

# H $\infty$ Filter Design for Anti-skid Braking System for Fighter Aircraft

Ritesh Singh, Vidya S. Rao, Dr. Shreesha C.

Instrumentation & Control Engineering Department, Manipal Institute of Technology,  
Manipal, Karnataka, India

**Abstract-** Aircraft brake system plays a very important role in the safe takeoff and landing. On landing, skid phenomena is observed due to runway surfaces, pilot brake demand or due to other factors which creates unsafe for the aircraft while landing. So we use anti-skid braking system which will prevent tires from slipping extremely on the track and ensure aircraft landing safety and stability with maximum braking efficiency. Estimate of the slip is estimated using a technique called H $\infty$  filter design. The purpose of estimation is to make the slip, error free, from the noises coming out from the sensors and other measuring devices which affect the system parameters. This paper presents modeling and simulation part of the work for the aircraft brake system.

**Index Terms-**  $\mu$ -slip curve, slip-ratio, anti-skid, H $\infty$  filter

## I. INTRODUCTION

IN the last few decades, a number of techniques have been used to control the skid of the fighter jets and as well as the commercial aircrafts during landing process by using skid control devices. Number of aircrafts using this antiskid braking system has been increased over the years for the safe landing over different landing conditions. Due to the safety reasons and to avoid skidding, various techniques were in design process and were used, such as velocity-rate-controlled, pressure-bias-modulated [9]; slip-velocity-controlled, pressure-bias-modulated [1]; hydro-mechanically-controlled system [2] and slip-ratio-controlled system [5]. To provide maximum braking and complete antiskid protection under all weather conditions is demanded by the modern skid control devices. New design approaches and control techniques are used for better performance of these systems during landing process on all runway conditions. The work presented here uses the slip-ratio-controlled design technique to solve the skidding problem.

Nose wheel steering system is widely used for modern aircraft steering. The facts shown as follows put up high requirements to it: side tumbling will not occur in aircraft ground steering; the modern civil aircraft rollout and turnoff should be quick and reduce the runway occupancy time, the fighter should land, taxi, and take-off on the runway of harsh conditions to enhance rapid reaction capability in future war; the aircraft should keep the correct heading when it taxiing under asymmetric loads, such as: cross-wind landing, one side main wheel bursting, and one side main wheel brake failure.

Tests on life-size aircraft are obviously expensive and risky, and experiments on test-rigs (namely drop-test facilities) allow only limited deduction of information about the landing

gear dynamics. Based on dynamics theory, the virtual prototyping software has been used widely for aircraft ground maneuvers dynamics study [2]. Simulation technique demonstrated here well suits for this kind of design and analysis. Simulation combines the ability to simulate models including a large number of degrees of freedom, arbitrary non-linear motion, and highly non-linear force laws with powerful analysis methods and a fast model set-up using components from libraries and from neighboring engineering disciplines as established in MATLAB/SIMULINK. A PID antiskid controller is designed and control algorithm simulation model is established in MATLAB/SIMULINK.

Skid control calculations are performed in the following manner. The instantaneous speed of each wheel is periodically updated and compared to a calculated aircraft velocity. The difference between wheel speed and aircraft velocity represents wheel slip. The nose wheel is not braked. When the slip exceeds a given magnitude, braking effectiveness begins to decrease. The control unit detects this excessive wheel slip and produces a brake release signal proportional to the skid severity. The release signal commands a brake pressure reduction until the wheel slip returns to the optimal level. When slip is below the amount required to produce maximum braking, no release signal is generated. The pilot's brake pedal input controls the braking level [3]. The complete architecture of the aircraft is shown in the

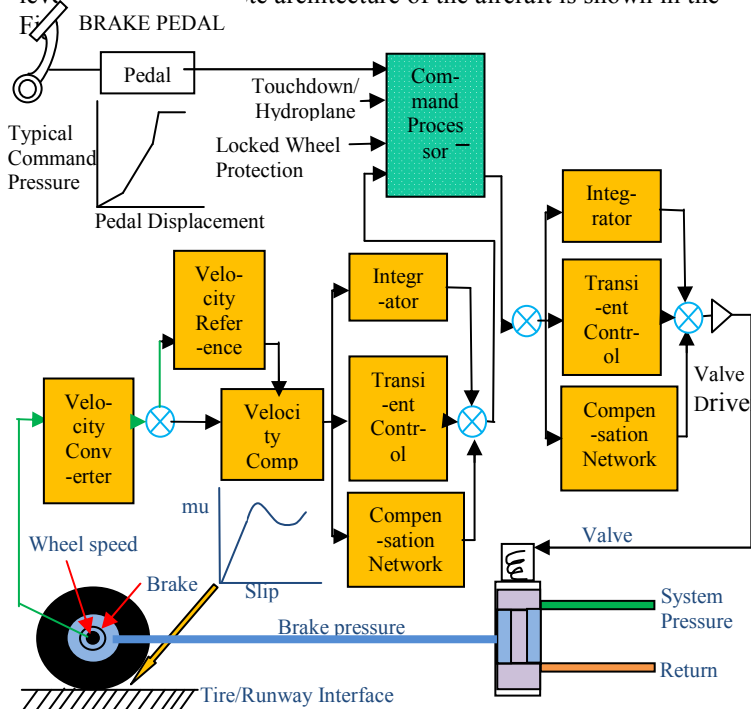


Figure 1 Architecture of Aircraft Brake System

II. CONTROL TECHNIQUE

The resultant friction coefficient between the tire base and the road surface is determined as a function of the resultant slip ratio. There is non-linear relationship between friction coefficient and slip ratio, and the runway conditions have great impact on that, as shown in Figure 2. The purpose of the anti-skid control is to make the slip ratio stay at the appropriate scope, so that the friction coefficient can reach the peak.

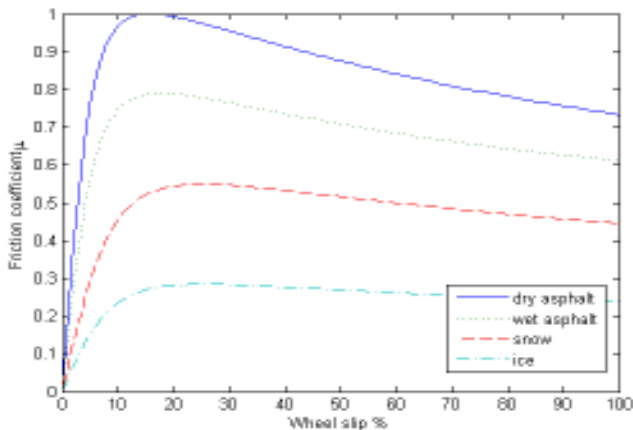


Figure 2 μ-slip curve

When braking force is applied to a rolling wheel, it will begin to slip. Slip ratio  $S$  is defined as the difference between aircraft velocity and wheel circumferential velocity, normalized to aircraft velocity –

$$S = \frac{V_x - \omega.r}{V_x} \quad (1)$$

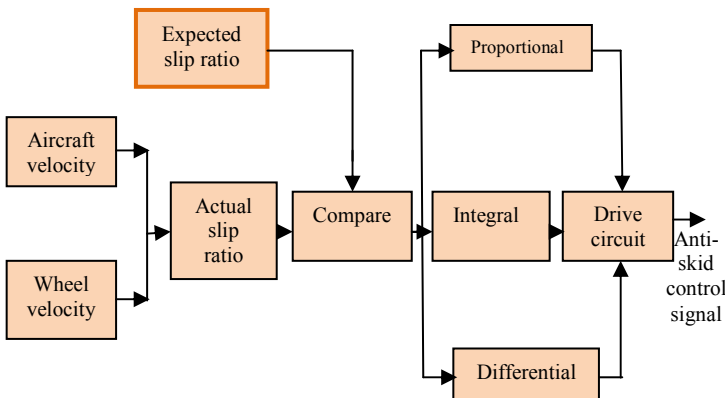


Figure 3 Principle diagram of slip-ratio control

The method used here is the slip-ratio-controlled mechanism as shown in Figure 3. In this we have the expected or desired slip ratio which we want to control. It is compared with the actual

slip ratio generated and is given to the control principle. And hence anti-skid control signal is generated.

The anti-skid control system consists of velocity sensor, control box, and electro hydraulic servo valve [4] as shown in Figure 4. The velocity sensor induces the velocity of the brake wheel and sends it to the control box. The control box outputs a current signal proportional to slip degree in

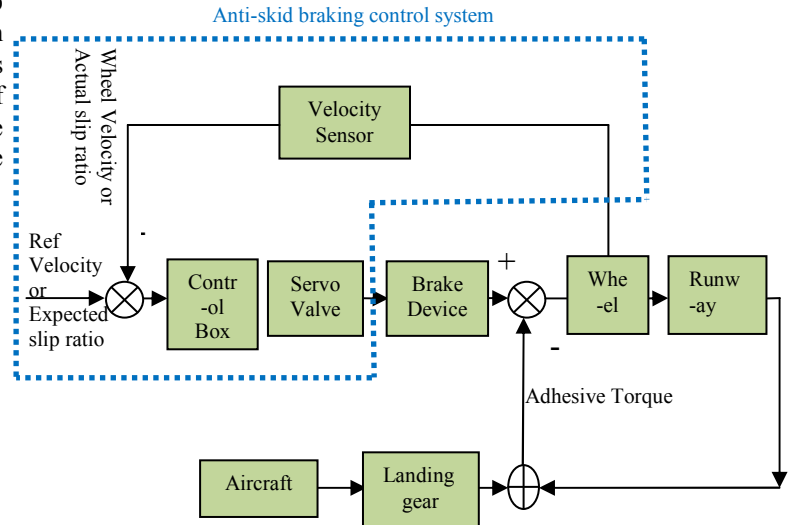


Figure 4 Principle diagram of anti-skid braking system

accordance with certain control strategy. The current signal regulates the output pressure of the electro hydraulic servo valve to change the brake force and lift slipping. The system obtains the best brake effect by regulating the brake. The control method is adopted to control brake force of the left and right wheels. Because the motion state of the two wheels is not very different from each other, the same control channel can be used to these two wheels. The control method, in Figure 4 is simplified to right and left wheels.

In the process of control, the difference of wheel velocity should be not too great, or else, that will lead to aircraft sideslip or roll phenomenon, and the movement will become the S-type, deviating from normal routes, threatening the safety of aircraft. For each side, the low wheel velocity is selected as the velocity of this group. Each side is controlled through a single channel. The cross protection doesn't work unless the velocity difference between left and right group is great.

III. DESIGN METHODOLOGY

A. Aircraft Brake System

Aircraft brake system is brake-by-wire systems consisting of hydraulic brakes mounted on left and right main wheels and are activated by electro-hydraulic servo valves. A digital electronics controller with embedded control law software is used to activate the servo valves. The brake pressure is maintained as demanded by the pilot through a (electrical) potentiometer mounted on a brake pedal for each brake. However, if the demand is high enough to result in tire skid, then, brake pressure is modulated to the maximum value that can be used without a tire skid.

**B. Aircraft Loading Factor**

Choice of the main wheel tire and nose wheel is made on the basis of the static and dynamic loading [10]. The total main gear load ( $F_m$ ) is calculated assuming that the aircraft is taxiing at low speed without braking. As shown in Figure 5, equilibrium gives -

$$F_m = \frac{l_n}{l_m + l_n} W \quad (2)$$

where  $W$  is the weight of the aircraft and  $l_m$  and  $l_n$  are the distance measured from the aircraft  $cg$  to the main and nose gear, respectively. The design condition occurs at maximum takeoff weight/mass of an aircraft with the aircraft  $cg$  at its after limit [10]. The choice of the nose wheel tires is based on the nose wheel load ( $F_n$ ) during braking at maximum effort, i.e., the steady braked load. Using the symbols shown in Figure 5, the total nose gear load under constant deceleration is calculated using -

$$F_m = \frac{l_n}{l_m + l_n} (W - L) + \frac{h_{cg}}{l_m + l_n} \left( \frac{a_x}{g} W - D + T \right) \quad (3)$$

where  $L$  is the lift,  $D$  is the drag,  $T$  is the thrust, and  $h_{cg}$  is the height of aircraft  $cg$  from the static ground line. Typical values for  $a_x/g$  on dry concrete vary from 0.35 for a simple brake system to 0.45 for an automatic brake pressure control system. As  $D$  and  $L$  are positive, the maximum nose gear load occurs at low speed. Reverse thrust affect can be given from [10], when  $T = 0$  we have value of  $F_n$  -

$$F_n = \frac{l_m + h_{cg} (a_x / g)}{l_m + l_n} W \quad (4)$$

The design condition occurs at maximum takeoff weight/mass of an aircraft with the aircraft  $cg$  at its forward limit. To ensure that the loads will not be exceeding in the static and braking conditions, a 7% safety factor is used in the calculation of the applied loads [10].

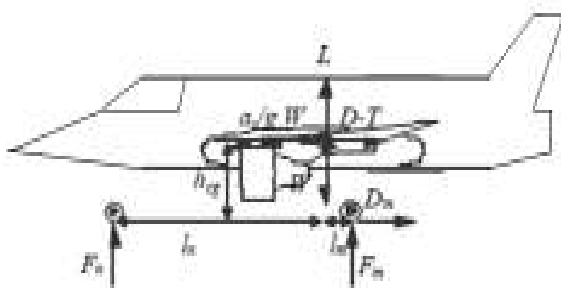


Figure 5 Forces acting on the aircraft during a braked roll

**C. Wheel Dynamics**

Although the dynamic model of aircraft taxiing is complex, by neglecting the air resistance and the suspension dynamics during the braking operation, a simplified mathematical model of single-wheel is adopted. The diagram of single-wheel is shown in

Figure 6. Where  $J$  is wheel inertia,  $\omega$  is angular speed of the wheel,  $F_b$  is tire friction force,  $r$  is wheel radius,  $T_b$  is applied brake torque.  $V_x$  is aircraft velocity relative to ground and  $F_z$  is vertical load on the wheels(N). The equation of the rotation of the wheel is expressed as -

$$J\dot{\omega} = F_b r - T_b \quad (5)$$

For braking at a steady rotational velocity, Eqn. 5 yields -

$$\mu F_z = \frac{T_b}{r}, \text{ where } F_b = \mu F_z.$$

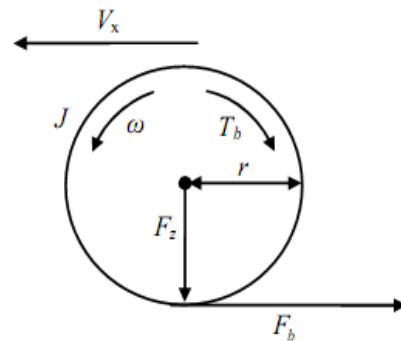


Figure 6 Wheel Dynamics

**D. Hydraulic System**

The reference model used for this study includes the elements to form a third order set (wheel dynamics, hydraulic system and its control). The mathematical representation of the behavior of the system is based on an intermediary modeling level allowing the main elements comprising its dynamics to be reconstructed.

The generation of the braking torque ( $C_{BRK}$ ) from a braking pressure applied in the pistons ( $P_{BRK}$ ) is relatively fast. For better representation, it is however in general represented by a first order transfer function -

$$C_{BRK}(s) = G_{BRK} \frac{1}{1 + \tau.S} P_{BRK}(s) \quad (6)$$

Behavior of the brake discs is modeled by a gain  $G_{BRK}$  and the hydraulic lag is given by  $- 1/ (1+\tau.s)$ . The braking pressure is generated by the hydraulic system from a flow rate ( $Q_{BRK}$ ) sent to the pistons [8] -

$$P_{BRK}(s) = \int_0^t \frac{\beta}{V_{BRK}} Q_{BRK} dt \quad (7)$$

Variable  $\beta$  represents a compressibility coefficient, which allows variations in the volumes of the pistons (and the nonlinear stiffness coefficient associated with it) to be compared to the compressibility of the hydraulic fluid. The value of  $\beta$  can vary according to the pressure applied in the pistons. The flow rate at piston inlet is generated by the difference between the supply pressure  $P_A$  and the return pressure  $P_R$  and is related to the position of the servo valve slide value ( $x_{BRK}$ ). This flow rate is written -

$$Q_{BRK} = \eta S (X_{BRK}) \sqrt{\square P} \quad (8)$$

$$\Delta P = 0, \text{if } -x_{BRK} = 0$$

where - 
$$\Delta P = P_a - P_{BRK}, \text{if } -x_{BRK} < 0 \quad (9)$$

$$\Delta P = P_{BRK} - P_a, \text{if } -x_{BRK} > 0$$

and  $\eta$  is the flow coefficient.  $S(.)$  represents the section for the passage of the hydraulic fluid through the servo valve. The position of the servo valve slide valve can be deduced from the following equation -

$$S. (X_{BRK}) = K. X_{BRK} \quad (10)$$

The position of the servo valve slide value can be deduced from the following equation -

$$X_{BRK} = \frac{K_2 S_2 - K_1 S_1}{r} (a_{SVBRK} - ISV_{BRK}) + X_{0BRK} + \frac{S_2 - S_1}{r} P_{BRK} \quad (11)$$

$ISV_{BRK}$  is the servo valve control current.  $S_1$  and  $S_2$  make reference to the slide valve sections in the left and right chambers of the servo valve.  $r$  is the stiffness of the spring re-centering the slide valve. Constants  $K_1$ ,  $K_2$  and  $a_{SVBRK}$  characterize the behavior of the first stage of the servo valve. The control of the braking pressure can be approximated by a first order corrector.

Finally, the anti-skid control is a complex design. Its aim is to prevent the wheels from skidding. It limits the braking pressure target so as to maintain the slip velocity close to its optimum value. To do so, a detection algorithm compares the wheel velocities with the velocity of the aircraft as shown in the Fig. 7.

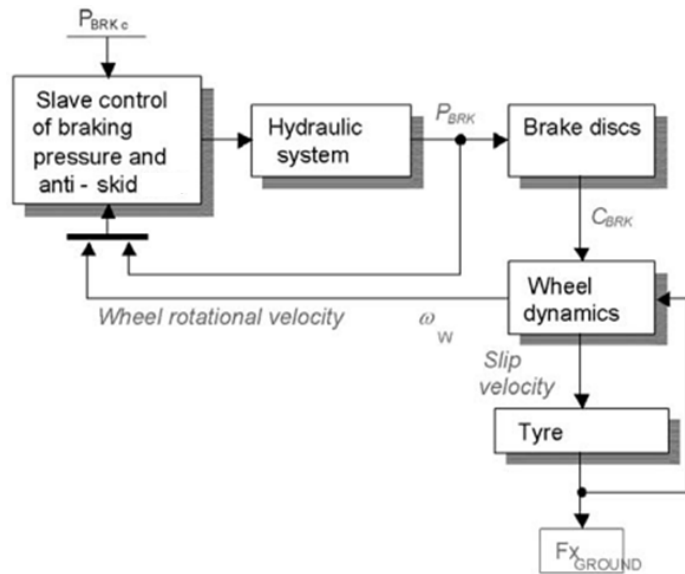


Figure 7 Schematic representation of braking system

#### IV. ESTIMATING SLIP CURVE BY $H_\infty$ FILTERING

The tire  $\mu$  slip curve is used to describe the developed friction of a braked wheel. It is empirical in nature. A formula that has been developed with many undetermined coefficients is so general that it has been called the “Magic Formula”. Unfortunately, the equipment needed to determine all the

parameters is beyond the means of modest wheel test facilities. For antiskid design, knowing only the amplitude and its location are sufficient. Therefore, we will use a much simpler formula that has only two parameters. They are the amplitude of the curve,  $\mu$  (mu), and the  $x$  axis location of the peak, slip as shown in Figure 2.

We consider the continuous-time system;

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + v \\ z &= Lx \end{aligned} \quad (12)$$

where  $L$  is a user-defined matrix and  $z$  is the vector that we want to estimate. Our estimate of  $z$  is denoted  $\hat{z}$ , and our estimate of the state at time 0 is denoted  $\hat{x}(0)$ . The vectors  $w$  and  $v$  are disturbances with unknown statistics; they may not even be zero-mean. In the game theory approach to  $H_\infty$  filtering we define the following cost function:

$$J_1 = \frac{\int_0^T \|z - \hat{z}\|_S^2 dt}{\|x(0) - \hat{x}(0)\|_{P_0}^2 + \int_0^T (\|w\|_{Q^{-1}}^2 + \|v\|_{R^{-1}}^2) dt} \quad (13)$$

$P_0$ ,  $Q$ ,  $R$  and  $S$  are positive definite matrices chosen by the engineer based on the specific problem [6]. Our goal is to find an estimator such that -

$$J_1 < \frac{1}{\theta} \quad (14)$$

The estimator that solves this problem is given by -

$$\begin{aligned} P(0) &= P_0 \\ \dot{P} &= AP + PA^T + Q - KCP + \theta PL^T SLP \\ K &= PC^T R^{-1} \\ \dot{\hat{x}} &= A\hat{x} + Bu + K(y - C\hat{x}) \\ \hat{z} &= L\hat{x} \end{aligned} \quad (15)$$

These equations are identical to the continuous-time Kalman filter equations, except for the  $\theta$  term in the  $\dot{P}$  equation. The inclusion of the  $\theta$  term in the  $\dot{P}$  equation tends to increase  $P$ , which tends to increase the gain  $K$ , which tends to make the estimator more responsive to measurements than the Kalman filter. This is a way of robustifying the filter to uncertainty in the system model [7]. The estimator given above solves the  $H_\infty$  estimation problem if and only if  $P(t)$  remains positive definite for all  $t \in [0, T]$ . As with the discrete-time filter, we can also obtain a steady-state continuous-time  $H_\infty$  filter. To do this we let  $\dot{P} = 0$  so that the differential Riccati equation above reduces to an algebraic Riccati equation.

### V. SIMULATION RESULTS

Simulation shown here shows how the aircraft brake system can be controlled using the slip-ratio-controlled design. Further PID controller is used to control the slip generated. The main purpose of the simulation is to regulate the slip generated due the aircraft tire in all runway conditions while landing process of the aircraft. The simulation work is carried out in MATLAB & SIMULINK (R2010a) work environment. And the slip generated, is estimated using  $H_\infty$  filter and hence is compared with the measured slip for the optimal performance.

To model this system in SIMULINK, several subgroups are used to avoid confusion. Appendix shows the aircraft brake system design to control the antiskid phenomena. It consists of the brake pedal both left and right to control the aircraft during landing and stop process. Signal from brake pedals is commanded to the hydraulic box to control the brake pressure and hence in proportion brake torque is applied. During landing, aircraft parameters with the aircraft velocity are sent to the control box were aircraft velocity and the wheel velocity is used to calculate the slip ratio using Eqn. 1 and;

$$\dot{s} = \frac{\dot{V}(1-s) - \dot{\omega} \cdot r}{V} \tag{16}$$

where  $\dot{\omega}$  can be manipulated using Eqn. 5. The generated slip is fed back to the antiskid control box, were the generated slip is subtracted from the desired slip. Hence the error is given to the PID controller, which in turn generates the control signal and is sent to the hydraulic control box for action.

The desired slip is kept at a value of 0.2, which is a constraint to avoid skidding. Initial value of the aircraft is taken as 90 m/sec with an angular velocity of 312 rad/sec. The wheel radius is taken as 0.341 m and with a rolling radius,  $r$  as 0.289 m. Moment of inertia in Eqn. 5 is taken as 1.84 kgm<sup>2</sup>. The initial brake pressure used is 270 Psi with a maximum of 1600 Psi. Aircraft mass is taken as 5000 kg. Actuator used here is a second order linear actuator which has a natural frequency of 150 rad/sec and a damping ratio of 0.7. The hydraulic system is modeled as a lag element with a first order system having the transfer function as,  $1 / (0.01s + 1)$ . And value of  $g$  is 9.8 m/sec<sup>2</sup> (32.2 ft/sec).

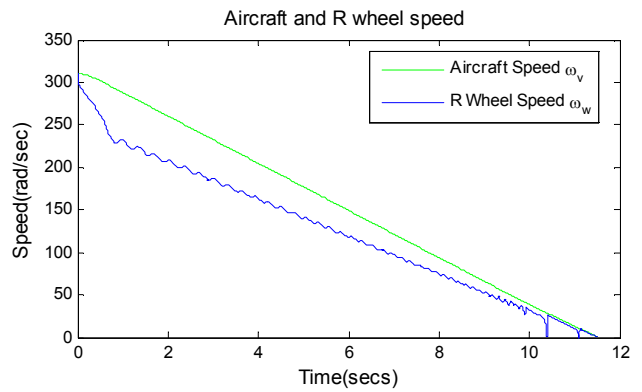


Figure 9 Aircraft and right wheel speed

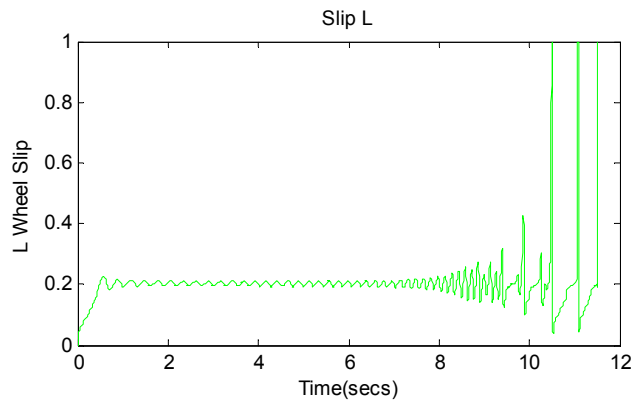


Figure 10 Left wheel slip

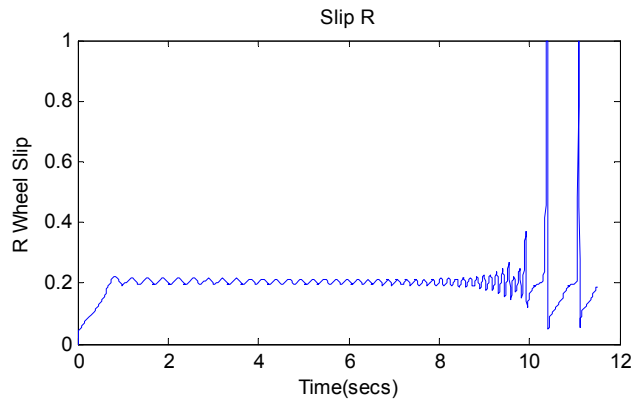


Figure 11 Right wheel slip

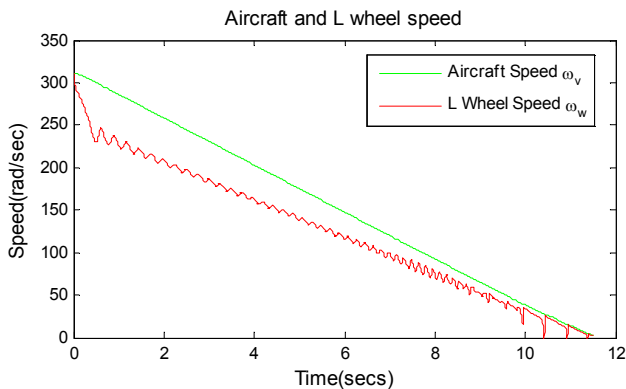


Figure 8 Aircraft and left wheel speed

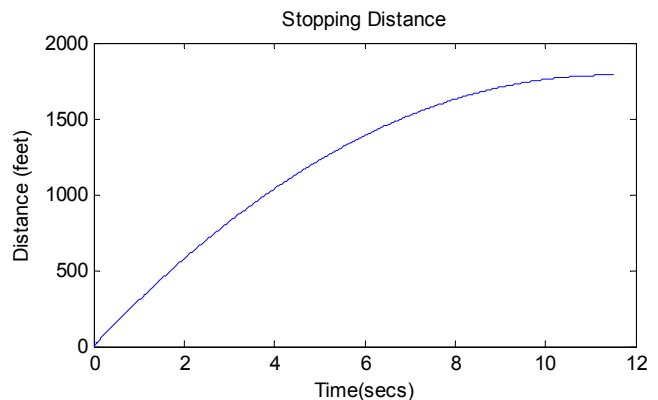


Figure 12 Aircraft stopping distance

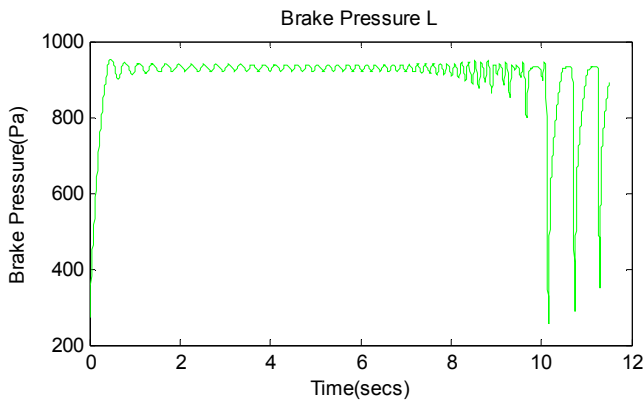


Figure 13 Left wheel brake pressure

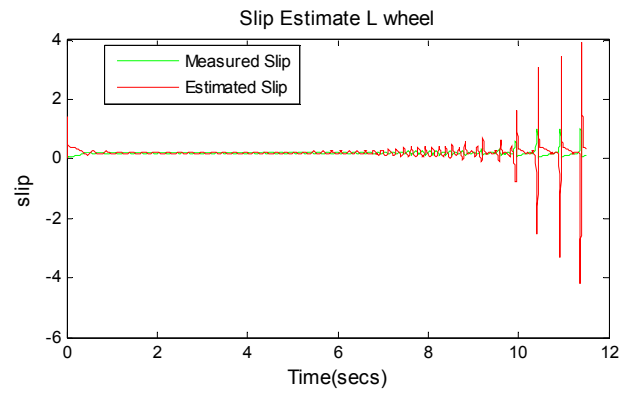


Figure 17 Left wheel slip estimate

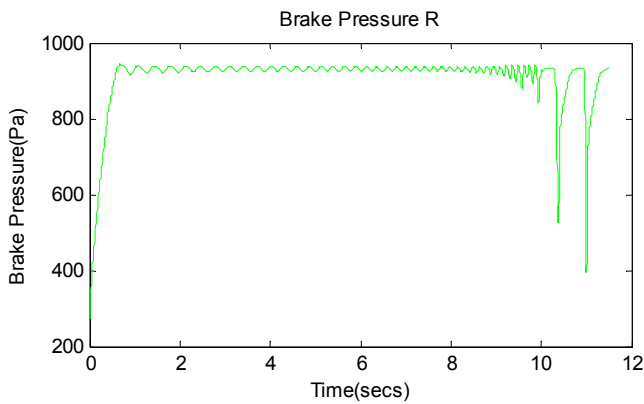


Figure 14 Right wheel brake pressure

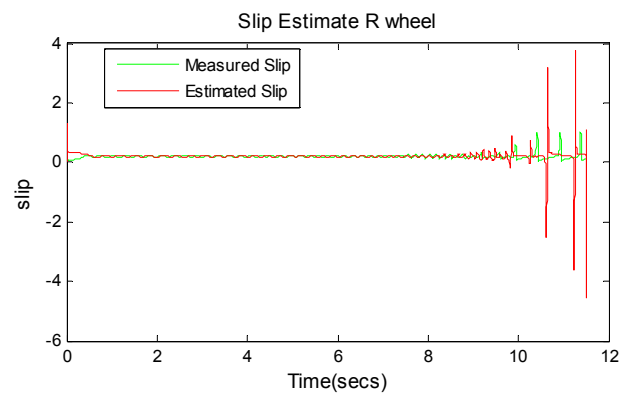


Figure 18 Right wheel slip estimate

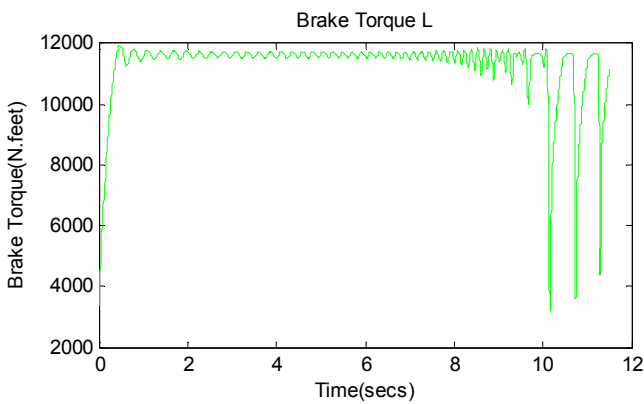


Figure 15 Left wheel brake torque

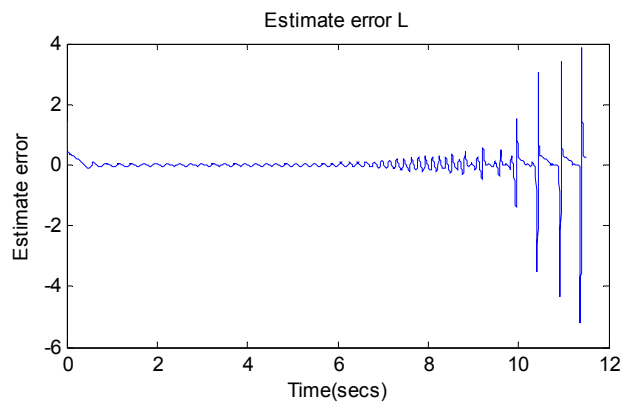


Figure 19 Left wheel estimate error

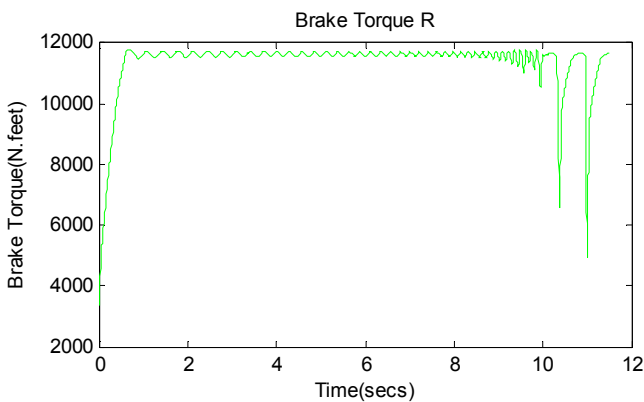


Figure 16 Right wheel brake torque

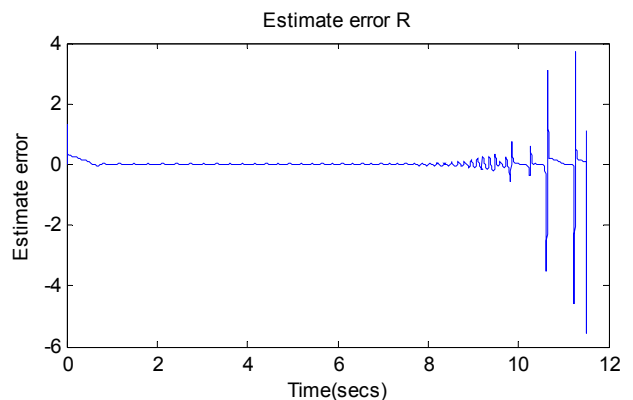


Figure 20 Right wheel estimate error



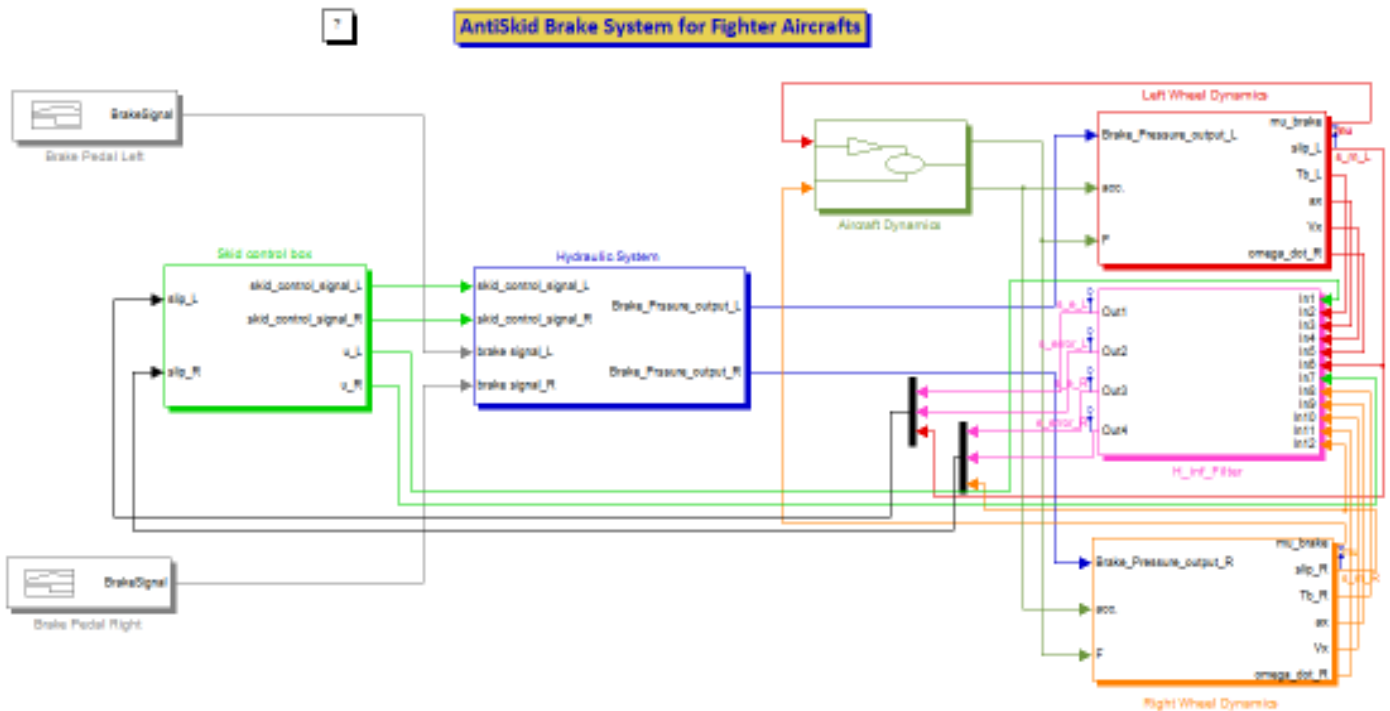
## VI. CONCLUSION

The wheel velocity and aircraft velocity show the aircraft retardation with the applied brake to the wheels. This shows that the applied brake pressure to the brake discs to stop the wheels result in the brake torque. This in turn provides the maximum friction coefficient, which is applied by the wheels on the runway to slow down the aircraft. The control mechanism, slip-ratio-controlled design is successfully implemented in controlling the slip on the runway. And hence the antiskid control technique is developed. The slip signal which is fed back to the control device gets noise attached to it. So, the estimation made in this work estimates the slip using  $H_\infty$  filter design.

The estimated slip generated from the filter is almost same as the measured slip. Hence the filter designed to make noise free slip is achieved.

Hence the design and objective performance of antiskid brake system for fighter aircrafts is successfully simulated using the simulation environment. Modeling of individual components of the system is validated through specific tests. Validation of all subsystem modeling is carried out through specific subsystem level tests. Simulation results match closely with dynamometer response during specific taxi tests or dynamometer tests provided in the open literature and in the research papers.

## APPENDIX



## ACKNOWLEDGMENT

My sincere thanks to Mr. Sayeed Ahmad and Mr. P S Udupa Aircraft Research & Design Centre (ADRC), HAL for their support and valuable suggestions. And I would like to acknowledge my other authors for their support and guidance.

## REFERENCES

- [1] NASA Technical Paper-1051, "Behaviour of aircraft antiskid braking systems on dry and wet runway surfaces", Dec. 1979.
- [2] NASA Technical Paper-1877, "Behaviour of aircraft antiskid braking systems on dry and wet runway surfaces", Aug. 1981.
- [3] Norman S Currey, "Aircraft Landing Gear Design: Principles and Practices", AIAA Education Series, 1998, ISBN 0930403-41-X.
- [4] Gregg Butterfield, "A brief introduction to aircraft antiskid" Crane Company, Hydr-Aire Division, ADIUS Conference 1993.
- [5] NASA Technical Paper-8455, "Behaviour of aircraft antiskid braking systems on dry and wet runway surfaces", Oct. 1977.
- [6] Dan Simon, "Optimal State Estimation – Kalman,  $H_\infty$ , and Nonlinear Approaches", Wiley Interscience, 2006, ISBN 13 978-0-471-70858-2.

- [7] Huijun Gao and Tongwen Chen, " $H_\infty$  Estimation for Uncertain Systems with Limited Communication Capacity", IEEE Transactions on Automatic Control, IEEE, Vol. 52, No. 11 Nov. 2007.
- [8] Zang, Hang, Wie & Zhou, "Modeling and simulation of aircraft anti-skid braking and steering using co-simulation method", The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Vol. 28 No. 6, 2009.
- [9] NASA Technical Paper-8332, "Behaviour of aircraft antiskid braking systems on dry and wet runway surfaces", Dec. 1976.
- [10] W. H. Mason, "Tires, Wheels, and Brakes", source: www.dept.aoe.vt.edu.

## AUTHORS

**First Author** – Ritesh Singh, M Tech. ( Control Systems ) Student, Dept. of I & C Engg., MIT Manipal, Karnataka, [ritesh.singh23@gmail.com](mailto:ritesh.singh23@gmail.com)  
**Second Author** – Asst. Prof. Vidya S Rao, Dept. of I & C Engg. MIT Manipal, Karnataka, [raovid@yahoo.co.in](mailto:raovid@yahoo.co.in)  
**Third Author** – Dr. Shreesha C., PhD, HOD, Dept. of I & C Engg., MIT Manipal, Karnataka, [shreesha.c@manipal.edu](mailto:shreesha.c@manipal.edu)