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SPECIAL PLANAR FUZZY GRAPH CONFIGURATIONS

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***ABSTRACT* -** In this paper new fuzzy graphs are constructed using special graph configurations. Hamiltonian fuzzy graph, fuzzy Hamiltonian circuit, cubic fuzzy graphs and its properties are introduced. Algorithmic development of spanning trees in graphs, matching in bipartite graphs, maximum flows in network are discussed. This paper concludes that if a fuzzy graph is planar, cubic connected then the Hamiltonian circuit problem for this graph is NP – complete.

Index Terms- Hamiltonian circuit fuzzy graph – Non deterministic polynomial

I. INTRODUCTION

Fuzzy set theory was introduced by zadeh in 1965 in a seminal article, Fuzzy set theory idea showed to fuzzy graph theory through Angiel Rosen field. He introduced the important concepts like paths, connectedness fuzzy bridges, fuzzy cut nodes, fuzzy forests, fuzzy trees etc. fuzzy end nodes was investigated by Butani. In this paper planar fuzzy Hamiltonian circuit problem is discussed.

A number of special graph configurations will be used to construct new fuzzy graphs. In a graph a path which passes through every vertex exactly once and returns to its starting point is called A Hamiltonian circuit. A fuzzy Hamiltonian circuit in a fuzzy graph is a fuzzy path which passes through every vertex exactly once and returns to its starting point. To characterize the fuzzy graphs which contain Hamiltonian circuits many attempts have been made. A fuzzy Hamiltonian line in a fuzzy directed graph is a fuzzy directed path which passes through each vertex exactly once, but need not return to its starting point.

This paper deals with the problem of recognition of fuzzy Hamiltonian circuit

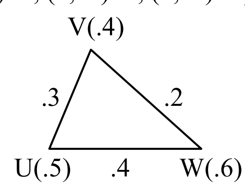
II. PRELIMINARIES

Definition 1:- A **fuzzy set** of a base set V is specified by its **membership function** σ where σ **:** v -> [0,1] assigning to each u Є v the degree or grade to which u belongs to σ

Definition 2:- A **fuzzy graph** G = (σ**,** μ**)** is a pair of function σ **:** V -> [0,1] and μ : v × v -> (0,1)where for all u, v Є V , we have μ (u, v) ≤ σ(u) ∩ σ(v)

μ is considered to be symmetric

Example:- A **fuzzy graph** G = (σ**,** μ) where
σ = {u/.5, v/.4, w/.6} and
μ ={(u, v)/.3, (u, w)/.4, (v, w)/.2}



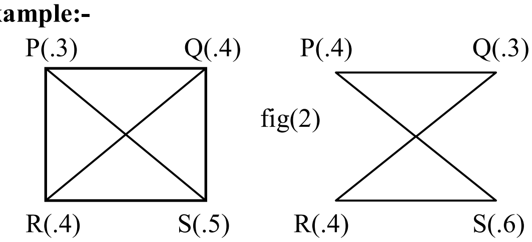
fig(1).

Definition 3:- A fuzzy graph that has neither self loops nor parallel edges is called a **simple fuzzy graph.**

Definition 4:- A simple fuzzy graph in which there exists an edge between every pair of vertices is called a **complete fuzzy graph**

Definition 5:- If fuzzy graph G has a spanning cycle Z, Then G is called a **Hamiltonian fuzzy graph.**

Example:-



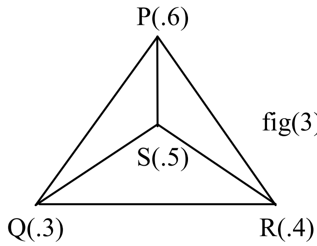
fig(2)

Definition 6:- A fuzzy graph is said to be **connected fuzzy graph** if there is at least one path between every pair of vertices in fuzzy graph.

Definition 7:- A **fuzzy Hamiltonian circuit** in a connected fuzzy graph is defined as a closed walk that traverse every vertex of G exactly once, except the starting vertex at which that walk also terminates.

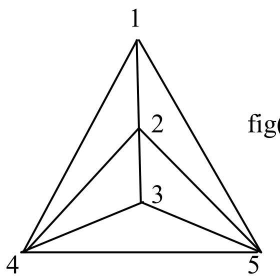
Definition 8:- A **cubic fuzzy graph** is 3 regular fuzzy graph (ie). Fuzzy graph whose vertices all have degree3.

Example:-



fig(3)

Definition 9:- A fuzzy graph G is said to be **planar** if there exists some geometric representation of G which can be drawn on plane such that no two of its edges intersect.

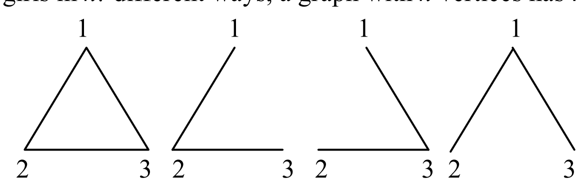


fig(4)

Definition 10:- If a fuzzy graph G can be drawn on a plane such that no two of its edges intersect than G is said to be **planar fuzzy graph.**

Algorithms are developed for minimum spanning trees in graphs, matching's in bipartite graphs, maximum increasing sub sequences, maximum flows in networks, and so on. All these algorithms are efficient, because in each case their time requirement grows as a polynomial function (such as *n*, n^2, or n^3) of the size of the input.

To better appreciate such efficient algorithms, consider the alternative: In all these problems we are searching for a solution (path, tree, matching, etc.) from among an exponential population of possibilities. Indeed, *n* boys can be matched with *n* girls in *n!* different ways, a graph with *n* vertices has n^{n-2}



fig(5)

Spanning trees, and a typical graph has an exponential number of paths from *s* to *t*. All these problems could in principle be solved in exponential time by checking through all candidate solutions, one by one. But an algorithm whose running time is 2^n, or worse, is all useless in practice. The quest for efficient algorithms is about finding clever ways to bypass this process of exhaustive search, using clues from the input in order to dramatically narrow down the search space.

III. DETERMINISTIC AND NON – DETERMINISTIC POLYNOMIAL

NP – COMPLETE PROBLEM

NP stands for "non-deterministic polynomial time" a term going back to the roots of complexity roots theory. Intuitively, it means that a solutions to any search problem can be found and verified in polynomial time by a special sort of algorithm, called a non-deterministic algorithm. Incidentally, the original definition of NP was not as a class of search problem, but as a class of decision problem: algorithm questions that can be answered by yes or no.

Example:-

Is there a truth assignment that satisfies the Boolean formula? But this too reflects a historical reality At the time theory of NP. Completeness was being developed by researches in the theory of computations were interested in formal languages a domain in which such decisions problems are of central importance.

We have problems that can be solved efficiently, they are easy problems minimum spanning tree, shortest path, bipartite matching, independent set on trees, linear programming, Euler path, minimum cut. We have a bunch of hard nuts that have escaped efficient solution over many decades or centuries. They are hard problems traveling salesman problem, longest path, 3D matching, independent set, integer linear programming, Rundrata path, balanced cut can be solved by algorithms that are specialized and diverse: dynamic programming, network flow, graph search, greedy. These problems are easy for a variety of different reasons. In stark contrast, the easy problems are all difficult for the same reason. At their core, they are all the same problem, just in different disguises. They are all equivalent. Each of them can be reduced to any of the others.

A **search problem** is **NP –** complete if all other search problems reduce to it.

Class of problems with polynomial size certificate for a yes answer that can be verified in polynomial time.

Example:- (Fuzzy Hamiltonian cycle problem) If a given fuzzy graph contain a Hamiltonian cycle, we can give a Hamiltonian cycle as certificate. The size of this certificate is bounded. We can check that it is a Fuzzy Hamiltonian cycle in polynomial time.

THE DOMINATING SET PROBLEM IN FUZZY LINE GRAPHS -THE TRAVELING SALESMAN PROBLEM (TSP)

In the traveling salesman problem (TSP) we are given *n* vertices 1,……,*n* and all *n*(*n*-1)/2 distances between them, as well as a budget. we are asked to find a tour, a cycle that passes through every vertex exactly once, of total cost or less-or to report that no such tour exists. That is, we seek a one to one correspondence *T*(1),….,*T*(n), T_1:V —>V of the vertices such that when they are toured in this order, the total distance covered is at most *b*:

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$d_{T(1),T(2)}+d_{T(2),T(3)}+ \ldots\ldots + d_{T(n),T(1)} \le$ b.

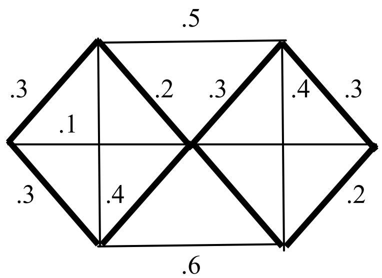

fig (6)

TSP as a search problem: given an instance, find a tour within the budget (or report that none exists). But we are expressing that traveling salesman problem in this way, when in reality it is an optimization problem, in which the shortest possible tour is sought. The framework of search problems is helpful to compare and relate problems. Because it encompasses optimization problems like the TSP in addition to true search problems.

Turning an optimization problem into a search problem does not change its difficulty at all, because the two versions reduce to one another. Any algorithm that solves the optimization TSP also readily solves the search problem: find the optimum tour and if it is within budget, return it; if not, there is no solution.

Some of these problems contain others as special cases (if we can solve one is polynomial time, we can solve both).

NP- complete special cases include the edge dominating set problem, is the dominating set problem in fuzzy line graphs.

NP- complete special cases include the minimum maximal matching problem which is essentially equal to the dominating set problem in fuzzy line graphs.

Example: If we can solve TSP problems in polynomial time, then we can solve all Hamiltonian cycle problems in polynomial time.

FUZZY HAMILTONIAN CIRCUIT GRAPH

- There is no known polynomial time algorithm that solves any single problem in the class.
- The existence of polynomial time algorithm for solving any particular problem in the class would imply that every problem in the class can be solved with a polynomial time algorithm.

It is widely believed that no NP-complete problem can be solved with a polynomial time algorithm and hence that all such problems are inherently computationally intractable.

The Hamiltonian line problem for directed planar fuzzy graphs is NP - complete. A fuzzy Hamiltonian line in a directed fuzzy graph is a directed path which passes through each vertex exactly once, but need not return to its starting point. The Hamiltonian circuit problems is NP-complete even for fuzzy graphs G satisfying the following conditions.

Let G is fuzzy graph in which no two of its edges intersect, which drawn on plane.

G is planar fuzzy graph.

In the graph G every vertex is of degree3

G is cubic fuzzy graph

There exists at least one path between every pair of vertices.

G is connected fuzzy graph.

Hence the Hamiltonian circuit problem is NP- complete.

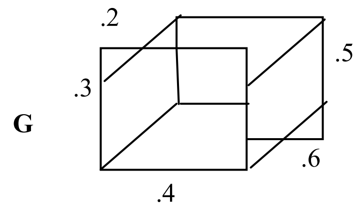

fig (7)

Thus the Hamiltonian circuit problem for these highly restricted fuzzy graphs seems to be essentially as difficult as that for arbitrary fuzzy graphs.

A number of special fuzzy graph configurations will be used in our new construction and are illustrated in Figs. Consider the fuzzy graph G, Any fuzzy Hamiltonian circuit in a fuzzy graph *G* that contains this fuzzy graph as a vertex-induced fuzzy subgraph must appear locally, and thus must use an edge. That is, this fuzzy subgraph acts like a single degree-3 vertex which has one specified edge that is required to be used in any fuzzy Hamiltonian circuit of *G* which is NP-complete.

Any fuzzy Hamiltonian circuit in a fuzzy graph G which contains this fuzzy graph as a vertex-induced fuzzy subgraph must appear locally in one of the two states. Thus this fuzzy subgraph acts like two separate edges. with the constraint that exactly one of these two edges must occur in any fuzzy Hamiltonian circuit of *G*. In this case, we say that the edges have been joined. Schematically, this will be represented by the abbreviation. which we shall call an exclusive-or line.

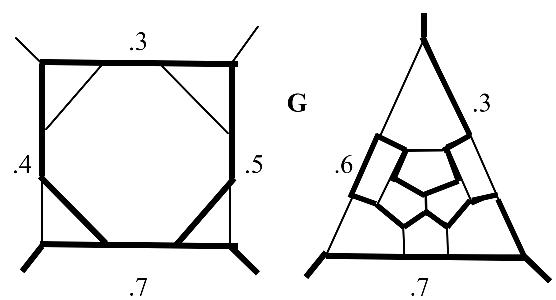

Fig (8)

The exclusive-or construction is crucial to the planarity of the fuzzy graph *G.* The key observation is that two exclusive-or lines joining different pairs of edges may cross each other without destroying the planarity of *G*. The property which permits this is that exclusive-or lines can be connected in series to cross over an edge of *G*, when that edge is required to occur in any fuzzy Hamiltonian circuit. The sequence of two exclusive-or's pictured there act like a single exclusive-or line joining the two outermost edges while permitting the required edge to pass between them. In particular, since all vertical edges in an exclusive-or fuzzy graph must occur in any fuzzy Hamiltonian circuit, we can use this property to allow two exclusive-or lines to cross each other. Figure shows schematically how this can be

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done and illustrates the possible states that can occur in a fuzzy Hamiltonian circuit graph which is NP- complete.

Result:

Thus the Hamiltonian circuit problem for planar fuzzy graphs is NP – complete.

IV. CONCLUSION

From our construction, we can conclude that the general planar Hamiltonian problems that were left open are all NP – complete. The undirected planar Hamiltonian circuit problem is NP - complete because it contains out problem as a special case. The directed planar Hamiltonian circuit problem is NP – complete because we can replace every edge in our construction with two directed edges and thus get a directed graph which has a Hamiltonian circuit if and only if our original undirected graph had one. Finally, the undirected planar fuzzy Hamiltonian line problem is NP – complete: A Hamiltonian line will exist if and only if the original graph had a Hamiltonian circuit. Note that the construction preserves connectivity and degree threeness, as well as planarity.

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