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A GENERAL THEORY OF THE SYSTEM 'QUANTUM INFORMATION-QUANTUM ENTANGLEMENT, SUBATOMIC PARTICLE DECAY-ASYMMETRIC SPIN STATES , NON LOCALITY- HIDDEN VARIABLES' -A CONCATENATED MODEL

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ABSTRACT: *One of the strengths of classical information theory is that physical representation of information can* ***be disregarded****: There is no need for an 'ink-on-paper' information theory or a 'DVD information' theory. This is because it is always possible* ***to efficiently (e&eb) transform*** *information from one representation to another. However, this is not the case for quantum information: it is not possible, for example, to write down on paper the previously unknown information* ***contained in*** *the polarization of a photon. In general, quantum mechanics* ***does not allow us to read out*** *the state of a quantum system with arbitrary precision. The existence of Bell correlations* ***between*** *quantum systems* ***cannot be e(e&eb) converted*** *into classical information. It is only possible* ***to (e&eb) transform*** *quantum information between quantum systems of sufficient information capacity. The information content of a message* $\mathcal{M}$ *can, for this reason, be measured in terms of the minimum number n of two-level systems which are needed to store the message:* $\mathcal{M}$ *consists of n qubits. In its original theoretical sense, the term qubit is thus a measure for the amount of information. A two-level quantum system can carry at most one qubit; in the same sense a classical binary digit can carry at most one classical bit. As a consequence of the noisy-channel coding theorem, noise* ***limits the*** *information content of an analog information carrier to be finite. It is very difficult* ***to protect*** *the remaining finite information content of analog information* ***carriers*** *against noise. The example of classical analog information shows that quantum information processing schemes must necessarily be tolerant against noise, otherwise there would not be a chance for them to be useful. It was a big breakthrough for the theory of quantum information, when quantum error correction codes and fault-tolerant quantum computation schemes were discovered. Thus there exists an inexorable link between Quantum Information, Quantum Entanglement, spin states and hidden variables. We given a General Theory for the system and investigate stability analysis, asymptotic analysis, Solutional behaviour. Surprisingly all the terms are positive indicating the stability of the asymptotic analysis. Ipso facto, the system seems to follow the characteristics of its constituents.*

INTRODUCTION:

Quantum information

In quantum mechanics, quantum information is (=) physical information that is held in the "state" of a quantum system. The most popular unit of quantum information is the qubit, a two-level quantum system. However, unlike classical digital states (which are discrete), a two-state quantum system can actually be in a superposition of the two states at any given time. Quantum information differs from classical information in several respects, among which we note the following: It cannot be read without the state becoming the measured value (1) an arbitrary state cannot be cloned, (2) The state may be in a superposition of basis values. However, despite this, the amount of information that can be retrieved (eb) in a single qubit is equal to one bit. It is in the processing of information (quantum computation) that the differentiation occurs.(eb)

2. The ability to manipulate (e&eb) quantum information enables us to perform tasks that would be unachievable in a classical context, such as unconditionally secure(-&+) transmission of information. Quantum information processing is the most general field that is concerned with quantum information. There are certain tasks which classical computers cannot perform "efficiently" (that is, in polynomial time) according to any known algorithm. However, a quantum computer can compute the answer to some of these problems in polynomial time; one well-known example of this is Shor's factoring algorithm. Other algorithms can (+&x) speed up a task less dramatically—for example, Grover's search algorithm which gives a quadratic speed-up over the best possible classical algorithm.

3. Quantum information, and **changes in** quantum information, can be **quantitatively measured** by using an analogue of Shannon entropy, called the von Neumann entropy. Given a statistical ensemble of

quantum mechanical systems with the density matrix ρ, it is given by

$$S(\rho) = -\operatorname{Tr}(\rho \ln \rho).$$

Many of the same entropy measures in classical information theory can also be generalized to the quantum case, such as Holevo entropy and the conditional quantum entropy.

Quantum Information Theory":

4. The theory of quantum information is a result of the effort to generalize classical information theory to the quantum world. Quantum information theory aims to answer the following question:

Time complexity

5. In computer science, the time complexity of an algorithm quantifies the amount of time taken by an algorithm to run as a function of the size of the input to the problem. The time complexity of an algorithm is commonly expressed using big O notation, which suppresses multiplicative constants and lower order terms. When expressed this way, the time complexity is said to be described asymptotically, i.e., as the input size goes to infinity. For example, if the time required by an algorithm on all inputs of size n is at most $5n^3 + 3n$, the asymptotic time complexity is O (n^3).Time complexity is commonly estimated by counting the number of elementary operations performed by the algorithm, where an elementary operation takes a fixed amount of time to perform. Thus the amount of time taken and the number of elementary operations performed by the algorithm differ by at most a constant factor. Since an algorithm's performance time may vary with different input sizes, the most commonly used measure of time complexity, the worst-case time complexity of an algorithm, denoted as **T (n)**, is the maximum amount of time taken on any input of size n. Time complexities are classified by the nature of the function T(n). For instance, an algorithm withT (n) = O (n) is called a linear time algorithm, and an algorithm with $T(n) = O(2^n)$ is said to be an exponential time algorithm.

6. **Computational complexity of mathematical operations**

The following table summarizes some classes of commonly encountered time complexities. In the table, $poly(x) = x^{O(1)}$, i.e., polynomial in x.

Name	**Complexity class**	**Running time (T(n))**	**Examples of running times**	**Example algorithms**
constant time		O(1)	10	Determining if an integer (represented in binary) is even or odd
Ackermann time		O(α(n))		Amortized time per operation using a disjoint set
logarithmic time		O(log* n)		Distributed coloring of cycles
log-logarithmic		O(log log n)		Amortized time per operation using a bounded priority queue[1]
logarithmic time	DLOGTIME	O(log n)	log n, $\log(n^2)$	Binary search
Poly logarithmic time		poly(log n)	$(\log n)^2$	
fractional power		$O(n^c)$ where $0 < c < 1$	$n^{1/2}$, $n^{2/3}$	Searching in a kd-tree
linear time		O(n)	n	Finding the smallest item in an unsorted array
"n log star n" time		O(n log* n)		Seidel's polygon

				triangulation algorithm.
Linearithmic time		O(n log n)	n log n, log n!	Fastest possible comparison sort
quadratic time		$O(n^2)$	n^2	Bubble sort; Insertion sort
cubic time		$O(n^3)$	n^3	Naive multiplication of two n×n matrices. Calculating partial correlation.
polynomial time	P	$2^{O(\log n)}$ = poly(n)	n, n log n, n^{10}	Karmarkar's algorithm for linear programming; AKS primality test
quasi-polynomial time	QP	$2^{poly(\log n)}$	$n^{\log \log n}$, $n^{\log n}$	Best-known $O(\log^2 n)$-approximation algorithm for the directed Steiner tree problem.
sub-exponential time (first definition)	SUBEXP	$O(2^{n^{\varepsilon}})$ for all $\varepsilon > 0$	$O(2^{\log n \log \log n})$	Assuming complexity theoretic conjectures, BPP is contained in SUBEXP.[2]
sub-exponential time (second definition)		$2^{o(n)}$	$2^{n1/3}$	Best-known algorithm for integer factorization and graph isomorphism
exponential time	E	$2^{O(n)}$	1.1^n, 10^n	Solving the traveling salesman problem using dynamic programming
factorial time		O(n!)	n!	Solving the traveling salesman problem via brute-force search
exponential time	EXPTIME	$2^{poly(n)}$	2^n, 2^{n2}	
double exponential time	2-EXPTIME	$2^{2^{poly(n)}}$	2^{2^n}	Deciding the truth of a given statement in Presburger arithmetic

Constant Time

7. An algorithm is said to be **constant time** (also written as **O (1)** time) if the value of T (n) is **bounded by** a value **that does not depend on** the size of the input. For example, **accessing** any single element in an array takes **constant time** as only one operation has to be performed to locate it. However, finding the minimal value in an unordered array is not a constant time operation as a scan over each element in the array is needed in order to **determine t**he minimal value. Hence it is a linear time operation, taking O (n) time. If the number of elements is known in advance and does not change, however, such an algorithm can still be said to run in constant time. Despite the name "constant time", the running time does not have to be independent of the problem size, but an upper bound for the running time has to be bounded independently of the problem size. For example, the task **"exchange the values of a and b if necessary so that a≤b**" is called constant time even though the time may depend on whether or not it is already true that a ≤ b. However, there is some constant t such that the time required is always at most t. Here are some examples of **code fragments** that run in constant time:

```
Int index = 5;
int item = list[index];
if (condition true) then
  perform some operation that runs in constant time
else
  perform some other operation that runs in constant time
for i = 1 to 100
  for j = 1 to 200
    perform some operation that runs in constant time
```

If T (n) is O (any constant value), this is equivalent to and stated in standard notation as T(n) being O(1).
Logarithmic time and Logarithmic growth:

8, An algorithm is said to take **logarithmic time** if T (n) = **O (log n)**. Due to the use of the binary numeral system by computers, the logarithm is frequently base 2 (that is, $\log_2$ n, sometimes written lg n). However, by the change of base equation for logarithms, $\log_a$ n and $\log_b$ n differ only by a constant multiplier, which in big-O notation is discarded; thus O (log n) is the standard notation for logarithmic time algorithms regardless of the base of the logarithm. Algorithms taking logarithmic time are commonly found in operations on binary trees or when using binary search. An algorithm is said to run in **poly logarithmic time** if T (n) = O $((\log n)^k)$, for some constant k. For example, matrix chain ordering can be solved in poly logarithmic time on a parallel. An algorithm is said to run in **sub-linear time** (often spelled **sub linear time**) if T(n) = o(n). In particular this includes algorithms with the time complexities defined above, as well as others such as the O $(n^{½})$ Grover's search algorithm. Typical algorithms that are exact and yet run in sub-linear time use parallel processing (as the NC_1 matrix determinant calculation does), non-classical processing (as Grover's search does), or alternatively have guaranteed assumptions on the input structure (as the logarithmic time binary search and many tree maintenance algorithms do). However, languages such as the set of all strings that have a 1-bit indexed by the first log(n) bits may depend on every bit of the input and yet be computable in sub-linear time. The specific term sub linear time algorithm is usually reserved to algorithms that are unlike the above in that they **are run over** classical serial machine models **and are not allowed** prior assumptions on the input. They are however allowed to be randomized, and indeed must be randomized for all but the most trivial of tasks. As such an algorithm must provide an answer without reading the entire input, its particulars heavily depend on the access allowed to the input. Usually for an input that is represented as a binary string $b_1,...,b_k$ it is assumed that the algorithm can in time O(1) request and obtain the value of b_i for any i. Sub-linear time algorithms are typically randomized, and provide only approximate solutions. In fact, the property of a binary string having only zeros (and no ones) can be **easily proved not to** be decidable by a (non-approximate) sub-linear time algorithm. Sub-linear time algorithms arise naturally in the investigation of property testing.
Linear time

9. An algorithm is said to take **linear time**, or **O (n)** time, if its time complexity is O(n). Informally, this means that for large enough **input sizes the** running time **increases linearly** with the size of the input. For example, a procedure that adds up all elements of a list requires time proportional to the length of the list. This description is slightly inaccurate, since the running time can significantly deviate from a precise proportionality; especially for small values of n. Linear time is often viewed as a **desirable attribute** for an algorithm. Much research has been invested into creating algorithms exhibiting (nearly) linear time or better. This research includes both software and hardware methods. In the case of hardware, some algorithms which, mathematically speaking, can never achieve linear time with standard computation models are able to run in linear time. There are several hardware technologies which exploit parallelism to provide this. An example is content-addressable memory. This concept of linear time **is used** in string matching algorithms such as the Boyer-Moore Algorithm and Ukkonen's Algorithm.

Linearithmic/quasi linear time

10. A **Linearithmic function** (portmanteau of linear and logarithmic) is a function of the form n · log n (i.e., a product of a linear and a logarithmic term). An algorithm is said to run in **Linearithmic time** if T (n) = **O (n log n)**. Compared to other functions, a Linearithmic function is ω(n), $o(n^{1+\varepsilon})$ for every ε > 0, and Θ(n · log n). Thus, a Linearithmic term grows faster than a linear term but slower than any polynomial in n with exponent strictly greater than 1.An algorithm is said to run in **quasi linear time** if T(n) = **O(n $\log^k$ n)** for any constant k. Quasilinear time algorithms are also $o(n^{1+\varepsilon})$ for every ε > 0, and thus run faster than any polynomial in n with exponent strictly greater than 1.In many cases, the n · log n running time is simply the result of performing a Θ(log n) operation n times. For example, binary tree sort creates a binary tree by inserting each element of the n-sized array one by one. Since the insert operation on a self-balancing binary search tree takes O(log n) time, the entire algorithm takes Linearithmic time. Comparison require at least Linearithmic number of comparisons in the worst case because log(n!) = Θ(n log n), by Stirling's approximation. They also frequently arise from the recurrence T(n) = 2 T(n/2) + O(n).
Some famous algorithms that run in Linearithmic time include:

Comb sort, in the average and worst case

- Quicksort in the average case
- Heap sort, merge sort, introsort, binary tree sort, smooth sort, patience sorting, etc. in the worst case
- Fast Fourier transforms
- Monge array calculation

An algorithm is said to be **sub quadratic time** if $T(n) = o(n^2)$.
For example, most naïve comparison-based sorting algorithms are quadratic (e.g. insertion sort), but more advanced algorithms can be found that are sub quadratic (e.g. Shell sort). No general-purpose sorts run in linear time, but the change from quadratic to sub-quadratic is of great practical importance.
Polynomial time

11. An algorithm is said to be of **polynomial time** if its running time is upper bounded by a polynomial expression in the size of the input for the algorithm, i.e., $T(n) = O(n^k)$ for some constant k Problems for which a polynomial time algorithm exists belong to the complexity class **P**, which is central in the field of computational complexity theory. Cobham's thesis states that polynomial time is a synonym for "tractable", "feasible", "efficient", or "fast". Some examples of polynomial time algorithms:
The Quicksort sorting algorithm on n integers performs at most An^2 operations for some constant A. Thus it runs in time $O(n^2)$ and is a polynomial time algorithm. All the basic arithmetic operations (addition, subtraction, multiplication, division, and comparison) can be done in polynomial time. Maximum matchings in graphs can be found in polynomial time.

Strongly and weakly polynomial time

12. In some contexts, especially in optimization, one differentiates between **strongly polynomial time** and **weakly polynomial time** algorithms. These two concepts are only relevant if the inputs to the algorithms **consist** of integers. Strongly polynomial time is **defined i**n the arithmetic model of computation. In this model of computation the basic arithmetic operations (addition, subtraction, multiplication, division, and comparison) take a **unit time step to** perform, regardless of the sizes of the operands. The algorithm runs in strongly polynomial time if
1. The number of operations in the arithmetic model of computation **is bounded by a** polynomial in the number of integers in the input instance; and
2. The space used by the algorithm **is bounded by a** polynomial in the size of the input.
Any algorithm with these two properties can be **converted to** a polynomial time algorithm by replacing the arithmetic operations by suitable algorithms for performing the arithmetic operations on a Turing machine. If the second of the above requirement is not met, then this is not true anymore. Given the integer 2^n (which takes up space proportional to n), it is possible to compute 2^{2^n} with n multiplications using repeated squaring. However, the space used to represent 2^{2^n} is proportional to 2^n, and thus exponential rather than polynomial in the space used to represent the input. Hence, it is not possible to carry out this computation in polynomial time on a Turing machine, but it is possible to compute it by polynomially many arithmetic operations. There are algorithms which run in polynomial time in the arithmetic model (where arithmetic operations take constant time), but not in the Turing machine model. The Euclidean algorithm for computing the greatest common divisor of two integers is one example. Given two integers a and b the running time of the algorithm in the arithmetic model is bounded by $O((\log a + \log b)^2)$. This is polynomial in the size of a binary representation of a and b as the size of such a representation is roughly $\log a + \log b$. However, the algorithm does not run in strongly polynomial time as the real running time also depends on the magnitudes of a and b and not only on the number of integers in the input (which is constant in this case, there is always only two integers in the input).An algorithm which runs in polynomial time but which is not strongly polynomial is said to run in **weakly polynomial time** A well-known example of a problem for which a weakly polynomial-time algorithm is known, **but is not known to admit** a strongly polynomial-time algorithm, is linear programming. Weakly polynomial-time should not be confused with pseudo-polynomial time.

Complexity classes

13. The concept of polynomial time leads to several complexity classes in computational complexity theory. Some important classes' defined using polynomial time is the following.

- **P**: The complexity class of decision problems that **can be solved on** a deterministic Turing machine in polynomial time.
- **NP**: The complexity class of decision problems that **can be solved** on a non-deterministic Turing machine in polynomial time.
- **ZPP**: The complexity class of decision problems that **can be solved** with zero error on a probabilistic Turing machine in polynomial time.
- **RP**: The complexity class of decision problems that **can be solved with** 1-sided error on a probabilistic Turing machine in polynomial time.
- **BPP**: The complexity class of decision problems that **can be solved with** 2-sided error on a probabilistic Turing machine in polynomial time.
- **BQP**: The complexity class of decision problems that can be solved with 2-sided error on a quantum Turing machine in polynomial time.

P is the smallest time-complexity class on a deterministic machine which is robust in terms of machine model changes. (For example, a change from a single-tape Turing machine to a multi-tape machine can lead to a quadratic speedup, but any algorithm that runs in polynomial time under one model also does so on the other.) Any given machine will have a complexity class corresponding to the problems which can be solved in polynomial time on that machine.
Super polynomial time

14. An algorithm is said to take **super polynomial time** if T (n) is not bounded above by any polynomial. It is ω (n^c) time for all constants c, where n is the input parameter, typically the number of bits in the input. For example, an algorithm that runs for 2^n steps on an input of size n requires super polynomial time (more specifically, exponential time).An algorithm that uses exponential resources is clearly super polynomial, but some algorithms are only very weakly super polynomial. For example, the Adleman–Pomerance–Rumely primality test runs for $n^{O(\log \log n)}$ time on n-bit inputs; this grows faster than any polynomial for large enough n, but the input size must become impractically large before it cannot be dominated by a polynomial with small degree. An algorithm that has been proven to require super polynomial time cannot be solved in polynomial time, and so is known to lie outside the complexity class **P**. thesis conjectures that these algorithms are impractical, and in many cases they are. Since the P versus NP problem is unresolved, no algorithm for a NP-complete problem is currently known to run in polynomial time.

Quasi-polynomial time

15. Quasi-polynomial time algorithms are algorithms which run slower than polynomial time, yet not so slow as to be exponential time. The worst case running time of a quasi-polynomial time algorithm is $2^{O((\log n)^c)}$ for some fixed c. The best-known classical algorithm for integer factorization, the general number field sieve, which runs in time about $2^{\tilde{O}(n^{1/3})}$ is not quasi-polynomial since the running time cannot be expressed as $2^{O((\log n)^c)}$ for some fixed c. If the constant "c" in the definition of quasi-polynomial time algorithms is equal to 1, we get a polynomial time algorithm, and if it less than 1, we get a sub-linear time algorithm. Quasi-polynomial time algorithms typically arise in reductions from an NP-hard problem to another problem. For example, one can take an instance of an NP hard problem, say3SAT, and convert it to an instance of another problem B, but the size of the instance becomes $2^{O((\log n)^c)}$. In that case, this reduction does not prove that problem B is NP-hard; this reduction only shows that there is no polynomial time algorithm for B unless there is a quasi-polynomial time algorithm for 3SAT (and thus all of NP). Similarly, there are some problems for which we know quasi-polynomial time algorithms, but no polynomial time algorithm is known. Such problems arise in approximation algorithms; a famous example is the directed Steiner tree problem, for which there is a quasi-polynomial time approximation algorithm achieving an approximation factor of $O(\log^3 n)$ (n being the number of vertices), but showing the existence of such a polynomial time algorithm is an open problem.
The complexity class **QP** consists of all problems which have quasi-polynomial time algorithms. It can be defined in terms of DTIME as follows

$$\mathrm{QP} = \bigcup_{c \in \mathbb{N}} \mathrm{DTIME}(2^{(\log n)^c})$$

Relation to NP-complete problems

16. In complexity theory, the unsolved P versus NP problem asks if all problems in NP have polynomial-time algorithms. All the best-known algorithms for NP-complete problems like 3SAT etc. take exponential time. Indeed, it is conjectured for many natural NP-complete problems that they do not have sub-exponential time algorithms. Here "sub-exponential time" is taken to mean the second definition presented above. (On the other hand, many graph problems represented in the natural way by adjacency matrices are **solvable in sub** exponential time simply **because the size** of the input is square of the number of vertices.) This conjecture (for the k-SAT problem) is known as the exponential time hypothesis. Since it is conjectured that NP-complete problems do not have quasi-polynomial time algorithms, some in approximability results in the field of approximation algorithms make the assumption that NP-complete problems do not have quasi-polynomial time algorithms. For example, see the known in approximability results for the cover problem. The term **sub-exponential time** is used to express that **the running time of** some algorithm may grow faster than any polynomial but is still significantly smaller than an exponential. In this sense, problems that have sub-exponential time algorithms are somewhat more tractable than those that only have exponential algorithms. The precise definition of "sub-exponential" is not generally agreed upon and we list the two most widely used ones below. A problem is said to be sub-exponential time solvable if it can be **solved in running** times whose logarithms grow smaller than any given polynomial. More precisely, a problem is in sub-exponential time if for every $\varepsilon > 0$ there exists an algorithm which solves the problem in time $O(2^{n^\varepsilon})$. The set of all such problems is the complexity class **SUBEXP** which can be defined in terms of DTIME as

$$\text{SUBEXP} = \bigcap_{\varepsilon > 0} \text{DTIME}\left(2^{n^\varepsilon}\right)$$

Note that this notion of sub-exponential is non-uniform in terms of ε in the sense that ε is not part of the input and each ε may have its own algorithm for the problem. Some authors define sub-exponential time as running times in $2^{o(n)}$. This definition allows larger running times than the first definition of sub-exponential time. An example of such a sub-exponential time algorithm is the best-known classical algorithm for integer factorization, the general number field sieve, which runs in time about $2^{\tilde{O}(n^{1/3})}$, where the length of the input is n. Another example is the best-known algorithm for the graph isomorphism problem, which runs in time $2^{O(\sqrt{n \log n})}$. It makes a difference whether the algorithm is allowed to be sub-exponential in the size of the instance, the number of vertices, or the number of edges. In parameterized complexity, this difference is made explicit by considering pairs (L, k) of decision problems and parameters k. **SUBEPT** is the class of all parameterized problems that run in time sub-exponential in k and polynomial in the input size n:[1]

$$\text{SUBEPT} = \text{DTIME}\left(2^{o(k)} \cdot \text{poly}(n)\right).$$

More precisely, SUBEPT is the class of all parameterized problems (L, k) for which there is a computable function $f : \mathbb{N} \to \mathbb{N}$ with $f \in o(k)$ and an algorithm that decides in time $2^{f(k)} \cdot \text{poly}(n)$.

]**Exponential time hypothesis**

17. The **exponential time hypothesis** (**ETH**) is that 3SAT, the satisfiability problem of Boolean formulas in conjunctive normal form with at most three literals per clause and with n variables, cannot be solved in time $2^{o(n)}$. More precisely, the hypothesis is that there is some absolute constant c>0 such that 3SAT cannot be decided in time 2^{cn} by any deterministic Turing machine. With m denoting the number of clauses, ETH is equivalent to the hypothesis that kSAT cannot be solved in time $2^{o(m)}$ for any integer $k \geq 3$. **The exponential time hypothesis implies P ≠ NP**. An algorithm is said to be **exponential time**, if T(n) is upper bounded by $2^{\text{poly}(n)}$, where poly(n) is some polynomial in n. More formally, an algorithm is exponential time if T(n) is bounded by $O(2^{n^k})$ for some constant k. Problems which admit exponential time algorithms on a deterministic Turing machine form the complexity class known as **EXP**.

$$\text{EXP} = \bigcup_{c \in \mathbb{N}} \text{DTIME}\left(2^{n^c}\right)$$

Sometimes, exponential time is used to refer to algorithms that have $T(n) = 2^{O(n)}$, where the exponent is at most a linear function of n. This gives rise to the complexity class **E**.

$$\text{E} = \bigcup_{c \in \mathbb{N}} \text{DTIME}\left(2^{cn}\right)$$

An algorithm is said to be double exponential time if T(n) is upper bounded by $2^{2^{poly(n)}}$, where poly(n) is some polynomial in n. Such algorithms belong to the complexity class 2-EXPTIME.

$$\text{2-EXPTIME} = \bigcup_{c \in \mathbb{N}} \text{DTIME}(2^{2^{n^c}})$$

Well-known double exponential time algorithms include: Decision procedures for Presburger arithmetic; Computing a Gröbner basis (in the worst case);Quantifier elimination on real closed fields takes at least doubly exponential time (but is not even known to be computable in ELEMENTARY)

Quantum entanglement

18. Quantum entanglement **occurs(eb)** when particles such as photons, electrons, molecules as large as buck balls and even small diamonds **interact physically** and then become **separated**; the type of **interaction** is such that each **resulting member** of a pair is properly described by the same quantum mechanical description (state), **which is indefinite** in terms of important factors such as position, momentum, spin, polarization, etc. According to the Copenhagen interpretation of quantum mechanics, their **shared state is indefinite** until measured Quantum entanglement is a form of **quantum superposition**. When a measurement is made and it **causes one member of such a pair** to take on a definite value (e.g., clockwise spin), the other member of this entangled pair will at any subsequent time be found to have taken the appropriately **correlated value** (e.g., counterclockwise spin). Thus, there **is a correlation** between the **results** of measurements performed on entangled pairs, and this correlation is observed even though the entangled pair may have been **separated b**y arbitrarily large distances. Th is behavior is **consistent** with quantum mechanical theory and has been demonstrated experimentally, and it is accepted by the physics community. However there is some debate about a possible underlying mechanism that enables this **correlation to occur** even when the separation distance is large. The difference in opinion derives from espousal of various interpretations of quantum mechanics.

19. The counterintuitive predictions of quantum mechanics **about strongly correlated** systems were first discussed by Albert Einstein in 1935, in a joint paper with Boris Podolsky and Nathan Rosen In this study, they formulated the EPR paradox, a thought experiment that attempted to show that quantum mechanical theory was incomplete. They wrote: We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete. Schrõdinger would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its **entire departure** from classical lines of thought. As with Einstein, Schrödinger was dissatisfied with the concept of entanglement, because it seemed **to violate the** speed limit on the transmission of information implicit in the theory of relativity Einstein later famously derided entanglement as "spukhafte Fernwirkung"[or "spooky action at a distance". So, despite the interest, the flaw in EPR's argument was not discovered until 1964, when John Stewart Bell demonstrated precisely how one of their key assumptions, **the principle of locality**, c**onflicted** with quantum theory. Specifically, he demonstrated an upper limit, known as Bell's inequality, regarding the strength of correlations that can **be produced** in any theory obeying local realism, and he showed that quantum theory predicts **violations** of this limit for certain entangled systems His inequality is experimentally testable, and there have been numerous relevant experiments, starting with the pioneering work of Freedman and Clauser in 1972 and Aspect's experiments in 1982. They have all shown agreement with quantum mechanics rather than the **principle of local realism**. However, the issue is not finally settled, for each of these experimental tests has left open at least one loophole by which it is possible to question the validity of the results. The work of Bell raised the possibility **of using these** super strong correlations as a **resource f**or communication. It led to the discovery of quantum key distribution protocols, most famously BB84 by Bennet and Brassard and E91 by Artur Ekert. Although BB84 does not use entanglement, Ekert's protocol uses **the violation (e) of a** Bell's inequality as a proof of security. Bell's inequality stirred interest on the possibilities of instantaneous and long range communication.

20. Quantum systems can **become entangled** through various types of i**nteractions**. If entangled, one object cannot be fully described without considering the other(s). They remain in a **quantum superposition** and share a single quantum state until a measurement is made. An example of entanglement **occurs** when **subatomic particles decay** into other particles. These decay events obey the various conservation laws, and as a result, pairs of particles can be **generated so that** they are in some specific quantum states. For instance, a pair of these particles may be **generated** having a two-state spin: one must be spin up and the other must be spin down. This type of entangled pair, where the particles

always have opposite spin, is known as the spin anti-correlated case, and if the probabilities for measuring each spin are equal, the pair is said to be in the singlet state. If each of two hypothetical experimenters, Alice and Bob, has one of the particles that form an entangled pair, and Alice measures the spin of her particle, the measurement will be entirely unpredictable, with a 50% probability of the spin being up or down. But if Bob subsequently measures the spin of his particle, the measurement will be entirely predictable—always opposite to Alice's, hence perfectly anti-correlated.

So far in this example experiment, the correlation seen with aligned measurements (i.e., up and down only) can be simulated classically. To make an analogous experiment, a coin might be sliced along the circumference into two half-coins, in such a way that each half-coin is either "heads" or "tails", and each half-coin put in a separate envelope and distributed respectively to Alice and to Bob, randomly. If Alice then "measures" her half-coin, by opening her envelope, for her the measurement will be unpredictable, with a 50% probability of her half-coin being "heads" or "tails", and Bob's "**measurement"** of his half-coin will always be opposite, hence perfectly **anti-correlated**.

21. However, with quantum entanglement, if Alice and Bob measure the spin of their particles in directions other than just up or down, with the directions chosen to form a Bell's inequality, they can now observe a **correlation** that is fundamentally stronger than anything that is achievable in classical physics. Here, the classical simulation of the experiment **breaks down** because there are no "directions" other than heads or tails to be measured in the coins. One might imagine that using a die instead of a coin could solve the problem, but the fundamental issue about measuring spin in different directions is that these measurements cannot have definite values at the same time—they are incompatible. In classical physics this does not make sense, since any number of properties can be measured simultaneously with arbitrary accuracy. Bell's theorem implies, and it has been proven mathematically, that compatible measurements **cannot show** Bell-like correlations, and thus entanglement is a fundamentally non-classical phenomenon.

Experimental results have demonstrated that effects due to entanglement travel at least thousands of times **faster than** the speed of light and that when measurements of the entangled particles are made in moving, relativistic reference frames in which each respective measurement occurs before the other, the measurement results remain **correlated.**
In a very recent experiment, "delayed-choice entanglement swapping" has been used to **decide** whether two particles were entangled or not after they had already been measured.

Entanglement, non-locality and hidden variables

22. There is much confusion about the meaning of entanglement, **non-locality and hidden** variables and how they **relate** to each other. As described above, entanglement is an experimentally verified and accepted property of nature. Non-locality and hidden variables are two proposed mechanisms that **enable the** effects of entanglement.

23. If the objects are indeterminate until one of them is measured, then the question becomes, "How can one account for something that was at one point indefinite with regard to its spin (or whatever is in this case the subject of investigation) suddenly becoming definite in that regard even though no physical interaction with the second object occurred, and, if the two objects are sufficiently far separated, could not even have had the time needed for such an interaction to proceed from the first to the second object?" The answer to the latter question involves the issue of locality, i.e., whether **for a change to occur** in something the agent of change has to be in physical contact (at least via some intermediary such as a field force) with the thing that changes. Study of entanglement brings into sharp focus the dilemma between locality and the completeness or lack of completeness of quantum mechanics.

24. In the media and popular science, quantum non-locality is often portrayed as being equivalent to entanglement. While it is true that a bipartite quantum state must be entangled in order for it to **produce** non-local correlations, there exist entangled states that **do not produce** such correlations. This is a case of dissipation of the entangled states. A well-known example of this is the **Werner state** that is entangled for certain values of p_{sym}, but can always be described using local hidden variables. In short, entanglement of a two-party state **is necessary but not sufficient for** that state to be non-local. It is important to recognize that entanglement is more commonly viewed as an algebraic concept, noted for being a **precedent to** non-locality as well as to quantum teleportation and to super dense, whereas non-locality **is defined** according to experimental statistics and is much more **involved with** the foundations and interpretations of quantum mechanics.

Methods of creating entanglement

25. Entanglement is usually **created** by direct interactions between subatomic particles. These interactions can take numerous forms. One of the most commonly **used methods** is spontaneous to **generate a** pair of photons entangled in polarization. Other methods include the use of a fiber coupler to confine and mix photons, the use of quantum dots to trap electrons until **decay** occurs, the use of the Hong-Ou-Mandel effect, etc. In the earliest tests of Bell's theorem, the entangled particles were **generated** using atomic cascades.
It is also possible to create entanglement between quantum systems that never directly interacted, through the use of **entanglement swappin**g.

Applications of entanglement

26. Entanglement has many applications in quantum information theory. With the aid of entanglement, otherwise impossible tasks may be achieved. Among the best-known applications of entanglement are super dense coding and **quantum teleportation** Not all researchers agree that entanglement is vital to the functioning of a quantum computer **.Entanglement is used** in some protocols of quantum cryptography. The Reeh-Schlieder theorem of quantum field theory is sometimes seen as an analogue of quantum entanglement.

Quantum mechanical framework

27. Consider two non interacting systems A and B, with respective Hilbert spaces H_A and H_B. The Hilbert space of the composite system is the tensor product

$$H_A \otimes H_B.$$

If the first system is in state $|\psi\rangle_A$ and the second in state $|\phi\rangle_B$, the state of the composite system is

$$|\psi\rangle_A \otimes |\phi\rangle_B.$$

States of the composite system which can be represented in this form are called separable states, or (in the simplest case) product states.

Not all states are separable states (and thus product states). Fix a basis $\{|i\rangle_A\}$ for H_A and a basis $\{|j\rangle_B\}$ for H_B. The most general state in $H_A \otimes H_B$ is of the form

$$|\psi\rangle_{AB} = \sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_B$$

This state is separable if $c_{ij}=c_i^A c_j^B$, yielding $|\psi\rangle_A = \sum_i c_i^A |i\rangle_A$ and $|\phi\rangle_B = \sum_j c_j^B |j\rangle_B$. It is inseparable if $c_{ij} \neq c_i^A c_j^B$. If a state is inseparable, it is called an entangled state.

For example, given two basis vectors $\{|0\rangle_A, |1\rangle_A\}$ of H_A and two basis vectors $\{|0\rangle_B, |1\rangle_B\}$ of H_B, the following is an entangled state:

$$\frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right).$$

If the composite system is in this state, it is impossible to attribute to either system A or system B a definite pure state. Another way to say this is that while the von Neumann entropy of the whole state is zero (as it is for any pure state), the entropy of the subsystems is greater than zero. In this sense, the systems are "entangled". This has specific empirical ramifications for interferometry. It is worthwhile to note that the above example is one of four Bell states, which are (maximally) entangled pure states (pure states of the $H_A \otimes H_B$ space, but which cannot be separated into pure states of each H_A and H_B).
Now suppose Alice is an observer for system A, and Bob is an observer for system B. If in the entangled state given above Alice makes a measurement in the $\{|0\rangle, |1\rangle\}$ Eigen basis of A, there are two

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possible outcomes, occurring with equal probability

1. Alice measures 0, and the state of the system collapses to $|0\rangle_A|1\rangle_B$.
2. Alice measures 1, and the state of the system collapses to $|1\rangle_A|0\rangle_B$.

If the former occurs, then any subsequent measurement performed by Bob, in the same basis, will always return 1. If the latter occurs, (Alice measures 1) then Bob's measurement will return 0 with certainty. Thus, system B has been altered by Alice performing a local measurement on system A. This remains true even if the systems A and B are spatially separated. This is the foundation of the EPR paradox.
The outcome of Alice's measurement is random. Alice cannot decide which state to collapse the composite system into, and therefore cannot transmit information to Bob by acting on her system. **Causality is thus preserved**, in this particular scheme. For the general argument, see no-communication theorem. As mentioned above, a state of a quantum system is given by a unit vector in a Hilbert space. More generally, if one has a large number of copies of the same system, then the state of this ensemble is described by a density matrix, which is a positive matrix, or a trace class when the state space is infinite dimensional, and has trace 1. Again, by the spectral, such a matrix takes the general form:

$$\rho = \sum_i w_i |\alpha_i\rangle\langle\alpha_i|,$$

Where the w_i's sum up to 1, and in the infinite dimensional case, we would take the closure of such states in the trace norm. We can interpret ρ as representing an ensemble where w_i is the proportion of the ensemble whose states are $|\alpha_i\rangle$. When a mixed state has rank 1, it therefore describes a pure ensemble. When there is less than total information about the state of a quantum system, we need density matrices to represent the state.

Following the definition in previous section, for a bipartite composite system, mixed states are just density matrices on $H_A \otimes H_B$. Extending the definition of separability from the pure case, we say that a mixed state is separable if it can be written as

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B,$$

where ρ_i^A's and ρ_i^B's are themselves states on the subsystems A and B respectively. In other words, a state is separable if it is probability distribution over uncorrelated states, or product states. We can assume without loss of generality that ρ_i^A and ρ_i^B are pure ensembles. A state is then said to be entangled if it is not separable. In general, finding out whether or not a mixed state is entangled is considered difficult. The general bipartite case has been shown to be NP-hard. For the 2×2 and 2×3 cases, a necessary and sufficient criterion for separability is given by the famous Positive Partial Transpose (PPT) condition.

28. Experimentally, a mixed ensemble might be realized as follows. Consider a "black-box" apparatus that spits electrons towards an observer. The electrons' Hilbert spaces are identical. The apparatus might produce electrons that are all in the same state; in this case, the electrons received by the observer are then a pure ensemble. However, the apparatus could produce electrons in different states. For example, it could produce two populations of electrons: one with state $|\mathbf{z}+\rangle$ with spins aligned in the positive $\mathbf{z}$ direction, and the other with state $|\mathbf{y}-\rangle$ with spins aligned in the negative $\mathbf{y}$ direction. Generally, this is a mixed ensemble, as there can be any number of populations, each corresponding to a different state.

Reduced density matrices

29. The idea of a reduced density matrix was introduced by Paul Dirac in 1930. Consider as above systems A and B each with a Hilbert space H_A, H_B. Let the state of the composite system be

$$|\Psi\rangle \in H_A \otimes H_B.$$

As indicated above, in general there is no way to associate a pure state to the component system A. However, it still is possible to associate a density matrix. Let

$$\rho_T = |\Psi\rangle\;\langle\Psi|.$$

which is the projection operator onto this state. The state of A is the partial trace of ρ_T over the basis of system B:

$$\rho_A \stackrel{\text{def}}{=} \sum_j \langle j|_B \left(|\Psi\rangle\langle\Psi|\right)|j\rangle_B = \mathrm{Tr}_B\, \rho_T$$
.

ρ_A is sometimes called the reduced density matrix of ρ on subsystem A. Colloquially, we "trace out" system B to obtain the reduced density matrix on A.

For example, the reduced density matrix of A for the entangled state $(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)/\sqrt{2}$ discussed above is

$$\rho_A = (1/2)\Big(|0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A\Big)$$

This demonstrates that, as expected, the reduced density matrix for an entangled pure ensemble is a mixed ensemble. Also not surprisingly, the density matrix of A for the pure product state $|\psi\rangle_A \otimes |\phi\rangle_B$ discussed above is

$$\rho_A = |\psi\rangle_A\langle\psi|_A.$$

In general, a bipartite pure state ρ is entangled if and only if its reduced states are mixed rather than pure. Reduced density matrices were explicitly calculated in different spin chains with unique ground state. An example is the one dimensional AKLT spin chain: the ground state can be divided into a block and an environment. The reduced density matrix of the block is proportional to a projector to a degenerate ground state of another Hamiltonian.

The reduced density matrix also was evaluated for XY spin chains, where it has full rank. It was proved that in the thermodynamic limit, the spectrum of the reduced density matrix of a large block of spins is an exact geometric sequence in this case.

30. Entropy of a mixed state can be viewed as a measure of quantum entanglement. In classical information theory, the Shannon entropy, H is associated to a probability distribution, $p_1, \cdots, p_n$, in the following way:

$$H(p_1, \cdots, p_n) = -\sum_i p_i \log_2 p_i$$
.

Since a mixed state ρ is a probability distribution over an ensemble, this leads naturally to the definition of the von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}\,(\rho \log_2 \rho)\,;$$

In general, one uses the Borel functional calculus to calculate $\log \rho$. If ρ acts on a finite dimensional Hilbert space and has Eigen values $\lambda_1, \cdots, \lambda_n$, the Shannon entropy is recovered:

$$S(\rho) = -\mathrm{Tr}\,(\rho \log_2 \rho) = -\sum_i \lambda_i \log_2 \lambda_i$$
.

Since an event of probability 0 should not contribute to the entropy, and given that $\lim_{p\to 0} p \log p = 0$, the convention is adopted that $0 \log 0 = 0$. This extends to the infinite dimensional case as well: if ρ has **spectral resolution**

$$\rho = \int \lambda dP_\lambda,$$

assume the same convention when calculating

$$\rho \log_2 \rho = \int \lambda \log_2 \lambda dP_\lambda.$$

As in statistical mechanics, the more uncertainty (number of microstates) the system should possess, the larger the entropy. For example, the entropy of any pure state is zero, which is unsurprising since there is no uncertainty about a system in a pure state. The entropy of any of the two subsystems of the entangled state discussed above is $\log 2$ (which can be shown to be the maximum entropy for 2×2 mixed states).

31. Entropy provides one tool which **can be used to** quantify entanglement, although other entanglement measures exist if the overall system is pure, the entropy of one subsystem **can be used** to measure its degree of entanglement with the other subsystems. For bipartite pure states, the von Neumann entropy of reduced states is the unique measure of entanglement in the sense that it is the only function on the family of states that satisfies certain axioms required of an entanglement measure.

It is a classical result that the Shannon entropy achieves its maximum at, and only at, the uniform probability distribution {1/n,...,1/n}. Therefore, a bipartite pure state

$$\rho \in H \otimes H$$

is said to be a **maximally entangled state** if the reduced state of ρ is the diagonal matrix

$$\begin{bmatrix} \frac{1}{n} & & \\ & \ddots & \\ & & \frac{1}{n} \end{bmatrix}.$$

For mixed states, the reduced von Neumann entropy is not the unique entanglement measure.
As an aside, the information-theoretic definition is closely related to entropy in the sense of statistical mechanics (comparing the two definitions, we note that, in the present context, it is customary to set the Boltzmann constant $k = 1$). For example, by properties of the Borel functional calculus, we see that for any unitary operator U,

$$S(\rho) = S(U\rho U^*).$$

Indeed, without the above property, the von Neumann entropy would not be well-defined. In particular, U could be the time evolution operator of the system, i.e.

$$U(t) = \exp\left(\frac{-iHt}{\hbar}\right)$$

where H is the Hamiltonian of the system. This associates the reversibility of a process with its resulting entropy change, i.e. a process is reversible if, and only if, it leaves the entropy of the system invariant. This provides a connection between quantum information theory and thermodynamics. Rényi entropy also can be used as a measure of entanglement.

Concurrence (quantum computing)

32. In quantum computing, the **concurrence** is an entanglement monotone **defined** for a mixed state of two qubits as.

$$C(\rho) \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

in which $\lambda_1, ..., \lambda_4$ are the Eigen values of the Hermitian matrix

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$$

With

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

the spin-flipped state of ρ, σ_y a Pauli spin matrix, and the Eigen values listed in decreasing order. Alternatively, the λ_i's represent the square roots of the Eigen values of the non-Hermitian matrix $\rho\tilde{\rho}$ from the concurrence, the **entanglement of formation** can be calculated.

For pure states, the concurrence is a polynomial $SL(2,\mathbb{C})^{\otimes 2}$ invariant in the state's coefficients. For mixed states, the concurrence can be defined by convex roof extension.

For the concurrence, there is monogamy of entanglement, that is, the concurrence of a qubit with the rest of the system **cannot ever exceed(or is less than)** the sum of the concurrences of qubit pairs which it is part of.

33. The **Ghirardi–Rimini–Weber theory**, or **GRW**, is a collapse theory in quantum mechanics. GRW differs from other collapse theories by proposing that wave function collapse happens spontaneously. GRW is an attempt **to avoid** the measurement problem in quantum mechanics. It was first reported in 1985

The Ghirardi–Rimini–Weber Theory

34.. GRW says that particles can undergo spontaneous wave-function collapses. For individual particles, these collapses happen probabilistically **and (eb)will occur** at a given rate with high probability but not with certainty; groups of particles behave in a statistically regular way, however. Since experimental physics has not already detected an unexpected spontaneous collapse, it can be argued that GRW collapses happen extremely rarely. Ghirardi, Rimini, and Weber suggest that the rate of spontaneous collapse for an individual particle is on the order of once every hundred million years. GRW and all collapse theories want to **reconcile the** mathematics of quantum mechanics, which suggests that subatomic **particles exist** in a **superposition** of two or more states, with the measured results, which only ever give us one state. We can easily prepare an electron to have a spin that is mathematically both up and down, for example, but any experimental **result will yield** either up or down and never a superposition of both states. The orthodox interpretation, or Copenhagen interpretation of quantum mechanics, posits **a wave-function collapse** every time one measures any feature of a subatomic particle. This would explain why we only get one value when we measure, but it doesn't explain why **measurement itself is such a special act**. More importantly, the orthodox interpretation doesn't define what counts as "measurement" and there is much dispute on the question GRW originated as an attempt to get away from the imprecise talk of **"measurement**" that plagues the orthodox interpretation.

35. By suggesting that particles spontaneously collapse into stable states, GRW **escapes the** ideas that measurement is a special act or that some specific part of measuring a subatomic particle causes the particle's wave function to collapse. At the same time, GRW theory is compatible with single-particle experiments that do not observe spontaneous wave-function collapses; this is because spontaneous collapse is posited to be extremely rare. However, since measurement entails quantum entanglement, GRW still **describes** the observed phenomenon of quantum collapses whenever we measure subatomic particles. This is because the measured particle becomes entangled with the very large number of particles that make up the measuring device. (For any macroscopic measuring device, there are sure to be very many orders of magnitude more than 10^8 entangled particles, so the likelihood of at least one particle in the entangled system collapsing at any given moment is extremely high.)

36. The **measurement problem** in quantum mechanics is the unresolved problem of how (or if) wave function collapse occurs. The inability to observe this process directly has **given rise to** different interpretations of quantum mechanics, and poses a key set of questions that each interpretation must answer. The wave function in quantum mechanics **evolves(eb) according to the** Schrödinger equation into a linear superposition of different states, but actual measurements always find the physical system in a definite state. Any future evolution is based on the state the system was discovered to be in when the measurement was made, meaning that the measurement "did something" to the process under examination. Whatever that "**something" may** be does not appear to be explained by the basic theory. To express matters differently (to paraphrase Steven Weinberg , the wave function **evolves** deterministically – knowing the wave function at one moment, the Schrödinger equation **determines the** wave function at any later time. If observers and their measuring apparatus are themselves described by a deterministic wave function, why can we not predict precise results for measurements, but only probabilities? As a general question: How can one establish a correspondence between quantum and classical reality?

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37. The best known is the "paradox" of the Schrödinger's cat: a cat is apparently evolving into a linear superposition of basis vectors that can be characterized as an "alive cat" and states that can be described as a "dead cat". Each of these possibilities is associated with a specific nonzero probability amplitude; the cat seems to be in some kind of "combination" state (specifically, a "superposition"). However, a single, particular observation of the cat does not measure the probabilities: it always finds either a living cat, or a dead cat. After the measurement the cat is definitively alive or dead. The question is: How are the probabilities converted into an actual, sharply well-defined outcome?

Interpretations

38. To avoid the problem by suggesting there is only one wave function, the superposition of the entire universe, and it never collapses—so there is no measurement problem. Instead, the act of measurement is actually an interaction between two quantum entities, which entangle to form a single larger entity, for instance living cat/happy scientist. Everett also attempted to demonstrate the way that in measurements the probabilistic nature of quantum mechanics would appear; work later extended by Bryce DeWitt. De Broglie–Bohm theory tries to solve the measurement problem very differently: this interpretation contains not only the wave function, but also the information about the position of the particle(s). The role of the wave function **is to generate the** velocity field for the particles. These velocities are such that the probability distribution for the particle remains consistent with the predictions of the orthodox quantum mechanics. According to de Broglie–Bohm theory, interaction with the environment during a measurement procedure separates the wave packets in configuration space which is where apparent wave function collapse comes from even though there is no actual collapse. Erich Joos and Heinz-Dieter Zeh claim that the latter approach was put on firm ground in the 1980s by the phenomenon of quantum decoherence. Zeh further claims that decoherence **makes it possible to** identify the fuzzy boundary between the quantum micro world and the world where the classical intuition is applicable. Quantum decoherence was proposed in the context of the many-worlds interpretation[ ], but it has also become an important part of some modern updates of the interpretation based on consistent histories Quantum decoherence does not describe the actual process of the wavefunction collapse, but it explains the conversion of the quantum probabilities (that exhibit interference effects) to the ordinary classical probabilities. See, for example, Zurek Zeh[and Schlosshauer

Elementary Particles

39. As described in the Quantum Mechanics essay, matter is made up of molecules and these, in turn are made up of atoms. A typical atom consists of a nucleus composed of positively charged **protons** and neutral **neutrons** surrounded by a cloud of orbiting negatively charged **electrons**. This model was first postulated by **Ernest Rutherford** in 1913. At the time, it was thought that all matter consisted of these three particles. They were referred to as **elementary particles**. These particles are tabulated below.

Property	**Electron**	**Proton**	**Neutron**
Symbol	e^-	p^+	n
Mass (kg)	9.109×10^{-31}	1.673×10^{-27}	1.675×10^{-27}
Mass (MeV)	0.51	938.2	939.6
Electric Charge	-1	+1	0

In 1928 Paul Dirac predicted that all particles should have opposites called anti-particles. The first of these was discovered in 1932 by Carl Anderson. This was an electron with a positive electric charge (+1). This particle is the anti-electron (also called a positron).It is identical in every respect to the electron apart from its electric charge. When an electron and positron come into contact, they mutually annihilate each other producing a flood of energy in accordance with Einstein's famous equation, $E = mc^2$.

40. Both the proton and the neutron have anti-particles. These also destroy each other if they meet with their particle. Ordinary matter is made up from particles. It appears that the Universe is made up of ordinary matter. Matter composed of anti-particles is known as anti-matter. Anti-matter can be created in the laboratory but does not last long as it quickly comes into contact with ordinary matter and is destroyed.It is now known that there are many more elementary particles than the six described so far. These have been created using modern high-technology equipment. These have been divided into a number of groups depending on their properties. Most of these newly discovered particles have their anti-particles.

Leptons

41. The electron (e) is the simplest of the leptons. There are two heavier leptons called the muon (m) and

the tau (t). Both are unstable and decay to simpler, more stable particles. Both have anti-particles. Muons are found in the air as cosmic rays enter the Earth's atmosphere and smash into atoms and molecules. Another type of lepton is the enigmatic neutrino (n). This was postulated in 1934 by Enrico Fermi to explain certain aspects of radioactive decay. There are three types of neutrino, each one associated with one of the three lepton described above (e, m, t). They are called the electron neutrino (ne), muon neutrino (nm), and tau neutrino (nt).Neutrinos hardly reacts with other types of matter. They can easily pass through the Earth. They have no electric charge. Each one has its anti-particle version so there are six types of neutrinos. Neutrinos have a very low mass and one type can change into one of the other two types. Leptons are never found in the nucleus of atoms. They are not subject to the Strong Nuclear Force which keeps the nucleus from flying apart. They are sometimes produced in the nucleus but are quickly expelled. Some radioactive atoms break down by a method called beta decay. During beta decay a neutron in the nucleus breaks down to give a proton (which remains in the nucleus), an electron (which flies out and causes the radioactivity of the atom) and an electron neutrino (which departs at the speed of light and is not usually detected). The atom changes to a new one since the number of protons (the Atomic Number) increases by one. Atomic Number is explained in The Elements. The reaction is shown below.
n ---> p+ + e- + ne

The six leptons are tabulated below.

Name of Lepton	**Symbol**	**Mass (MeV)**
Electron	e	0.511
Electron Neutrino	n_e	~ 0
Muon	m	106
Muon Neutrino	n_m	~ 0
Tau	t	1,777
Tau Neutrino	n_t	~ 0

Baryons

42. The two most common baryons are the proton and neutron. They are both of similar mass but the proton has a single positive charge. They are collectively known as nucleons. Both are found in the nuclei of atoms, being kept there by the **Strong Nuclear Force** that binds them together. In recent years it has been suggested that baryons are made up of even more elementary particles called **quarks**. Quarks are found in six types (called **flavors**). In 1989 it was shown that only three pairs of quarks can exist. These correspond with the three leptons and the three neutrinos. **Quarks are unusual in having fractional electric charges.**

Name of Quark	**Symbol**	**Charge**	**Mass (MeV)**
Up	u	+(2/3)	2 - 8
Down	d	-(1/3)	5 - 15
Strangeness	s	-(1/3)	100 - 300
Charm	c	+(2/3)	1,000 - 1,600
Bottom (or Beauty)	b	-(1/3)	4,100 - 4,500
Top (or Truth)	t	+(2/3)	180,000

Baryons are made up of quark triplets. The proton is composed of two u quarks and a d quark. These quark charges of +(2/3) +(2/3) -(1/3) add up to the proton's charge of +1.
The neutron is made from two d quarks and a u quark. These quark charges of -(1/3) -(1/3) +(2/3) add up to the neutron's charge of 0.
The proton and neutron are stable particles in the most nuclei. Outside the nucleus or in certain unstable nuclei, neutrons decay as shown above. There exist other baryons, produced in high energy experiments that are less stable. These too are made up of quark triplets. Hundreds of these particles are known. Some of them are tabulated below.

Baryon Particle	Quark Triplet	Charge
p (proton)	uud	+(2/3)+(2/3)-(1/3) = +1
n (neutron)	udd	+(2/3)-(1/3)-(1/3) = 0
D^-	ddd	-(1/3)-(1/3)-(1/3) = -1
L^0	uds	+(2/3)-(1/3)-(1/3) = 0
S^+	uus	+(2/3)+(2/3)-(1/3) = +1
W^-	sss	-(1/3)-(1/3)-(1/3) = -1
C_1^{++}	cuu	+(2/3)+(2/3)+(2/3) = +2

All six quarks have their anti-quarks with charges opposite in value to their quark counterparts. The (u) anti-quark has a charge of -(2/3) while the (d) anti-quark has a charge of +(1/3). The anti-proton is made up of (u)(u)(d) and has a charge of -1.

Mesons

43. Mesons are particles only discovered when the forces binding nucleons together were investigated. In a nucleus, the protons and neutrons are not really separate entities, each with its own distinct identity. They change into each other by rapidly passing particles called pions (p) between themselves. Pions are the most common of the mesons. Mesons are composed of quarks. Mesons are composed of a quark / anti-quark pair. The positive pion (p+) is made from a u quark and a (d) anti quark. The negative pion (p-) is made from a d quark and a (u) anti quark. Some of the many known mesons are tabulated below.

Meson Particle	Quark Pair	Charge
p^+ (positive pion)	u(d)	+(2/3)+(1/3) = +1
p^- (negative pion)	(u)d	-(2/3)-(1/3) = -1
K^0 (neutral kaon)	d(s)	-(1/3)+(1/3) = 0
f	s(s)	-(1/3)+(1/3) = 0
D^-	d(c)	-(1/3)-(2/3) = -1
J (or j)	c(c)	+(2/3)-(2/3) = 0

Kaons are short lived mesons that decay into simpler particles. Normally, particles and anti-particles decay in a similar way. The example below shows the decay of the neutron and the anti-neutron.
n ---> p+ + e- + ne
(n) ---> p- + e+ + (ne)
The decays are mirror images of each other. Kaons are unique in that the matter and anti-matter forms occasionally decay in slightly different modes. This is referred to as a breakdown of a property called parity. This breakdown of parity conservation may account for the fact that the Universe is mainly matter rather than a 50-50 mixture of matter and anti-matter. A mixed matter Universe would not last long as the matter and anti-matter would destroy each other.

Forces

44. Leptons and Baryons are referred to as Fermions. Particles have a property called spin. The spin of Fermions has half-integer values (1/2, 3/2, etc). Because of this type of spin, Fermions obey the Pauli Exclusion Principle. This means that two fermions cannot occupy the same energy states. With electrons this gives rise to atoms whose electrons are distributed in shells. These shells give atoms their differing chemical properties. Mesons are another type of particle. These are called Bosons. Bosons have integer spin (0, 1, 2). Bosons do not obey the Pauli Exclusion Principle. The best known Boson is the massless photon, a quantum of light. Bosons are known as the force carriers. When two particles interact they exchange a Boson. The photon is the force carrier for the Electromagnetic Force. Three bosons (W+, W- and Z0) carry the Weak Nuclear Force. This is the force responsible for beta decay. Various gluons carry the Strong Nuclear Force. Some people suggest the existence of a graviton to carry the Gravitational

Force. New theories called Superstrings, Twisters and M Theory are attempting to link relativity (especially gravity) and predict the properties of all the sub-atomic particles and the forces of nature.

Hidden variable theory

45. For hidden variables in economics, see latent variable. For other uses historically, in physics, hidden variable theories were espoused by some physicists who argued that quantum mechanics is incomplete. These theories argue against the orthodox interpretation of quantum mechanics, which is the Copenhagen Interpretation. Albert Einstein, the most famous proponent of hidden variables, objected to the fundamentally probabilistic nature of quantum mechanics,[and famously declared "I am convinced God does not play dice" Einstein, Podolsky, and Rosen argued that "elements of reality" (hidden variables) must be added to quantum mechanics to explain entanglement without action at a distance. Later, Bell's theorem would suggest (in the opinion of most physicists and contrary to Einstein's assertion) that local hidden variables are impossible.

The most famous such theory (because it gives the same answers as quantum mechanics, thus providing a supposed counterexample to the famous proof by von Neumann that was generally believed to demonstrate that no hidden variable theory reproducing the statistical predictions of QM is possible) is that of David Bohm. It is most commonly known as the Bohm interpretation or the Causal Interpretation of quantum mechanics. In Bohm's interpretation, the (nonlocal) quantum potential constitutes an implicate (hidden) order, and may itself be the result of yet a further implicate order (super implicate order). Nowadays Bohm's theory is considered to be one of many interpretations of quantum mechanics which give a realist interpretation, and not merely a positivistic one, to quantum-mechanical calculations. By some it is considered the simplest theory to explain the orthodox quantum mechanics formalism Nevertheless it is a hidden variable theory.

46. Under the Copenhagen interpretation, quantum mechanics is nondeterministic, meaning that it generally does not predict the outcome of any measurement with certainty. Instead, it tells us what the probabilities of the outcomes are. This leads to the situation where measurements of a certain property done on two apparently identical systems can give different answers. The question arises whether there might be some deeper reality hidden beneath quantum mechanics, to be described by a more fundamental theory that can always predict the outcome of each measurement with certainty. In other words if the exact properties of every subatomic particle and smaller were known the entire system could be modeled exactly using deterministic physics similar to classical physics. In other words, the Copenhagen interpretation of quantum mechanics might be an incomplete description of reality. The designation of variables as underlying "hidden" variables depends on the level of physical description (so for example "if a gas is described in terms of temperature, pressure, and volume, then the velocities of the individual atoms in the gas would be hidden variables".). Physicists supporting the Bohmian interpretation of quantum mechanics maintain that underlying the probabilistic nature of the universe is an objective foundation/property — the hidden variable. Others, however, believe that there is no deeper reality in quantum mechanics — experiments have shown a vast class of hidden variable theories to be incompatible with observations. Kirchmair and colleagues show that, in a system of trapped ions, quantum mechanics conflicts with hidden variable theories regardless of the quantum state of the system. Although determinism was initially a major motivation for physicists looking for hidden variable theories, nondeterministic theories trying to explain what the supposed reality underlying the quantum mechanics formalism looks like are also considered hidden variable theories; for example Edward Nelson's stochastic mechanics.

EPR Paradox & Bell's Theorem

47.In 1935, Einstein, Podolsky and Rosen wrote a four-page paper titled "Can quantum-mechanical description of physical reality be considered complete?" that argued that such a theory was in fact necessary, proposing the EPR Paradox as proof. In 1964, John Bell showed through his famous theorem that if local hidden variables exist, certain experiments could be performed where the result would satisfy a Bell inequality. If, on the other hand, Quantum entanglement is correct the Bell inequality would be violated. Another no-go theorem concerning hidden variable theories is the theorem. Physicists such as Alain Aspect and Paul Kwiat have performed experiments that have found violations of these inequalities up to 242 standard deviations (excellent scientific certainty). This rules out local hidden variable theories, but does not rule out non-local ones (which would refute quantum entanglement). Theoretically, there could be experimental problems that affect the validity of the experimental findings.

Non-local hidden-variable theory

48. Assuming the validity of Bell's theorem, any classical hidden-variable theory which is consistent with quantum mechanics would have to be **non-local**, maintaining the existence of instantaneous or faster-than-light acausal relations (correlations) between physically separated entities. The first hidden-variable theory was the pilot wave theory of Louis de Broglie, dating from 1927. The currently best-known hidden-variable theory, the Causal Interpretation, of the physicist and philosopher David Bohm, created in 1952, is a non-local hidden variable theory. Those who believe the Bohm interpretation to be actually true (rather than a mere model or interpretation), and the quantum potential to be real, refer to Bohmian mechanics What Bohm did, unknowingly rediscovering (and extending) the idea that Louis de Broglie had proposed and abandoned, was to posit both the quantum particle, e.g. an electron, and a hidden 'guiding wave' that governs its motion. Thus, in this theory electrons are quite clearly particles. When you perform a **double-slit experiment** (see wave-particle duality), they go through one slit rather than the other. However, their choice of slit is not random but is governed by the guiding wave, resulting in the wave pattern that is observed.

49. Such a view does not contradict the idea of local events that is used in both classical atomism and relativity theory as Bohm's theory (and indeed quantum mechanics, with which it is exactly equivalent) are still locally causal but allow nonlocal correlations (that is information travel is still restricted to the speed of light). It points to a view of a more holistic, mutually interpenetrating and interacting world. Indeed Bohm himself stressed the holistic aspect of quantum theory in his later years, when he became interested in the ideas of Jiddu Krishnamurti. Nevertheless this nonlocality is seen as a weakness of Bohm's theory by some physicists. It was deliberately designed to give predictions that are in all details identical to conventional quantum mechanics Bohm's aim was not to make a serious counterproposal but simply to demonstrate that hidden-variable theories are indeed possible.[His hope was that this could lead to new insights and experiments that would lead beyond the current quantum theories Gerard has disputed the validity of Bell's theorem on the basis of the super determinism loophole and proposed some ideas to construct local deterministic models. In August 2011, Roger Colbeck and Renato Renner published a proof that any extension of quantum mechanical theory, whether using hidden variables or otherwise, cannot provide a more accurate prediction of outcomes, assuming that observers can freely choose the measurement settings Colbeck and Renner write: "In the present work, we have ... excluded the possibility that any extension of quantum theory (not necessarily in the form of local hidden variables) can help predict the outcomes of any measurement on any quantum state. In this sense, we show the following: under the assumption that measurement settings can be chosen freely, quantum theory really is complete".

QUANTUM INFORMATION AND QUANTUM ENTANGLEMENT SYSTEM

NOTATION :

G_{24} : **Category 1 of Quantum Information for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 1 is representative and constitutive of Quantum Information, which gets dissipated by Category 1 Quantum Entanglement 1

G_{25} : **Category 2 of Quantum Information for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality - Hidden Variables . This category 2 is representative and constitutive of Quantum Information, which gets dissipated by Category 2 Quantum Entanglement 2

G_{26} : **Category 3 of Quantum Information for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables . This category 3 is representative and constitutive of Quantum Information, which gets dissipated by Category 3 Quantum Entanglement 3

T_{24} : **Category 1 of Quantum Entanglement for the system** Quantum Information-,Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality - Hidden Variables . 4

This category 1 belongs to Quantum Entanglement, which dissipates category 1of Quantum Information.

T_{25} : **Category 2 of Quantum Entanglement for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 2 belongs to Quantum Entanglement, which dissipates category 2 of Quantum Information. 5

T_{26} : **Category 3 of Quantum Entanglement for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables . This category 3 belongs to Quantum Entanglement, which dissipates category 3 of Quantum Information. 6

SUBATOMIC PARTCILE DECAY AND ASSYMMETRIC SPIN STATES SYSTEM:

G_{28} : **Category 1 of Subatomic Particle Decay for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, non-Locality- Hidden Variables. This category 1 is representative and constitutive of subatomic particle decay, which gets dissipated by category 1 of asymmetric spin states. 7

G_{29} : **Category 2 of Subatomic Particle Decay for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 2 is representative and constitutive of subatomic particle decay, which gets dissipated by category 2 of asymmetric spin states 8

G_{30} : **Category 3 of Subatomic Particle Decay for the system** Quantum Information- Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non-Locality-Hidden Variables. This category 1 is representative and constitutive of subatomic particle decay, which gets dissipated by category 3 of asymmetric spin states 9

T_{28} : **Category 1 of Asymmetric Spin States for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, non Locality-Hidden Variables. This category 1 belongs to asymmetric spin states which dissipates category 1 of subatomic particle decay 10

T_{29} : **Category 2 of Asymmetric Spin States for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 2 belongs to asymmetric spin states which dissipates category 2 of subatomic particle decay 11

T_{30} : **Category 3 of Asymmetric Spin States for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality- Hidden Variables. This category 3 belongs to asymmetric spin states, which dissipates category 3 of subatomic particle decay. 12

NONLOCALITY AND HIDDEN VARIABLES SYSTEM:

G_{32} : **Category 1 of Non Locality for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 1 belongs to and is representative and constitutional of non-locality, which is dissipated by category 1 of hidden variables 13

G_{33} : **Category 2 of Non Locality for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 2 belongs to and is representative and constitutional of non-locality, which is dissipated by category 2 of hidden variables 14

G_{34} : **Category 3 of Non Locality for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 3 15 belongs to and is representative and constitutional of non-locality, which is dissipated by category 3 of hidden variables

16

T_{32} : **Category 1 of Hidden Variables for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 1 belongs to and is representative and constitutional of Hidden Variables, which dissipates category 1 Non Locality

T_{33} : **Category 2 of Hidden Variables for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 2 belongs to and is representative and constitutional of Hidden Variables, which dissipates category 2 Non Locality 17

T_{34} : **Category 3 of Hidden Variables for the system** Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non Locality-Hidden Variables. This category 3 belongs to and is representative and constitutional of Hidden Variables, which dissipates category 3 Non Locality 18

ACCENTUATION COEFFCIENTS of The Total System 'Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non-Locality-Hidden Variables'

$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}$ 19

$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$

$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

DISSIPATION COEFFCIENTS of The Total System 'Quantum Information-Quantum Entanglement, Subatomic Particle Decay-Asymmetric Spin States, Non-Locality-Hidden Variables'

$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)} , (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}$ 20

$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}(b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

GOVERNING EQUATIONS OF QUANTUM INFORMATION-QUANTUM ENTANGLEMENT SYSTEM

The differential system of this model is now

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24}$$ 21

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25}$$ 22

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26}$$ 23

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24}$$ 24

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25}$$ 25

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26}$$ 26

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation coefficient** 27

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detrition coefficient** 28

GOVERNING EQUATIONS OF SUBATOMIC PARTICLE DECAY - ASYMMETRIC SPIN STATES SYSTEM

The differential system of this model is now

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$ 29

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$ 30

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$ 31

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28}$ 32

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29}$ 33

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30}$ 34

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation coefficient** 35

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detrition coefficient** 36

GOVERNING EQUATIONS OF NONLOCALITY AND HIDDEN VARIABLES SYSTEM

The differential system of this model is now

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32}$ 37

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33}$ 38

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34}$ 39

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\big((G_{35}),t\big)\right]T_{32}$ 40

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\big((G_{35}),t\big)\right]T_{33}$ 41

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\big((G_{35}),t\big)\right]T_{34}$ 42

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation coefficient** 43

$-(b''_{32})^{(6)}\big((G_{35}),t\big) =$ **First detrition coefficient** 44

FINAL GOVERNING EQUATIONS OF THE HOLISTIC SYSTEM :
QUANTUM INFORMATION-QUANTUM ENTANGLEMENT, SUBATOMIC PARTICLE DECAY-ASYMMETRIC SPIN STATES, NON-LOCALITY-HIDDEN VARIABLES

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\right] G_{24}$ 45

$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \right] G_{25}$ 46

$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \right] G_{26}$ 47

Where $\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ are first augmentation coefficients for category 1, 2 and 3 48

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ are second augmentation coefficient for 8category 1, 2 and 3 49

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ are third augmentation coefficient for category 1, 2 and 3 50

$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[(b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \right] T_{24}$ 51

$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[(b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \right] T_{25}$ 52

$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[(b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \right] T_{26}$ 53

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ are first detrition coefficients for category 1, 2 and 3 54

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ are second detritions coefficients for category 1, 2 and 3 55

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ are second detritions coefficients for category 1, 2 and 3 56

$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[(a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \right] G_{28}$ 57

$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[(a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \right] G_{29}$ 58

$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[(a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \right] G_{30}$ 59

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ are first augmentation coefficients for category 1, 2 and 3 60

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ are second augmentation coefficient 61 for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ are third augmentation coefficient for category 1, 2 and 3 62

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\right]T_{28} \quad 63$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\right]T_{29} \quad 64$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\right]T_{30} \quad 65$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ are first detrition coefficients for category 1, 2 and 3 66

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ are second detrition coefficients for category 1,2 and 3 67

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ are third detrition coefficients for category 1,2 and 3 68

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\right]G_{32} \quad 69$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\right]G_{33} \quad 70$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\right]G_{34} \quad 71$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ are first augmentation coefficients for category 1, 2 and 3 72

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ are second augmentation coefficients for category 1, 2 and 3 73

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ are third augmentation coefficients for category 1, 2 and 3 74

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\right]T_{32} \quad 75$$

$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[(b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \right] T_{33}$ 76

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35},t)} \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \right] T_{34}$ 77

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ are first detrition coefficients for category 1, 2 and 3 78

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ are second detrition coefficients for category 1, 2 and 3 79

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ are third detritions coefficients for category 1,2 and 3 80

$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[(a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \right] G_{24}$ 81

$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \right] G_{25}$ 82

$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \right] G_{26}$ 83

Where $\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ are first augmentation coefficients for category 1, 2 and 3 84

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ are second augmentation coefficient for category 1, 2 and 3 85

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ are third augmentation coefficient for category 1, 2 and 3 86

$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[(b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \right] T_{24}$ 87

$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[(b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \right] T_{25}$ 88

$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[(b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \right] T_{26}$ 89

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ are first detrition coefficients for category 1, 2 and 3 90

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ are second detritions coefficients for category 1, 2 and 3 91

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ are second detritions coefficients for category 1, 2 and 3 92

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\right]G_{28}$ 93

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\right]G_{29}$ 94

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\right]G_{30}$ 95

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ are first augmentation coefficients for category 1, 2 and 3 96

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ are second augmentation coefficient for category 1, 2 and 3 97

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ are third augmentation coefficient for category 1, 2 and 3 98

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\right]T_{28}$ 99

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\right]T_{29}$ 100

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\right]T_{30}$ 101

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ are first detrition coefficients for category 1, 2 and 3 102

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ are second detrition coefficients for category 1,2 and 3 103

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ are third detrition coefficients for category 1,2 and 3 104

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\right]G_{32}$ 105

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\right]T_{33}$ 106

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35},t)} \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \right] T_{34}$ 107

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ are first detrition coefficients for category 1, 2 and 3 108

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ are second detrition coefficients for category 1, 2 and 3 109

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ are third detrition coefficients for category 1,2 and 3 110

$(a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i,j = 24,25,26$ 111

(A) The functions $(a''_i)^{(4)}, (b''_i)^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a''_i)^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$ 112

$(b''_i)^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b'_i)^{(4)} \leq (\hat{B}_{24})^{(4)}$ 113

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(B) $lim_{T_2 \to \infty} (a''_i)^{(4)} (T_{25},t) = (p_i)^{(4)}$

$lim_{G \to \infty} (b''_i)^{(4)} \big((G_{27}),t\big) = \ (r_i)^{(4)}$ 115

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

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Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition:

$|(a''_i)^{(4)}(T'_{25},t) - (a''_i)^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T'_{25}| e^{-(\hat{M}_{24})^{(4)} t}$ 117

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$|(b''_i)^{(4)}((G_{27})',t) - (b''_i)^{(4)}\big((G_{27}),T\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(4)}(T'_{25},t)$ and$(a''_i)^{(4)}(T_{25},t)$. (T'_{25},t) And (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a''_i)^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a''_i)^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 119

(C) $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 120

(D) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ and $(\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}(b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}$ $i = 24,25,26,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$ 121

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$ 122

(E) $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30$ 123

(F) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

<u>**Definition of**</u> $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$ 124

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$ 125

(G) $lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$ 126

$lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = \ (r_i)^{(5)}$ 127

<u>**Definition of**</u> $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}$ are positive constants and $i = 28,29,30$

They satisfy Lipschitz condition:

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$ 128

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), (T_{31})\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$ 129

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) And (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient**, would be absolutely continuous.

<u>**Definition of**</u> $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 130

(H) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

<u>**Definition of**</u> $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 131

(I) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$ 132

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$ 133

Where we suppose

(J) $(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$ 134

(K) The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$ 135

$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$ 136

(L) $lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$

$lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = \ (r_i)^{(6)}$ 137

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$: 138

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$ 139

$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), (T_{35})\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$ 140

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 141

(M) $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 142

(N) There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)} \ (\hat{k}_{32})^{(6)}] < 1$$ 143

$$\frac{1}{(\hat{M}_{32})^{(6)}} [\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)} \ \ (\hat{k}_{32})^{(6)}] < 1$$ 144

Theorem 1: if the conditions IN THE FOREGOING ARE fulfilled, there exists a solution satisfying the conditions 145

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$ 146

$T_i(t) \leq \ (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$ 147

Definition of $G_i(0), T_i(0)$: 148

$G_i(t) \leq \left(\hat{P}_{28} \right)^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$ 149

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0), T_i(0)$:

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$G_i(t) \leq \left(\hat{P}_{32} \right)^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$ 152

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$ 153

Proof:
Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$, 154

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 155

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 156

By 157

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

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$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} \left(T_{25}(s_{(24)}), s_{(24)} \right) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 159$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 160$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 161$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} \left(G(s_{(24)}), s_{(24)} \right) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 162$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:
Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$, 163

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 164

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 165

By 166

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

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$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} \left(T_{29}(s_{(28)}), s_{(28)} \right) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 168$$

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$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} \left(G(s_{(28)}), s_{(28)} \right) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$ 170

$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$ 171

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$, 173

$0 \leq G_i(\mathrm{t}) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 174

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 175

By 176

$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$

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$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$

$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} \mathrm{G}_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$ 178

179

$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$

$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$ 180

$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$ 181

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying THE GOVERNING EQUATIONS OF THE HOLISTIC SYSTEM into itself .Indeed it is obvious that 182

$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}} \right) \right] ds_{(24)} =$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1 \right)$$

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From which it follows that

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0 \right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0} \right)} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 4

(b) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying systemic equations into itself .Indeed it is obvious that

$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$ 184

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 185

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)}+G_{29}^{0}\right)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying systemic equations into itself .Indeed it is obvious that 186

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]\,ds_{(32)} = \left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 187

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)}+G_{33}^{0}\right)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$ 188

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose $(\hat{P}_{24})^{(4)}$ *and* $(\hat{Q}_{24})^{(4)}$ large to have

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$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^{0}\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^{0}}{G_j^{0}}\right)}\right] \leq (\hat{P}_{24})^{(4)}$$

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$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^{0}\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^{0}}{T_j^{0}}\right)}+(\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself 191

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 192

$$d\left(\left((G_{27})^{(1)},(T_{27})^{(1)}\right),\left((G_{27})^{(2)},(T_{27})^{(2)}\right)\right) = \sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

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Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

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$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$
$$\int_0^t\{(a_{24}')^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$
$$(a_{24}'')^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$
$$G_{24}^{(2)}\left|(a_{24}'')^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a_{24}'')^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\; e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\widehat{M}_{24})^{(4)} t} \leq \frac{1}{(\widehat{M}_{24})^{(4)}} \left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)} \right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)} \right) \right)$$ 195

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{24})^{(4)}$ *and* $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. 196

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ *and* $(b''_i)^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 197

From the governing equation set up here in before it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{ (a'_i)^{(4)} - (a''_i)^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \} ds_{(24)} \right]} \geq 0$$

$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(4)} t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: 198

Remark 3: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating 199

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$ 200

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$ 201

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 202

Remark 5: If T_{24} is bounded from below and $lim_{t\to\infty}((b''_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 203

Definition of $(m)^{(4)}$ *and* ε_4 : 204

Indeed let t_4 be so that for $t > t_4$ 205

$(b_{25})^{(4)} - (b''_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 206

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $lim_{t\to\infty}(b''_{26})^{(4)}\left((G_{27})(t), t\right) = (b'_{26})^{(4)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations for the governing system:

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$ 207

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose
$(\hat{P}_{28})^{(5)}$ *and* $(\hat{Q}_{28})^{(5)}$ large to have

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$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)}$$

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$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying systemic equations into itself 210

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 211

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$$d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right), \left((G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

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Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$
It results

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$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)}\left|G_{29}^{(1)} - G_{29}^{(2)}\right|e^{-(\hat{M}_{28})^{(5)}s_{(28)}}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)} +$$
$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\hat{M}_{28})^{(5)}s_{(28)}}e^{-(\hat{M}_{28})^{(5)}s_{(28)}} +$$
$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right|e^{-(\hat{M}_{28})^{(5)}s_{(28)}}e^{(\hat{M}_{28})^{(5)}s_{(28)}} +$$
$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)|\ e^{-(\hat{M}_{28})^{(5)}s_{(28)}}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

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$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right|e^{-(\hat{M}_{28})^{(5)}t} \leq$$
$$\frac{1}{(\hat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 5: The fact that we supposed $(a''_{28})^{(5)}$ *and* $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 216

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ *and* $(b''_i)^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on

$(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark : There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 217

From the governing equations of the totalistic system, it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$ 218

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 219

Remark: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if 220

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$ 221

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$ 222

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark : If $G_{28}\ is$ bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 223

Remark : If T_{28} is bounded from below and $lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 224

Definition of $(m)^{(5)}\ and\ \varepsilon_5$:

Indeed let t_5 be so that for $t\ > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$ 225

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 226

$T_{29}\ \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \ \frac{1}{2}$ it results 227

$T_{29}\ \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of SYSTEMAL equations

Analogous inequalities hold also for $G_{33}\ , G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\ \hat{M}_{32}\ )^{(6)}}\ , \frac{(b_i)^{(6)}}{(\ \hat{M}_{32}\ )^{(6)}} < 1$ and to choose $(\ \hat{P}_{32}\ )^{(6)}\ and\ (\ \hat{Q}_{32}\ )^{(6)}$ large to have 228

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\ \hat{P}_{32})^{(6)} + \left((\ \hat{P}_{32}\ )^{(6)} + G_j^0\right)e^{-\left(\frac{(\ \hat{P}_{32}\ )^{(6)} + G_j^0}{G_j^0}\right)}\right] \leq (\ \hat{P}_{32}\ )^{(6)}$$ 229

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$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying SYSTEMAL EQUATIONS 231

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 232

$$d\left(\left((G_{35})^{(1)},(T_{35})^{(1)}\right),\left((G_{35})^{(2)},(T_{35})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$ 233

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$ 234

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$
$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$
$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$
$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$ 235

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(\left((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)}\right)\right)$$ 236

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ and $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 237

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 238

From 69 to 32 it results

$$G_i(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\geq 0$$
$T_i(t)\geq T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0$ for $t>0$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ and $\left((\widehat{M}_{32})^{(6)}\right)_3$: 239

Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$G_{32}<(\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt}\leq\left((\widehat{M}_{32})^{(6)}\right)_1-(a'_{33})^{(6)}G_{33}$ and by integrating 240

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$ 241

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a'_{34})^{(6)}$ 242

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 243

Remark 5: If T_{32} is bounded from below and $lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 244

Definition of $(m)^{(6)}$ *and* ε_6 :

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Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(\mathrm{t}) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 246

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t), t(t), t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of Solutional equations

Behavior of the solutions of systemic equations 247

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(a) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$ 248

$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \leq -(\tau_1)^{(4)}$ 249

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$:

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(b) By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ 251

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and 252

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

253

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$ 254

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ 255

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(c) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by 256

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if} \ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

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$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)} \ , \boldsymbol{if} \ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

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$(\ m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}, \ \boldsymbol{if} \ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \ \boldsymbol{if} \ (u_0)^{(4)} < (u_1)^{(4)}$

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$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)} \ , \boldsymbol{if} \ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$ 260

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$(\ \mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if} \ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of systemic equations satisfies the inequalities 262

$$G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined in the foregoing

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$ 263

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}\left((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)}\right)} \left[e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}\left((S_1)^{(4)} - (a'_{26})^{(4)}\right)} \left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

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$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}}$$

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$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}$$

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$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}\left((R_1)^{(4)} - (b'_{26})^{(4)}\right)} \left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}\left((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)}\right)} \left[e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

<u>Definition of</u> $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 268

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

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$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

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$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

<u>Behavior of the solutions of SYSTEMAL equations</u>

<u>Theorem 2:</u> If we denote and define

<u>Definition of</u> $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$: 280

(d) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$ 281

$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \leq -(\tau_1)^{(5)}$ 282

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 283

(e) By $(\nu_1)^{(5)} > 0, (\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ 284
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and 285

Definition of $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{\nu}_1)^{(5)} > 0, (\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the 286
roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ 287
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:- 288

(f) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$ 289

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ 290

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$ 291

and analogously

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$ 292

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$ 293

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$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of the universal system in question satisfies the inequalities 295

$G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$

where $(p_i)^{(5)}$ is defined by equation 29

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$ 296

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}\left((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq\right.$ 297

$\left.\frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}\left((S_1)^{(5)} - (a'_{30})^{(5)}\right)}\left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}\right)$ 298

$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{\left((R_1)^{(5)} + (r_{28})^{(5)}\right)t}}$ 299

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}$ 300

$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq$ 301

$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^0 e^{-(R_2)^{(5)}t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 302

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$ 303

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$ 304

Behavior of the SYSTEMAL solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$: 305

(g) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$ 306

$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$ 307

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$:

(h) By $(\nu_1)^{(6)} > 0, (\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the 308
equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ 309
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 310

By $(\bar{\nu}_1)^{(6)} > 0, (\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the
roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ 311
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:- 312

(i) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \; \boldsymbol{if} \; (\nu_0)^{(6)} < (\nu_1)^{(6)}$ 313

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if} \; (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ 314

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \; \boldsymbol{if} \; (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \; \boldsymbol{if} \; (u_0)^{(6)} < (u_1)^{(6)}$

315

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$ 316

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined by 59 and 66 respectively 317

Then the solution of THE SYSTEM SATISFIES the inequalities

$$G_{32}^0 e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$ 318

where $(p_i)^{(6)}$ is defined UIN THE FOREGOING

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)}-(p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$ 319

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)}-(a_{34}')^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a_{34}')^{(6)}t}\right] + G_{34}^0 e^{-(a_{34}')^{(6)}t}\right)$$ 320

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}}$$ 321

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$ 322

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}')^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b_{34}')^{(6)}t}\right] + T_{34}^0 e^{-(b_{34}')^{(6)}t} \leq T_{34}(t) \leq$$ 323

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

<u>Definition of</u> $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 324

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32}')^{(6)}$ 324

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32}')^{(6)}$ 325

$(R_2)^{(6)} = (b_{34}')^{(6)} - (r_{34})^{(6)}$

<u>Proof</u> : From Governing Equations Of the System we obtain

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$ 326

<u>Definition of</u> $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$ 327

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$ 328

From which one obtains

<u>Definition of</u> $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

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(a) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$ 330

$it\ follows\ (\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$ 331

In the same manner , we get 332

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$ 333

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(b) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 334

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$ 335

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

(c) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 336

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$ 337

And so with the notation of the first part of condition (c) , we have
Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$ 338

In a completely analogous way, we obtain
Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$ 339

Now, using this result and replacing it in Solutional equations of the governing set, we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

From Governing Equations of the System we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right) - (a''_{29})^{(5)}(T_{29},t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$ 340

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$ 341

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(d) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$ 342

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

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$it\ follows\ (\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 344

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(e) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 345

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(f) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 346

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have
Definition of $\nu^{(5)}(t)$:-

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$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain
Definition of $u^{(5)}(t)$:-

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$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in Solutional equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then
$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

FROM THE GOVERNING EQUATIONS OF THE SYSTEM WE OBTAIN:

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

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<u>Definition of</u> $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$ 350

From which one obtains

<u>Definition of</u> $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(g) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$ 351

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$ 352

$it\ follows\ (\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 353

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(h) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 354

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(i) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 355

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$ 356

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

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$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in governing equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

We can prove the following 358

Theorem 3: If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0 ,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

$with\ (p_{24})^{(4)}, (r_{25})^{(4)}$ satisfied , then the system

If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions 359

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0 ,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

$with\ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation 25 are satisfied , then the system

If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions 360

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

$with\ (p_{32})^{(6)}, (r_{33})^{(6)}$ are satisfied , then the system

$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0$ 361

$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0$ 362

$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0$ 363

$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0$ 364

$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0$ 364

$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0$ 366

has a unique positive solution , which is an equilibrium solution for the system

$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0$ 367

$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0$ 368

$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0$ 369

$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0$ 370

$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0$ 371

$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0$ 372

has a unique positive solution , which is an equilibrium solution for the system

$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0$ 373

$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0$ 374

$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0$ 375

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0$ 376

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0$ 377

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0$ 378

has a unique positive solution , which is an equilibrium solution for the system

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$ 379

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if 380

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if 381

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Definition and uniqueness of T_{25}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations 382

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations 383

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ are being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

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$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(d) By the same argument, the equations GOVERNING THE SYSTEM admit solutions G_{24}, G_{25} if

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - \left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

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Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(e) By the same argument, the equations of the system admit solutions G_{28}, G_{29} if

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - \left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

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Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(f) By the same argument, the equations GOVERNING THE SYSTEM admit solutions G_{32}, G_{33} if

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - \left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$ 387

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of THE SYSTEM: 388

G_{25}^* *given by* $\varphi(G_{27}) = 0$, T_{25}^* *given by* $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]}$$ 389

Obviously, these values represent an equilibrium solution of the Solutional equations
Finally we obtain the unique solution of Governing equations of the system in question

G_{29}^* *given by* $\varphi((G_{31})^*) = 0$, T_{29}^* *given by* $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$ 390

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]}$$ 391

Obviously, these values represent an equilibrium solution of the governing equations of the system
Finally we obtain the unique solution of the system in question:

G_{33}^* *given by* $\varphi((G_{35})^*) = 0$, T_{33}^* *given by* $f(T_{33}^*) = 0$ and 392

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]}$$ 393

Obviously, these values represent an equilibrium solution of the given system

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ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(4)}$ *and* $(b''_i)^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$ 395

$$\frac{\partial (a''_{25})^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \quad \frac{\partial (b''_i)^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$ 396

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25}$ 397

$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25}$ 398

$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25}$ 399

$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right)$ 400

$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right)$ 401

$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^*\mathbb{G}_j\right)$ 402

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}$ *and* $(b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

_Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$ 403

$\frac{\partial (a''_{29})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b''_i)^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$ 404

Then taking into account systemic equations and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29}$ 405

$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29}$ 406

$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29}$ 407

$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right)$ 408

$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right)$ 409

$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right)$ 410

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}$ *and* $(b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. 411

_Denote

<u>Definition of</u> $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a''_{33})^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b''_i)^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$ 412

413

Then taking into account sytemal equations and neglecting the terms of power 2, we obtain

$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33}$ 414

$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33}$ 415

$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33}$ 416

$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right)$ 417

$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right)$ 418

$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right)$ 419

The characteristic equation of this system is

$$\left((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)}\right)\{\left((\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)}\right)$$
$$\left[\left(\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25} + (b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$
$$+\left(\left((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24} + (a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25} + (b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^2 + \left((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$
$$+\left(\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$
$$+\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25} + (b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\} = 0$$

420

$+$

$$\left((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}\right)\{\left((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}\right)$$
$$\left[\left(\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29} + (a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$
$$\left(\left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29} + (b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$
$$+\left(\left((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28} + (a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$
$$\left(\left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29} + (b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^2 + \left((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$+\left(\left((\lambda)^{(5)}\right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$
$$+\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29} + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$
$$\left(\left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29} + (b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\} = 0$$

421

$+$

$$\left((\lambda)^{(6)}+(b_{34}')^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a_{34}')^{(6)}+(p_{34})^{(6)}\right)$$
$$\left[\left(\left((\lambda)^{(6)}+(a_{32}')^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$
$$+\left(\left((\lambda)^{(6)}+(a_{33}')^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G_{32}^{*}+(a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$ 422
$$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}')^{(6)}+(a_{33}')^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b_{32}')^{(6)}+(b_{33}')^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$
$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}')^{(6)}+(a_{33}')^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$
$$+\left((\lambda)^{(6)}+(a_{32}')^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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