Selection of Bayesian Chain Sampling Attributes Plans Based On Geometric Distribution

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Abstract- A formula to compute the average probability of acceptance of chain sampling plans (ChSP-1) by attributes under Beta- Geometric distribution is provided. The performance/ discriminating power of the beta-geometric sampling plans is also discussed by determining the operating characteristic curve. The Average Probabilities of Acceptance are compared with conventional sampling plans.

Index Terms- Bayesian Chain Sampling Plan, Beta-Geometric distribution, Operating Characteristic curve.

I. Introduction

Inspection of raw materials or semi-finished products, or finished products is one of the aspects of quality assurance. When inspection is done for the purpose of acceptance or rejection of lots of finished products based on adherence to a standard, the type of inspection procedure employed is usually called acceptance sampling. The acceptance sampling is used primarily for incoming or receiving inspection and it has become typical to work with suppliers to improve their performance. Acceptance sampling is also useful in situation where testing is destructive, the cost of 100% inspection is extremely high, the time taken would be too long, the inspection error rate is too high and the product liability risks are serious.

Dodge (1969) has given the major areas of acceptance sampling as

- Sampling by the methods of attributes, in which each unit in a sample is inspected on a go-no-go basis for one or more characteristics.
- Sampling by the methods of variables, in which each unit in a sample is measured for a single characteristics, such as weight or strength.
- Continuous sampling of a flow of units by the methods of attributes.
- Special purpose plans includes chain sampling, skip-lot sampling, small sample plans etc,

Bayesian Acceptance Sampling Plan

Bayesian statistics is directed towards the use of sample information. Thomas Baye's (1702 – 1761) was the first to use prior information in inductive inference and the approach to statistics, which formally seeks to utilize prior information, is called Bayesian analysis. Suppose a product due to random fluctuation these lots will differ in quality even though the process is stable and in control. This fluctuation can be separated into within lot (sampling) variation of individual units and between (sampling and process) variation.

Bayesian Acceptance Sampling approach is associated with utilization of prior process history for the selection of distribution viz., Gamma-Poisson, Beta-Binomial to describe the random fluctuation involved in an Acceptance sampling. Bayesian plans require the user to specify explicitly the distribution of defectives from lot to lot.

The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The combination of prior knowledge, represented with the prior distribution and the empirical knowledge based on the sample leads to the decision of the lot.

Beta-Binomial distribution.

Let 'X' be the outcome of 'n' Bernoulli trials with a fixed probability 'p' and let the corresponding probability be denoted by

$$b(x, n, p) = nc_x\, p^x\, q^{n-x},$$

Assuming that 'p' has a prior distribution with density w (p), the marginal distribution of X, the mixed binomial distribution is given as

$$b_w(x, n, p) = \int_0^1 b(x, n, p)\, w(p)\, dp$$

Nicholas Lauer (1978) proposed a procedure for Acceptance Probabilities for Single Sampling Plans where the Proportion Defective has a Beta Distribution.Rajagopal, Loganathan, Vijayaraghavan (2009) provided a procedure for the selection of Bayesian Single Sampling Attributes Plans Based on Polya Distribution

Chain sampling plan (ChSP –1)

Sampling inspection in which the criteria for acceptance and non -acceptance of the lot depends on the results of the inspection of immediately preceding lot is adopted in chain sampling plan (ChSP–1) which is proposed by Dodge (1955).This plan make use of cumulative results of several samples help to overcome the short coming of the single sampling plan.

Conditions for Application of (ChSP –1).

The cost of destructiveness of testing is such that a relatively small sample size necessary, although other factors make a large sample desirable.

The product comprises a series of successive lots produced by a continuing process.

All the lots are expected to be of essentially the same quality.

The consumer has faith in the integrity of the producer.

Operating procedure:

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The plan is implemented in the following way.

- For each lot, select sequence of sample of 'n' units and test each unit for conformance to the specified requirements.
- Accept the lot if d (the observed number of defectives) is zero in the sequence of sample of 'n' units, and reject if d>1.
- Accept the lot, if'd' is equal to 1 and if no defectives are formed in the immediately preceding 'i' sequence of sample size 'n'.

Dodge (1955) has given the operating characteristics function of ChSP-1 as

$$P_a (P) = P_o + P_1 (P_o)^i$$

The chain sampling plan is characterized by the parameters n & i, where 'n' is the sample size and 'i' is the number of preceding samples with zero defective.

Geometric Distribution:

Thomas Ryan (2011) proclaims that, in recent years the Geometric distribution has been used as an alternative to the binomial, and also as an alternative to the Poisson distribution. In this study instead of binomial distribution geometric distribution is used to calculate the Average Probability of Acceptance.

Suppose if an experiment is repeated under binomial trails until getting the first failure, let 'n' be the number of trails for the first failure. In this context, 'n' in not a fixed number, In fact 'n' could be any of this numbers 1, 2, 3 ……What is the probability that our first failure comes on the 'n' trails. The answer is given by the Geometric Probability Distribution.

Conditions to apply Geometric Distribution:

- The problem presents a sequence of independent trails of some random process.
- All the trails must occur in a sequence.
- The outcomes of each trail are success and failure
- The outcomes of all trails are statistically independent.
- All trails have the same probability of success.

Beta- Geometric Distribution:

The Beta-Geometric (s, t) distribution models the no of success that will occur in a binomial process before the first failure is observed and where, the Geometric probability 'p' is itself a random variable taking Beta (s, t) distribution.

Geometric variable'x' giving the number of success that will occur before the first failure is given by

$$g (x, p) = p^x (1 – p).$$

Where 'n' is the number of trails in which the first failure occurs (n = 1, 2…) and 'p' is the probability of success on each trail assuming 'p' has Beta-Geometric distribution is given by

$$g_w (x, p) = \int_0^1 g (x, p) \; w (p) \, dp$$

Bayesian chain sampling plan:

The operating characteristic function of ChSP – 1 given by Dodge (1955) is

$$P_a (p) = P_o + P_1 (Po)^i$$

The probability of Acceptance based on Geometric distribution will be

$$P (n, i/p) = (1 – p) + p (1 – p)^{1+i}$$

Using the past history of inspection, it is observed that 'p' follows a Beta distribution with density function.

$$w (p)dp = \frac{1}{\beta (S,t)} p^{s-1} (1-p)^{t-1} dp$$

The Average probability of acceptance is given by

$$\overline{P} = \int_0^1 p(n, i/p) w(p) dp$$

Can be written in terms of a beta distribution as

$$\overline{P} = \frac{t}{s+t} + \frac{s(t+i)(t+i-1)(t+i-2)\ldots\ldots}{(s+t+i+1)(s+t+i)(s+t+i-1)\ldots\ldots(s+t)} \quad \ldots\text{(A)}$$

Construction of Tables:

The Average probability of acceptance for Bayesian chain sampling plan (ChSP-1) using Beta-Geometric Distribution in given by the equation (A)

For i = 1, $\overline{P}$ reduces to

$$\overline{P} = (1 - \mu) \; 1 + \frac{s\mu \; (s - \mu s + \mu)}{(s+2\mu)(s+\mu)}$$

Where μ = s/s + t

For i = 2, $\overline{P}$ reduces

$$\overline{P} = (1-\mu) \left(1 + \frac{s\mu(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+3\mu)(s+2\mu)(s+\mu)} \right)$$

For i = 3, $\overline{P}$ reduces to

$$\overline{P} = (1 - \mu) \; 1 + \frac{s\mu \; (s - \mu s + 3\mu)(s - \mu s + 2\mu) \; (s - \mu s + \mu)}{(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}$$

For $i = 4$, $\overline{P}$ reduces to

$$\overline{P} = (1 - \mu) \quad 1 + \frac{s\mu\,(s-\mu s+4\mu)(s-\mu s+3\mu)\,(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+5\mu)(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}$$

For $i = 5$, $\overline{P}$ reduces to

$$\overline{P} = (1 - \mu)\left(1 + \frac{s\mu\,(s-\mu s+5\mu)(s-\mu s+4\mu)\,(s-\mu s+3\mu)(s-\mu s+2\mu)(s-\mu s+\mu)}{(s+6\mu)(s+5\mu)(s+4\mu)(s+3\mu)(s+2\mu)(s+\mu)}\right)$$

Tables (shown below) give the Average Probabilities of Acceptance for different values of 'μ' and the figures show the discriminating power of OC Curves for different values of 'i' and for fixed value of s = 1,s = 2,s = 3,s = 4, s = 5.

Comparison with conventional plans

The values obtained in BChSP-1 are compared with conventional sampling plan.

Illustration-1

For i = 1, s = 1, μ=0.1 the average probability of acceptance in BChSP-1 is 0.9682 where as in conventional plan it is 0.9810

Illustration-2

For i = 2, s = 1, μ=0.1 the average probability of acceptance in BChSP-1 is 0.9577 where as in conventional plan it is 0.9729

Illustration-3

For i = 4, s = 1, μ=0.1 the average probability of acceptance in BChSP-1 is 0.9429 where as in conventional plan it is 0.9590

On comparison it is observed that Bayesian chain sampling plan is more advantageous to the consumer than the conventional sampling plan.

II. CONCLUSION

Bayesian Acceptance Sampling is the technique which deals with procedures in which decision to accept or reject the lot or process is based in the examination of past history or knowledge of samples. The present work mainly relates to the construction of tables of average probability of acceptance and OC curve for Bayesian chain sampling plan using Geometric distribution. Compared to the Binomial distribution, Geometric distribution is very easy to calculate and the tables provided here are tailor-made, handy and ready to use to the industrial shop-floor condition. It is also observed this plan will be more advantageous to the consumer.

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Table 1.1: Average Probability of Acceptance (i=1)

s / μ	1	2	3	4	5	g(x,p)
.1	0.9682	0.9740	0.9762	0.9774	0.9781	0.9810
.2	0.8952	0.9091	0.9147	0.9177	0.9197	0.9280
.3	0.8010	0.8194	0.8273	0.8316	0.8344	0.8470
.4	0.6952	0.7143	0.7226	0.7273	0.7303	0.7440
.5	0.5833	0.6000	0.6071	0.6111	0.6136	0.6250

.6	0.4682	0.4808	0.4857	0.4883	0.4899	0.4960
.7	0.3515	0.3595	0.3619	0.3629	0.3633	0.3630
.8	0.2342	0.2381	0.2384	0.2381	0.2376	0.2320
1.0	0	0	0	0	0	0

Table 1.2: Average Probability of Acceptance (i=2)

s / μ	1	2	3	4	5	g(x,p)
.1	0.9577	0.9644	0.9670	0.9684	0.9692	0.9729
.2	0.8714	0.8839	0.8892	0.8922	0.8940	0.9024
.3	0.7691	0.7823	0.7881	0.7913	0.7934	0.8029
.4	0.6606	0.6714	0.6759	0.6783	0.6798	0.6864
.5	0.5500	0.5571	0.5595	0.5606	0.5612	0.5625
.6	0.4390	0.4425	0.4429	0.4426	0.4423	0.4384
.7	0.3282	0.3290	0.3279	0.3268	0.3259	0.3189
.8	0.2181	0.2173	0.2157	0.2143	0.2132	0.2064
1.0	0	0	0	0	0	0

Table 1.3: Average Probability of Acceptance ((i=3)

s / μ	1	2	3	4	5	g(x,p)
.1	0.9495	0.9563	0.9591	0.9606	0.9615	0.9656
.2	0.8556	0.8659	0.8704	0.8730	0.8746	0.8819
.3	0.7502	0.7592	0.7629	0.7650	0.7663	0.7720
.4	0.6420	0.6476	0.6495	0.6503	0.6508	0.6518
.5	0.5333	0.5357	0.5357	0.5354	0.5350	0.5313
.6	0.4252	0.4251	0.4238	0.4226	0.4217	0.4154
.7	0.3178	0.3163	0.3144	0.3130	0.3119	0.3057
.8	0.2112	0.2093	0.2076	0.2063	0.2055	0.2013
1.0	0	0	0	0	0	0

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Table 1.4: Average Probability Acceptance (i=4)

s / μ	1	2	3	4	5	g(x,p)
.1	0.9429	0.9496	0.9523	0.9539	0.9548	0.9590
.2	0.8444	0.8527	0.8563	0.8584	0.8597	0.8655
.3	0.7382	0.7440	0.7462	0.7473	0.7479	0.7504
.4	0.6308	0.6333	0.6337	0.6336	0.6334	0.6311
.5	0.5238	0.5238	0.5227	0.5218	0.5210	0.5156
.6	0.4176	0.4161	0.4143	0.4129	0.4119	0.4061
.7	0.3123	0.3101	0.3082	0.3069	0.3060	0.3017
.8	0.2076	0.2056	0.2041	0.2032	0.2026	0.2003
1.0	0	0	0	0	0	0

Table 1.5: Average Probability Acceptance (i=5)

s / μ	1	2	3	4	5	g(x,p)
.1	0.9375	0.9438	0.9465	0.9480	0.9489	0.9531
.2	0.8364	0.8429	0.8456	0.8471	0.8481	0.8524
.3	0.7300	0.7336	0.7346	0.7350	0.7352	0.7353
.4	0.6235	0.6242	0.6237	0.6231	0.6226	0.6187
.5	0.5179	0.5167	0.5152	0.5140	0.5131	0.5078
.6	0.4130	0.4109	0.4091	0.4078	0.4069	0.4025
.7	0.3090	0.3067	0.3050	0.3040	0.3033	0.3005
.8	0.2055	0.2036	0.2024	0.2017	0.2013	0.2001
1.0	0	0	0	0	0	0

Figure 2.1: Comparison of OC curve for various values of i with s = 1

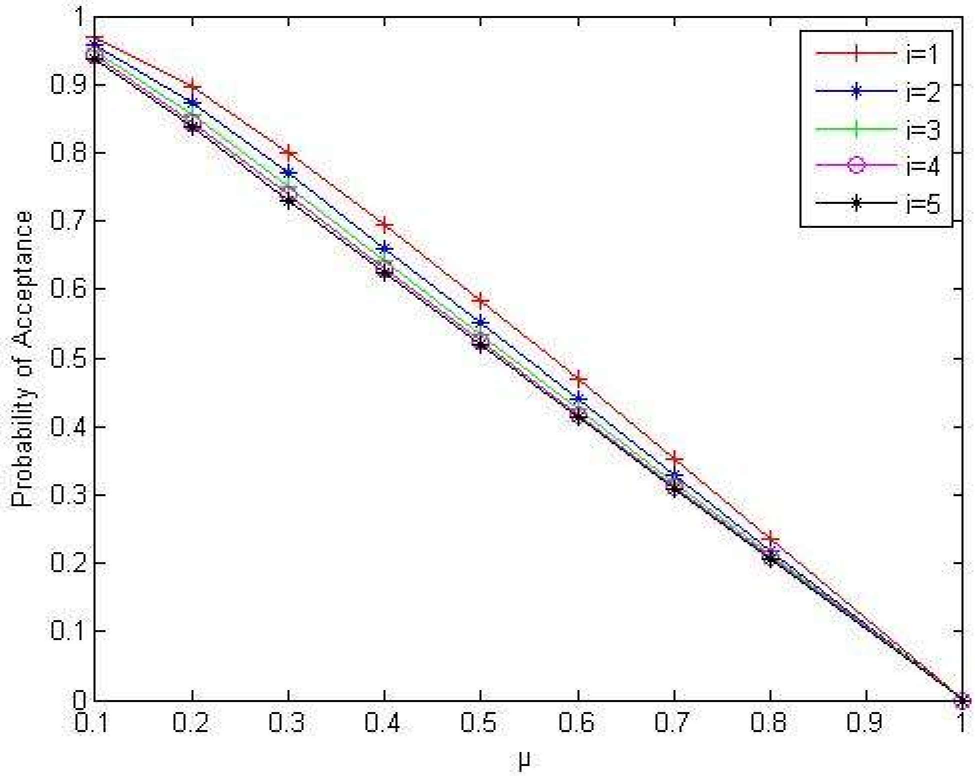

Figure 2.2: Comparison of OC curve for various values of i with s = 2

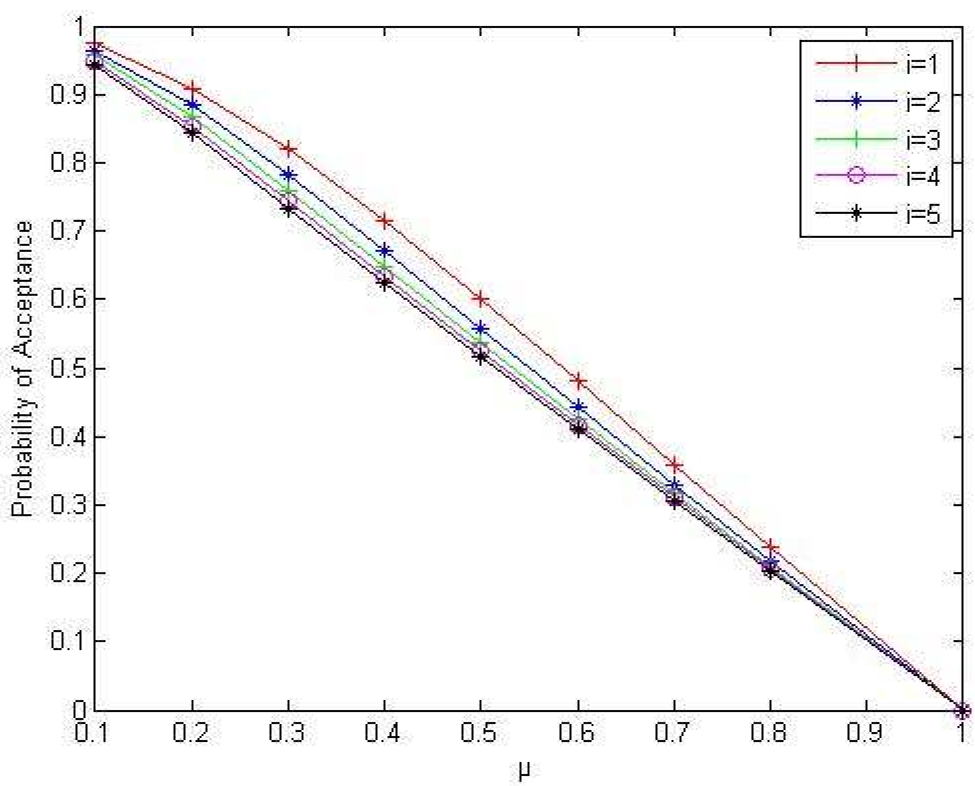

Figure 2.3: Comparison of OC curve for various values of i with s = 3

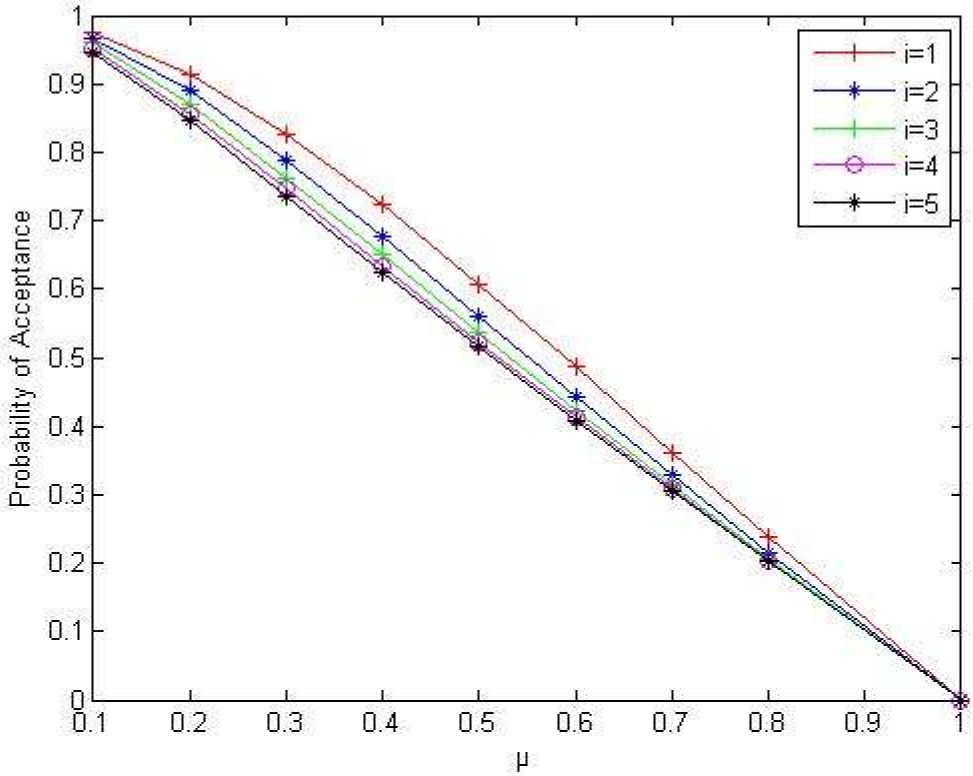

Figure 2.4: Comparison of OC curve for various values of i with s = 4

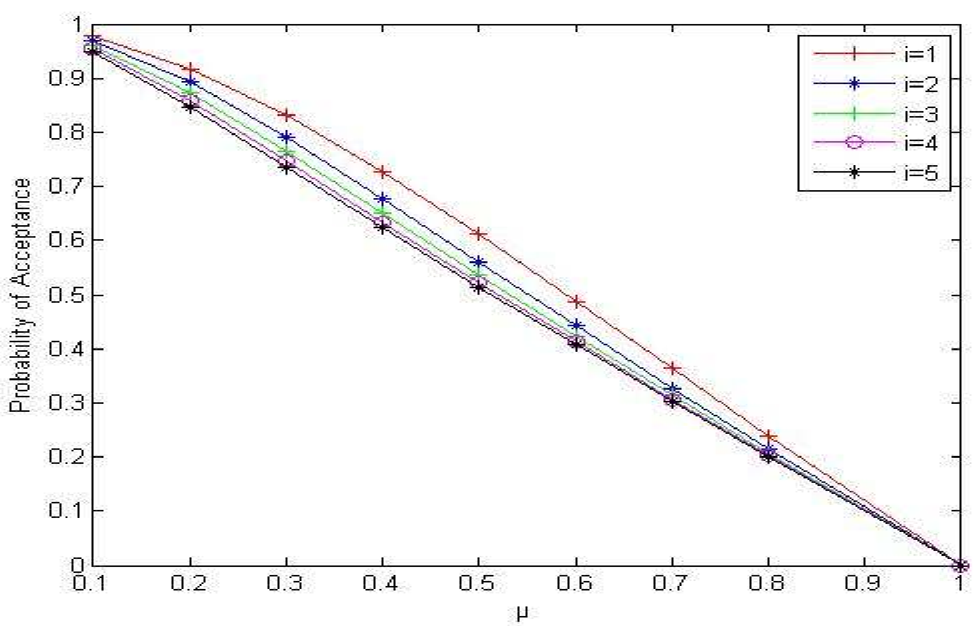

Figure 2.5: Comparison of OC curve for various values of i with s = 5