3 & \text{if } x = \pm i \end{cases}$$

Now we calculate the trilevel subset G_μ^t for t = 0.5

$G_\mu^t = G_{3\mu 3}^t \cup G_{2\mu 2}^t \cup G_{1\mu 1}^t$

$= \{x \in G_1 / \mu_1(x) \geq t\} \cup \{x \in G_2 / \mu_2(x) \geq t\} \cup \{x \in G_3 / \mu_3(x) \geq t\}$

$G_\mu^t = \{0\} \cup \{0, \pm 1\} \cup \{0\}$

$= \{0, \pm 1\}$

$\therefore \quad G^t = \{0, \pm 1\}$ is a sub-trigroup of the trigroup G.

Theorem: 2.3

Let $\mu = \mu_1 U \mu_2 U \mu_3$ be a fuzzy sub – trigroup of the trigroup G.Where μ_1, μ_2 and μ_3 are fuzzy sub groups of the group G.The trilevel subset G_μ^t of μ for

$t \in [0, \min \mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ can be represented as the union of three subgroups of the group G.

That is $G_\mu^t = G_{3\mu 3}^t \cup G_{2\mu 2}^t \cup G_{1\mu 1}^t$.

Proof:-

Let μ be a fuzzy sub – trigroup of the group G with respect to addition and

$t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$

$\Rightarrow$ there exists fuzzy subgroups μ_1, μ_2 and μ_3 of the group G such that $\mu = \mu_1 \cup \mu_2 \cup \mu_3$.

Let G_μ^t be the level subset of μ,

Then we have

$x \in G_\mu^t \Leftrightarrow \mu(x) \geq t$

$\Leftrightarrow \max\{\mu_1(x), \mu_2(x), \mu_3(x)\} \geq t$

$\Leftrightarrow \mu_1 \geq t$ or $\mu_2(x) \geq t$ or $\mu_3(x) \geq t$

$\Leftrightarrow x \in G_{1\mu 1}^t$ or $x \in G_{2\mu 2}^t$ or $x \in G_{3\mu 3}^t$

$\Leftrightarrow x \in G_{1\mu 1}^t \cup G_{2\mu 2}^t \cup G_{3\mu 3}^t$

Hence $G_\mu^t = G_{1\mu 1}^t \cup G_{2\mu 2}^t \cup G_{3\mu 3}^t$

III. CONCLUSION

In this paper with the basic concepts of group theory,we define related concepts of fuzzy sub-trigroup and further we proved properties of fuzzy sub-trigroup.

REFERENCES

[1] S. AbouZaid, On fuzzy subnearrings and ideals, Fuzzy Sets and Sys. 44 (1991), 139146.

[2] Yuan Bo and Wu Wangming, Fuzzy ideals on a distributive lattice, Fuzzy Sets and Sys. 35 (1990), 231240.

[3] P. S. Das, Fuzzy groups and level subgroups, J. Math. Anal. and Appl. 84 (1981), 264269.

[4] V. N. Dixit, R. Kumar and N. Ajal, On fuzzy rings, Fuzzy Sets and Sys. 49 (1992), 205213.

[5] N. Kuroki, Fuzzy biideals in semigroups, Comment.Math. Univ. St. Pauli. 28 (1979), 1721.

[6] N. Kuroki, On fuzzy ideals and fuzzy biideals in semigroups, Fuzzy Sets and Sys. 5 (1981), 203215.

[7] G.Nirmala,S.Suganthi, fuzzy algebra –presended the paper in national conference on Recent advances in pure and applied mathematics –Govt.Arts college for women (Aut),Pudukkottai.

[8] G.Nirmala,S.Suganthi published the paper-Lagrange's theorem in fuzzy approach-The pmu journal of Humanities and Sciences-july-december 2011.

[9] W. B. Vasantha Kandasamy, Smaramdache fuzzy Algebra American Research Press Rehoboth 2003. 10.Zimmermann H., Fuzzy Set Theory and its Applications (2001), ISBN 978 – 0 – 7923- 7435 – 0

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