"NOISE" OR "DISCORDANCE" AND QUANTUM COMPUTATION- A STRANGE MENAGE A TROIS- AN INVOLUTION EVOLUTION MODEL

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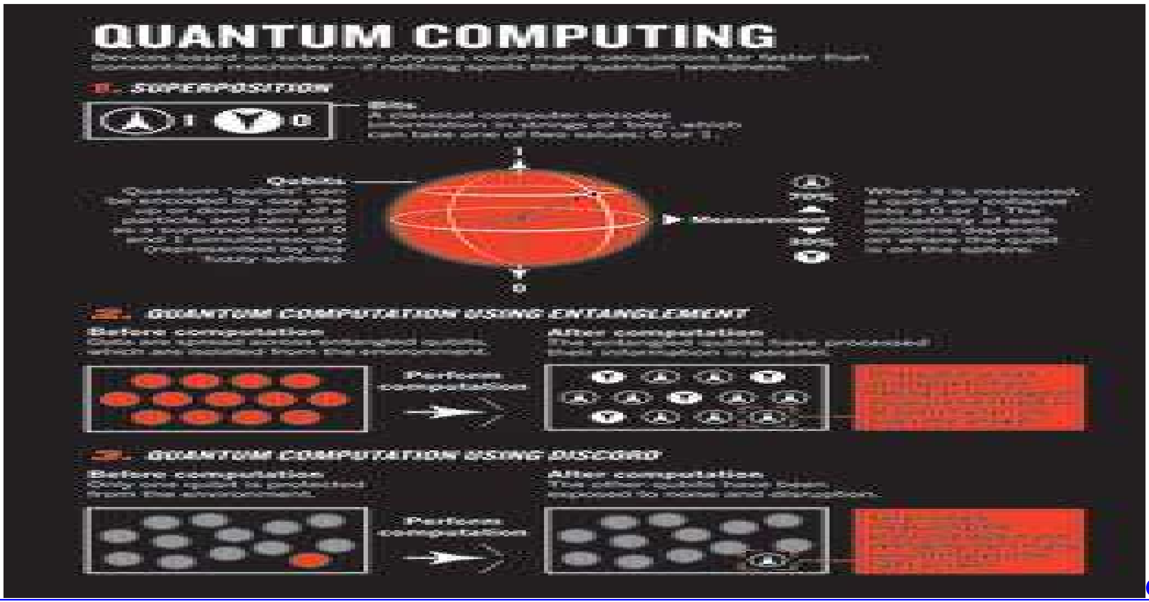
ABSTRACT: This is a story of autumn sonata and rocky razzmatazz, a combination of angst saga of taciturn bay and enduring silence on one hand and combination of Byzantine logic and sedative humour on the other. It is at this point the anecdote of life meets aphorism of thought! Courage of conviction and will for vindication have made Physicists to do what usually others do: If you cannot beat them join them philosophy! Since noise gets in the way of quantum computing, cure it by adding more noise. Reports in "The register" states that startling proposal is the work of a team led by the Australian National University's Dr André Carvalho, along with collaborators from Brazil and Spain. Noise is normally treated as the enemy in quantum-level experiments, because it destroys the useful characteristics of qubits (quantum computing elements). As Dr Carvalho explains "in the quantum world, these operations work because the qubits can be in a state of superposition" (that is, multiple possible states existing at the same time – as did the famous cat belonging to Herr Schrödinger'). "Those coherence properties disappear with time, and that means the main resource, entanglement, disappears with noise."The particular kind of noise Carvalho's work deals with is the spontaneous emission of photons. This means, simply, that if a qubit starts in a "1" state, it will eventually emit a photon and fall to a "0" state. Without a solution to the noise problem, Dr Carvalho says, computation becomes impossible. "Because we have no control on the outcomes of the measurement – they are totally random – if we just passively wait it would take an infinite amount of time to extract even a very simple computation."Dr Carvalho's solution is two-fold: first, to add noise (as photons, using a laser) back into a qubit before the state-decay takes place; second, to perform measurement on the system in just the right way. If, after adding the extra noise, the system were simply left alone, decoherence will happen more quickly, he said. "But if we measure the photons that are coming out, and measure them in the right way, then at the end of the measurement, we have a quantum gate."In other words, under the right circumstances, the act of measurement is what creates the quantum gate that can perform computation. Unfortunately, measurement, like noise, is an enemy of entanglement and superposition. Just as in Schrödinger's thought experiment, if you "look in the box", the quantum system will resolve itself into a classical state. "When you measure a quantum system, you destroy the system," he said. "If you measure the state of a photon, you will know that it's in a particular state."However, because of the added noise, there are two kinds of photons available for measurement – those that are created by spontaneous emission, and those added in the excitation process. There are two kinds of photons, the spontaneous and the 'noisy channel'," – and since the detector "can't tell whether the photon is coming from one process or the other, that creates a superposition. Metaphorically it is the case of million monkeys typing Shakespeare Problem. Here we give a Model an Involution and Evolution and discuss the properties and Solutional behaviour Of the problem. Concatenated in the Annexure re the equations of "Measurement of Quantum Gates " and "collapse of Quantum States" which should round off the explanation on that score with inclusion of Gates."

Quantum Computing:

Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along? (Zeeya Merali)Quantum computation, attempting to exploit subatomic physics to create a device with the potential to outperform its best macroscopic counterparts IS A Gordian knot with the Physicists. . Quantum systems are fragile, vulnerable and susceptible both in its thematic and discursive form and demand immaculate laboratory conditions to survive long enough to be of any use. Now White was setting out to test an unorthodox quantum algorithm that seemed to turn that lesson on its head. Energetic franticness, ensorcelled frenzy, entropic entrepotishness, Ergodic erythrism messiness and disorder would be virtues, not vices — and perturbations in the quantum system would drive computation, not disrupt it.
Conventional view is that such devices should get their computational power from quantum entanglement — a phenomenon through which particles can share information even when they are separated by arbitrarily large distances. But the latest experiments suggest that entanglement might not be needed after all. Algorithms could instead tap into a quantum resource called discord, which would be far cheaper and easier to maintain in the lab.
Classical computers have to encode their data in an either/or fashion: each bit of information takes a value of 0 or 1,

and nothing else. But the quantum world is the realm of both/and. Particles can exist in 'superpositions' — occupying many locations at the same time, say, or simultaneously (e&eb)spinning clockwise and anticlockwise.



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So, Feynman argued, computing in that realm could use quantum bits of information — qubits — that exist as superpositions of 0 and 1 simultaneously. A string of 10 such qubits could represent all 1,024 10-bit numbers simultaneously. And if all the qubits shared information through entanglement, they could race through myriad calculations in parallel — calculations that their classical counterparts would have to plod through in a languorous, lugubrious and lachrymososhish manner sequentially (see 'Quantum computing').

The notion that quantum computing can be done only through entanglement was cemented in 1994, when Peter Shor, a mathematician at the Massachusetts Institute of Technology in Cambridge, devised an entanglement-based algorithm that could factorize large numbers at lightning speed — potentially requiring only seconds to break the encryption currently used to send secure online communications, instead of the years required by ordinary computers. In 1996, Lov Grover at Bell Labs in Murray Hill, New Jersey, proposed an entanglement-based algorithm that could search rapidly through an unsorted database; a classical algorithm, by contrast, would have to laboriously search the items one by one.

But entanglement has been the bane of many a quantum experimenter's life, because the slightest interaction of the entangled particles with the outside world — even with a stray low-energy photon emitted by the warm walls of the laboratory — can destroy it. Experiments with entanglement demand ultra-low temperatures and careful handling. "Entanglement is hard to prepare, hard to maintain and hard to manipulate," says Xiaosong Ma, a physicist at the Institute for Quantum Optics and Quantum Information in Vienna. Current entanglement record-holder intertwines just 14 qubits, yet a large-scale quantum computer would need several thousand. Any scheme that bypasses entanglement would be warmly welcomed, without any hesitation, reservation, regret, remorse, compunction or contrition. Says Ma.

Clues that entanglement isn't essential after all began to trickle in about a decade ago, with the first examples of rudimentary regimentation and seriotological sermonisations and padagouelogical pontifications quantum computation. In 2001, for instance, physicists at IBM's Almaden Research Center in San Jose and Stanford University, both in California, **used a 7**-qubit system to implement Shor's algorithm, factorizing the number 15 into 5 and 3. But controversy erupted over whether the experiments deserved to be called quantum computing, says Carlton Caves, a quantum physicist at the University of New Mexico (UNM) in Albuquerque.

The trouble was that the computations were done at room temperature, using liquid-based nuclear magnetic resonance (NMR) systems, in which information is encoded in atomic **nuclei using(e) an internal quantum property known as spin**. Caves and his colleagues had already shown that entanglement could not be sustained in these conditions. "The nuclear spins would be jostled about too much for them to stay lined up neatly," says Caves. According to the orthodoxy, **no entanglement meant** any quantum computation. The NMR community gradually accepted that they had no entanglement, yet the computations were producing real results. Experiments were explicitly performed for a quantum search without (e(e))exploiting entanglement. These experiments really called into question what gives quantum computing its power,.

Order out of disorder

Discord, an obscure measure of quantum correlations. Discord quantifies (=) how much a system can be disrupted when people observe it to gather information. Macroscopic systems are not e(e&eb)affected by observation, and so have zero discord. But quantum systems are unavoidably (e&eb) affected because measurement forces them to settle on one of their many superposition values, so any possible quantum correlations, including entanglement, give (eb) a positive value for discord. Discord is connected (e&eb)to quantum computing."An algorithm challenged the idea that quantum computing requires (e) to painstakingly prepare(eb) a set of pristine qubits in the lab.

In a typical optical experiment, the pure qubits might (e) consist of horizontally polarized photons representing 1 and vertically polarized photons representing 0. Physicists can entangle a stream of such pure qubits by passing them through a (e&eb) processing gate such as a crystal that alters (e&eb) the polarization of the light, and then read off the state of the qubits as they exit. In the real world, unfortunately, qubits rarely stay pure. They are far more likely to become messy, or 'mixed' — the equivalent of unpolarized photons. The conventional wisdom is that mixed qubits are(e) useless for computation because they e(e&eb) cannot be entangled, and any measurement of a mixed qubit will yield a random result, providing little or no useful information.

If a mixed qubit was sent through an entangling gate with a pure qubit. The two could not become entangled but, the physicists argued, their interaction might be enough to carry (eb) out a quantum computation, with the result read from the pure qubit. If it worked, experimenters could get away with using just one tightly controlled qubit, and letting the others be badly battered sadly shattered by environmental noise and disorder. "It was not at all clear why that should work," says White. "It sounded as strange as saying they wanted to measure someone's speed by measuring the distance run with a perfectly metered ruler and measuring the time with a stopwatch that spits out a random answer."

Datta supplied an explanation he calculated that the computation could be(eb) driven by the quantum correlation between the pure and mixed qubits — a correlation given mathematical expression by the discord."It's true that you must have entanglement to compute with idealized pure qubits," "But when you include mixed states, the calculations look very different."Quantum computation without (e) the hassle of entanglement," seems to have become a point where the anecdote of life had met the aphorism of thought. Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful.

The team confirmed that the qubits were not entangled at any point. Intriguingly, when the researchers tuned down the polarization quality of the one pure qubit, making (eb) it almost mixed, the computation still worked. "Even when you have a system with just a tiny fraction of purity, that is (=) vanishingly close to classical, it still has power," says White. "That just blew our minds." The computational power only disappeared when the amount of discord in the system reached zero. "It's counter-intuitive, but it seems that putting noise and disorder in your system gives you power," says White. "Plus, it's easier to achieve."For Ma, White's results provided the "wow! Moment" that made him takes discord seriously. He was keen to test discord-based algorithms that used more than the two qubits used by White, and that could perform more glamorous tasks, but he had none to test. "Before I can carry out any experiments, I need the recipe of what to prepare from theoreticians," he explains, and those instructions were not forthcoming.

Although it is easier for experimenters to handle noisy real-world systems than pristinely glorified ones, it is a lot harder for theoretical physicists to analyse them mathematically. "We're talking about messy physical systems, and the equations are even messier," says Modi. For the past few years, theoretical physicists interested in discord have been trying to formulate prescriptions for new tests. It is not proved that discord is (eb) essential to computation — just that it is there. Rather than being the engine behind computational power, it could just be along for the ride, he argues. Last year, Acín and his colleagues calculated that almost every quantum system contains discord. "It's basically everywhere," he says. "That makes it difficult to explain why it causes power in specific situations and not others." It is almost like we can perform our official tasks amidst all noise, subordination pressure, superordinational scatological pontification, coordination dialectic deliberation, but when asked to do something different we want “peace”.”Silence”, “No disturbance” .Personally, one thinks it is a force of habit. And habits die hard. Modi shares the concern. "Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful. We need to identify what that way is," he says.Du and Ma are independently conducting experiments to address these points. Both are attempting to measure the amount of discord at each stage of a computation — Du using liquid NMR and electron-spin resonance systems, and Ma using photons. The very ‘importance giving’, attitude itself acts as an anathema, a misnomer.

A finding that quantifies how and where discord acts would strengthen the case for its importance, says Acín. We suspect it acts only in cases where there is ‘speciality’like in quantum level. Other ‘mundane ‘world’ happenings take place amidst all discord and noise. Nobody bothers because it is ‘run of the mill’ But for ‘selective and important issues’ one needs ‘calm’ and ‘non disturbance’ and doing’ all ‘things’ amidst this worldly chaos we portend is

'Khuda' 'Allah" or 'Brahman" And we feel that Quantum Mechanics is a subjective science and teaches this philosophy much better than others. But if these tests find discord wanting, the mystery of how entanglement-free computation works will be reopened. "The search would have to begin for yet another quantum property," he adds. Vedral notes that even if Du and Ma's latest experiments are a success, the real game-changer will be discord-based algorithms for factorization and search tasks, similar to the functions devised by Shor and Grover that originally ignited the field of quantum computing. "My gut feeling is that tasks such as these will ultimately need entanglement," says Vedral. "Though as yet there is no proof that they can't be done with discord alone."
Zurek says that discord can be thought of as a complement to entanglement, rather than as a usurper. "There is no longer a question that discord works," he declares. "The important thing now is to find out when discord without entanglement can be (eb)exploited most usefully, and when entanglement is essential. ,and produces 'Quantum Computation'

How Noise Can Help Quantum Entanglement

Electrons would be in two places at once – Double Salary- Sine qua non benefits etal., it must sound interesting,— in the modern view that has gained traction in the past decade, you don't see quantum effects in everyday life not because you are big, per se, but because those effects are camouflaged by their own sheer complexity. They are there if you know how to look, and physicists have been realizing that they show up in the macroscopic world more than they thought. "The standard arguments may be too pessimistic as to the survival of quantum effects," physicist Anthony Leggett.
In the most distinctive such effects, called entanglement, two electrons establish a kind of telepathic link that transcends space and time. And not just electrons: you, too, retain a quantum bond with your loved ones that endures no matter how far apart you may be. If that sounds hopelessly romantic, the flip side is that particles are incurably promiscuous but not parsimonious; complex yet not curmudgeonly; permissive yet not Yiddish schmaltz hooking up with every other particle they meet. So you also retain a quantum bond with every loser who ever bumped into you on the street and every air molecule that ever brushed your skin. The bonds you want are overwhelmed by those you don't. Entanglement thus foils entanglement, a process known as decoherence.
To preserve entanglement for use in, say, quantum computers, physicists use all the tactics of a parent trying to control a amorous escapades, extramural activities, salacious coveting, diurnal dynamics and nocturnal predications teenager's love life, such as isolating the particle from its environment or chaperoning the particle and undoing any undesired entanglements. And they typically have about as much success. But if you can't beat the environment, why not use it? "The environment can act more positively," says physicist Vlatko Vedral of the National University of Singapore and the University of Oxford.
One approach has been suggested by Jianming CAI and Hans J. Briegel of the Institute for Quantum Optics and Quantum Information. Suppose you have a V-shaped molecule you can open and close like a pair of tweezers. When the molecule closes, two electrons on the tips become entangled. If you just keep them there, the electrons will eventually decohere as particles from the environment bombard them, and you will have no way to reestablish entanglement.
The answer is to open up the molecule and, counter intuitively, leave the electrons even more exposed to the environment. In this position, decoherence resets the electrons back to a default, lowest-energy state. Then you can close the molecule again and reestablish entanglement afresh. If you open and close fast enough, it is as though the entanglement was never broken. The team calls this "dynamic entanglement," as opposed to the static kind that endures as long as you can isolate the system from bombardment. The oscillation notwithstanding, the researchers say dynamic entanglement can do everything the static sort can.
A different approach uses a group of particles that act collectively as one. Because of the group's internal dynamics, it can have multiple default, or equilibrium, states, corresponding to different but comparably energetic arrangements. A quantum computer can store data in these equilibrium states rather than in individual particles. This approach, first proposed a decade ago by Alexei Kitaev, then at the Landau Institute for Theoretical Physics in Russia, is known as passive error correction, because it does not require physicists to supervise the particles actively. If the group deviates from equilibrium, the environment does the work of pushing it back. Only when the temperature is high enough does the environment disrupt rather than stabilize the group. "The environment both adds errors as well as removes them," says Michal Horodecki of the University of Gdansk in Poland.

Quantum decoherence

In quantum mechanics, quantum decoherence is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition. A consequence of this dephasing leads to classical or probabilistically additive behavior. Quantum decoherence gives the appearance of wave function collapse (the reduction of the physical possibilities into a single possibility as seen by an observer) and justifies the framework and intuition of classical physics as an acceptable approximation: decoherence is the mechanism by which the classical limit emerges out of a quantum starting point and it determines the location of the quantum-classical boundary. Decoherence occurs when a system interacts with its environment in a thermodynamically irreversible way. This prevents different elements in the quantum superposition of the system+environment's wavefunction from interfering with each other. Decoherence has been a subject of active research since the 1980s.

Decoherence can be viewed as the loss of information from a system into the environment (often modeled as a heat bath), since every system is loosely coupled with the energetic state of its surroundings. Viewed in isolation, the system's dynamics are non-unitary (although the combined system plus environment evolves in a unitary fashion) Thus the dynamics of the system alone are irreversible. As with any coupling, entanglements are generated between the system and environment, which have the effect of sharing quantum information with—or transferring it to—the surroundings.

Decoherence does not generate actual wave function collapse. It only provides an explanation for the appearance of the wavefunction collapse, as the quantum nature of the system "leaks" into the environment. That is, components of the wavefunction are decoupled from a coherent system, and acquire phases from their immediate surroundings. A total superposition of the global or universal wavefunction still exists (and remains coherent at the global level), but its ultimate fate remains an interpretational issue. Specifically, decoherence does not attempt to explain the measurement problem. Rather, decoherence provides an explanation for the transition of the system to a mixture of states that seem to correspond to those states observers perceive. Moreover, our observation tells us that this mixture looks like a proper quantum ensemble in a measurement situation, as we observe that measurements lead to the "realization" of precisely one state in the "ensemble".Decoherence represents a challenge for the practical realization of quantum computers, since they are expected to rely heavily on the undisturbed evolution of quantum coherences. Simply put; they require(e)_ that coherent states be preserved and that decoherence is managed, in order to actually perform quantum computation.

Mechanisms

To examine how decoherence operates, an "intuitive" model is presented. The model requires some familiarity with quantum theory basics. Analogies are made between visualisable classical phase spaces and Hilbert spaces. A more rigorous derivation in Dirac notation shows how decoherence destroys interference effects and the "quantum nature" of systems. Next, the density matrix approach is presented for perspective.

Phase space picture

An N-particle system can be represented in non-relativistic quantum mechanics by a wavefunction, $\psi(x_1, x_2, \ldots, x_N)$. This has analogies with the classical phase space. A classical phase space contains a real-valued function in 6N dimensions (each particle contributes 3 spatial coordinates and 3 momenta). Our "quantum" phase space conversely contains a complex-valued function in a 3N dimensional space. The position and momenta do not commute but can still inherit much of the mathematical structure of a Hilbert space. Aside from these differences, however, the analogy holds.

Different previously-isolated, non-interacting systems occupy different phase spaces. Alternatively we can say they occupy different, lower-dimensional subspaces in the phase space of the joint system. The effective dimensionality of a system's phase space is the number of degrees of freedom present which—in non-relativistic models—is 6 times the number of a system's free particles. For a macroscopic system this will be a very large dimensionality. When two systems (and the environment would be a system) start to interact, though, their associated state vectors are no longer constrained to the subspaces. Instead the combined state vector time-evolves a path through the "larger volume", whose dimensionality is the sum of the dimensions of the two subspaces. A square (2-d surface) extended by just one dimension (a line) forms a cube. The cube has a greater volume, in some sense, than its component square and line axes. The extent two vectors interfere with each other is a measure of how "close" they are to each other (formally, their overlap or Hilbert space scalar product together) in the phase space. When a system couples to an external environment, the dimensionality of, and hence "volume" available to, the joint state vector increases enormously. Each environmental degree of freedom contributes an extra dimension.

The original system's wavefunction can be expanded arbitrarily as a sum of elements in a quantum superposition. Each expansion corresponds to a projection of the wave vector onto a basis. The bases can be chosen at will. Let us choose any expansion where the resulting elements interact with the environment in an element-specific way. Such elements will—with overwhelming probability—be rapidly separated from each other by their natural unitary time evolution along their own independent paths. After a very short interaction, there is almost no chance of any further interference. The process is effectively irreversible. The different elements effectively become "lost" from each other in the expanded phase space created by coupling with the environment; in phase space, this decoupling is monitored through the Wigner quasi-probability distribution. The original elements are said to have decohered. The environment has effectively selected out those expansions or decompositions of the original state vector that decohere (or lose phase coherence) with each other. This is called "environmentally-induced-super selection", or einselection The decohered elements of the system no longer exhibit quantum interference between each other, as in a double-slit experiment. Any elements that decohere from each other via environmental interactions are said to be quantum entangled with the environment. The converse is not true: not all entangled states are decohered from each other.

Any measuring device or apparatus acts as an environment since, at some stage along the measuring chain, it has to be large enough to be read by humans. It must possess a very large number of hidden degrees of freedom. In effect, the interactions may be considered to be quantum measurements. As a result of an interaction, the wave functions of the system and the measuring device become entangled with each other. Decoherence happens when different portions of the system's wavefunction become entangled in different ways with the measuring device. For two einselected elements of the entangled system's state to interfere, both the original system and the measuring in both elements device must significantly overlap, in the scalar product sense. If the measuring device has many degrees of freedom, it is very unlikely for this to happen.

As a consequence, the system behaves as a classical statistical ensemble of the different elements rather than as a single coherent quantum superposition of them. From the perspective of each ensemble member's measuring device, the system appears to have irreversibly collapsed onto a state with a precise value for the measured attributes, relative to that element.

Dirac notation

Using the Dirac notation, let the system initially be in the state $|\psi\rangle$ where

$$|\psi\rangle = \sum_i |i\rangle\langle i|\psi\rangle$$

where the $|i\rangle$s form an einselected basis (environmentally induced selected eigen basis; and let the environment initially be in the state $|\epsilon\rangle$. The vector basis of the total combined system and environment can be formed by tensor multiplying the basis vectors of the subsystems together. Thus, before any interaction between the two subsystems, the joint state can be written as:

$$|before\rangle = \sum_i |i\rangle|\epsilon\rangle\langle i|\psi\rangle.$$

where $|i\rangle|\epsilon\rangle$ is shorthand for the tensor product: $|i\rangle \otimes |\epsilon\rangle$. There are two extremes in the way the system can interact with its environment: either (1) the system loses its distinct identity and merges with the environment (e.g. photons in a cold, dark cavity get converted into molecular excitations within the cavity walls), or (2) the system is not disturbed at all, even though the environment is disturbed (e.g. the idealized non-disturbing measurement). In general an interaction is a mixture of these two extremes, which we shall examine:

System absorbed by environment

If the environment absorbs the system, each element of the total system's basis interacts with the environment such that:

$|i\rangle|\epsilon\rangle$ evolves into $|\epsilon_i\rangle$

and so

$|before\rangle$ evolves into $|after\rangle = \sum_i |\epsilon_i\rangle\langle i|\psi\rangle$

where the unitarity of time-evolution demands that the total state basis remains orthonormal and in particular their scalar or inner products with each other vanish, since $\langle i|j\rangle = \delta_{ij}$:

$$\langle \epsilon_i|\epsilon_j\rangle = \delta_{ij}$$

This orthonormality of the environment states is the defining characteristic required for einselection

System not disturbed by environment

This is the idealized measurement or undisturbed system case in which each element of the basis interacts with the environment such that:

$|i\rangle|\epsilon\rangle$ Evolves into the product $|i,\epsilon_i\rangle = |i\rangle|\epsilon_i\rangle$

i.e. the system disturbs the environment, but is itself undisturbed by the environment.
and so:

$|before\rangle$ evolves into $|after\rangle = \sum_i |i, \epsilon_i\rangle\langle i|\psi\rangle$

where, again, unitarity demands that:

$$\langle i, \epsilon_i | j, \epsilon_j\rangle = \langle i|j\rangle\langle\epsilon_i|\epsilon_j\rangle = \delta_{ij}\langle\epsilon_i|\epsilon_j\rangle = \delta_{ij}$$

and additionally decoherence requires, by virtue of the large number of hidden degrees of freedom in the environment, that

$$\langle\epsilon_i|\epsilon_j\rangle \approx \delta_{ij}$$

As before, this is the defining characteristic for decoherence to become einselection. The approximation becomes more exact as the number of environmental degrees of freedom affected increases.

Note that if the system basis $|i\rangle$ were not an einselected basis then the last condition is trivial since the disturbed environment is not a function of i and we have the trivial disturbed environment basis $|\epsilon_j\rangle = |\epsilon'\rangle$. This would correspond to the system basis being degenerate with respect to the environmentally-defined-measurement-observable. For a complex environmental interaction (which would be expected for a typical macro scale interaction) a non-einselected basis would be hard to define.

Loss of interference and the transition from quantum to classical

The utility of decoherence lies in its application to the analysis of probabilities, before and after environmental interaction, and in particular to the vanishing of interference terms after decoherence has occurred. If we ask what is the probability of observing the system making a transition or quantum leap from ψ to ϕ before ψ has interacted with its environment, then application of the Born probability rule states that the transition probability is the modulus squared of the scalar product of the two states:

$$prob_{before}(\psi \rightarrow \phi) = |\langle\psi|\phi\rangle|^2 = |\sum_i \psi_i^* \phi_i|^2 = \sum_i |\psi_i^* \phi_i|^2 + \sum_{ij;i\neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

where $\psi_i = \langle i|\psi\rangle, \psi_i^* = \langle\psi|i\rangle$ and $\phi_i = \langle i|\phi\rangle$ etc.

Terms appear in the expansion of the transition probability above which involve $i \neq j$; these can be thought of as representing interference between the different basis elements or quantum alternatives. This is a purely quantum effect and represents the non-additivity of the probabilities of quantum alternatives.

To calculate the probability of observing the system making a quantum leap from ψ to ϕ after ψ has interacted with its environment, then application of the Born probability rule states we must sum over all the relevant possible states of the environment, E_i, before squaring the modulus:

$$prob_{after}(\psi \rightarrow \phi) = \sum_j |\langle after|\phi, \epsilon_j\rangle|^2 = \sum_j |\sum_i \psi_i^* \langle i, \epsilon_i|\phi, \epsilon_j\rangle|^2 = \sum_j |\sum_i \psi_i^* \langle i|\phi\rangle\langle\epsilon_i|\epsilon_j\rangle|^2$$

The internal summation vanishes when we apply the decoherence / einselection condition $\langle\epsilon_i|\epsilon_j\rangle \approx \delta_{ij}$ and the formula simplifies to:

$$prob_{after}(\psi \rightarrow \phi) \approx \sum_j |\psi_j^* \langle j|\phi\rangle|^2 = \sum_i |\psi_i^* \phi_i|^2$$

If we compare this with the formula we derived before the environment introduced decoherence we can see that the effect of decoherence has been to move the summation sign Σ_i from inside of the modulus sign to outside. As a result all the cross- or quantum interference-terms:

$$\sum_{ij;i\neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

have vanished from the transition probability calculation. The decoherence has irreversibly converted quantum behaviour (additive probability amplitudes) to classical behaviour (additive probabilities)In terms of density matrices, the loss of interference effects corresponds to the diagonalization of the "environmentally traced over" density matrix.

Density matrix approach

The effect of decoherence on density matrices is essentially the decay or rapid vanishing of the off-diagonal elements of the partial trace of the joint system's density matrix, i.e. thetrace, with respect to any environmental basis, of the density matrix of the combined system and its environment. The decoherence irreversibly converts the "averaged" or "environmentally traced over" density matrix from a pure state to a reduced mixture; it is this that gives the appearance of wavefunction collapse. Again this is called "environmentally-induced-super selection", or einselection. The advantage of taking the partial trace is that this procedure is indifferent to the environmental basis chosen.

The density matrix approach has been combined with the Bohmian approach to yield a reduced trajectory approach, taking into account the system reduced density matrix and the influence of the environment

Operator-sum representation

Consider a system S and environment (bath) B, which are closed and can be treated quantum mechanically. Let $\mathcal{H}_S$ and $\mathcal{H}_B$ be the system's and bath's Hilbert spaces, respectively. Then the Hamiltonian for the combined system is

$$\hat{H} = \hat{H}_S \otimes \hat{I}_B + \hat{I}_S \otimes \hat{H}_B + \hat{H}_I$$

where $\hat{H}_S, \hat{H}_B$ are the system and bath Hamiltonians, respectively, and $\hat{H}_I$ is the interaction Hamiltonian between the system and bath, and $\hat{I}_S, \hat{I}_B$ are the identity operators on the system and bath Hilbert spaces, respectively. The time-evolution of the density operator of this closed system is unitary and, as such, is given by

$$\rho_{SB}(t) = \hat{U}(t)\rho_{SB}(0)\hat{U}^{\dagger}(t)$$

where the unitary operator is $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$. If the system and bath are not entangled initially, then we can write $\rho_{SB} = \rho_S \otimes \rho_B$. Therefore, the evolution of the system becomes

$$\rho_{SB}(t) = \hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^{\dagger}(t).$$

The system-bath interaction Hamiltonian can be written in a general form as

$$\hat{H}_I = \sum_i \hat{S}_i \otimes \hat{B}_i,$$

where $\hat{S}_i \otimes \hat{B}_i$ is the operator acting on the combined system-bath Hilbert space, and $\hat{S}_i, \hat{B}_i$ are the operators that act on the system and bath, respectively. This coupling of the system and bath is the cause of decoherence in the system alone. To see this, a partial trace is performed over the bath to give a description of the system alone:

$$\rho_S(t) = Tr_B[\hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^{\dagger}(t)].$$

$\rho_S(t)$ is called the reduced density matrix and gives information about the system only. If the bath is written in terms ofitssetof orthogonal basis kets, that is, if it has been initially diagonal zed then $\rho_B(0) = \sum_j a_j |j\rangle\langle j|$.

Computing the partial trace with respect to this (computational)basis gives:

$$\rho_S(t) = \sum_l \hat{A}_l \rho_S(0) \hat{A}_l^{\dagger}$$

where $\hat{A}_l, \hat{A}_l^{\dagger}$ are defined as the Kraus operators and are represented as

$$\hat{A}_l = \sqrt{a_j}\langle k|\hat{U}|j\rangle.$$

This is known as the operator-sum representation (OSR). A condition on the Kraus operators can be obtained by using the fact that $Tr(\rho_S(t)) = 1$; this then gives

$$\sum_l \hat{A}_l^{\dagger}\hat{A}_l = \hat{I}_S.$$

This restriction determines if decoherence will occur or not in the OSR. In particular, when there is more than one term present in the sum for $\rho_S(t)$ then the dynamics of the system will be non-unitary and hence decoherence will take place.

Semi group approach

A more general consideration for the existence of decoherence in a quantum system is given by the master equation, which determines how the density matrix of the system alone evolves in time. This uses the Schrödinger picture, where evolution of the state (represented by its density matrix) is considered. The master equation is:

$$\rho_S'(t) = \frac{-i}{\hbar}\left[\tilde{\mathbf{H}}_{\mathbf{S}}, \rho_S(t)\right] + L_D\left[\rho_S(t)\right]$$

where $\tilde{\mathbf{H}}_{\mathbf{S}} = \mathbf{H}_{\mathbf{S}} + \Delta$ is the system Hamiltonian, $\mathbf{H}_{\mathbf{S}}$, along with a (possible) unitary contribution from the bath, Δ and L_D is the Lindblad decohering term. The Linblad decohering term is represented as

$$L_D\left[\rho_S(t)\right] = \frac{1}{2}\sum_{\alpha,\beta=1}^{M} b_{\alpha\beta}\Big(\left[\mathbf{F}_\alpha, \rho_S(t)\mathbf{F}_\beta^\dagger\right] + \left[\mathbf{F}_\alpha\rho_S(t), \mathbf{F}_\beta^\dagger\right]\Big).$$

They $\{\mathbf{F}_\alpha\}_{\alpha=1}^{M}$ are basis operators for the M-dimensional space of bounded operators that act on the system Hilbert space $\mathcal{H}_S$-these are the error generators-and $b_{\alpha\beta}$represent the elements of a positive semi-definite Hermitian matrix-these matrix elements characterize the decohering processes and, as such, are called the noise parameters. The semi group approach is particularly nice, because it distinguishes between the unitary and decohering (non-unitary) processes, which are not the case with the OSR. In particular, the non-unitary dynamics are represented by L_D, whereas the unitary dynamics of the state are represented by the usual Heisenberg commutator. Note that when $L_D[\rho_S(t)] = 0$, the dynamical evolution of the system is unitary. The conditions for the evolution of the system density matrix to be described by the master equation are:
(1) the evolution of the system density matrix is determined by a one-parameter semi group
(2) the evolution is "completely positive" (i.e. probabilities are preserved)
(3) the system and bath density matrices are initially decoupled
Examples of non-unitary modeling of decoherence
Decoherence can be modeled as a non-unitary process by which a system couples with its environment (although the combined system plus environment evolves in a unitary fashion). Thus the dynamics of the system alone, treated in isolation, are non-unitary and, as such, are represented by irreversible transformations acting on the system's Hilbert, $\mathcal{H}$. Since the system's dynamics are represented by irreversible representations, then any information present in the quantum system can be lost to the environment or heat bath. Alternatively, the decay of quantum information caused by the coupling of the system to the environment is referred to as decoherence. Thus decoherence is the process by which information of a quantum system is altered by the system's interaction with its environment (which form a closed system), hence creating an entanglement between the system and heat bath (environment). As such, since the system is entangled with its environment in some unknown way, a description of the system by itself cannot be made without also referring to the environment (i.e. without also describing the state of the environment).

Quantum dissipation

Quantum Dissipation is the branch of physics that studies the quantum analogues of the process of irreversible loss of energy observed at the classical level. Its main purpose is to derive the laws of classical dissipation from the framework of quantum mechanics. It shares many features with the subjects of quantum decoherence and quantum theory of measurement.

Models of Quantum Dissipation

The main problem to address to study dissipation at the quantum level is the way to envisage the mechanism of irreversible loss of energy. Quantum mechanics usually deal with the Hamiltonian formalism, where the total energy of the system is a conserved quantity. So in principle it would not be possible to describe dissipation in this framework.
The idea to overcome this issue consists on splitting the total system in two parts: the quantum system where dissipation occurs and a so-called environment or bath where the energy of the former will flow towards. The way both systems are coupled depends on the details of the microscopic model, and hence, the description of the bath. To include an irreversible flow of energy (i.e., to avoid Poincaré recurrences in which the energy eventually flows back to the system), requires that the bath contain an infinite number of degrees of freedom. Notice that by virtue of the principle of universality, it is expected that the particular description of the bath will not affect the essential features of the dissipative process, as far as the model contains the minimal ingredients to provide the effect.
The simplest way to model the bath was proposed by Feynman and Vernon in a seminal paper from 1963 . In this

description the bath is a sum of an infinite number of harmonic oscillators, that in quantum mechanics represents a set of free bosonic particles.

The Caldeira-Leggett model

In 1981 Amir Caldeira and Anthony J. Leggett proposed a simple model to study in detail the way dissipation arises from a quantum point of view. It describes a quantum particle in one-dimension coupled to a bath. The Hamiltonian reads:

$$H = \frac{P^2}{2M} + V(X) + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) + X \sum_i C_i q_i + X^2 \sum_i \frac{C_i^2}{2m_i \omega_i^2},$$

The first two terms correspond to the Hamiltonian of a quantum particle of mass M and momentum P, in a potential V at position X. The third term describes the bath as a sum of infinite harmonic oscillators with masses m_i and momentum p_i, at positions q_i. ω_i are the frequencies of the harmonic oscillators. The next term describes the way system and bath is coupled. In the Caldeira - Leggett model the bath is coupled to the position of the particle. C_i are coefficients which depend on the details of the coupling. The last term is a counter-term which must be included to ensure that dissipation is homogeneous in all space. As the bath couples to the position, if this term is not included the model is not translation ally invariant, in the sense that the coupling is different wherever the quantum particle is located. This gives rise to an unphysical renormalization of the potential, which can be shown to be suppressed by including the counter-term.

To provide a good description of the dissipation mechanism, a relevant quantity is the bath spectral function, defined as follows:

$$J(\omega) = \frac{\pi}{2} \sum_i \frac{C_i^2}{m_i \omega_i} \delta(\omega - \omega_i)$$

The bath spectral function provides a constraint in the choice of the coefficients C_i. When this function has the form $J(\omega) = \eta\omega$, the corresponding classical kind of dissipation can be shown to be Ohmic. A more generic form is $J(\omega) \propto \omega^s$. In this case, if $s > 1$ the dissipation is called "super-Ohmic", while if $s < 1$ is sub-Ohmic. An example of a super-Ohmic bath is the electro-magnetic field under certain circumstances.

As mentioned, the main idea in the field of quantum dissipation is to explain the way classical dissipation can be described from a quantum mechanics point of view. To get the classical limit of the Caldeira - Leggett model, the bath must be integrated out (or traced out), which can be understood as taking the average over all the possible realizations of the bath and studying the effective dynamics of the quantum system. As a second step, the limit $\hbar \rightarrow 0$ must be taken to recover classical mechanics. To proceed with those technical steps mathematically, the path integral description of quantum mechanics is usually employed. The resulting classical equations of motion are:

$$M \frac{d^2}{dt^2} X(t) = -\frac{\partial V(X)}{\partial X} - \int_0^T dt' \alpha(t-t')(X(t) - X(t'))$$

where:

$$\alpha(t-t') = \frac{1}{2\pi} \int_0^\infty J(\omega) e^{-\omega|t-t'|} d\omega$$

is a kernel which characterizes the effective force that affects the motion of the particle in the presence of dissipation. For so-called Markovian baths, which do not keep memory of the interaction with the system, and for Ohmic dissipation, the equations of motion simplify to the classical equations of motion of a particle with friction:

$$M \frac{d^2}{dt^2} X(t) = -\frac{\partial V(X)}{\partial X} - \eta \frac{dX(t)}{dt}$$

Hence, one can see how Caldeira-Leggett model fulfills the goal of getting classical dissipation from the quantum mechanics framework. The Caldeira-Leggett model has been used to study quantum dissipation problems since its introduction in 1981, being extensively used as well in the field of quantum decoherence.

The dissipative two-level system

This particular realization of the Caldeira - Leggett model deserves special attention due to its interest in the field of Quantum Computation. The aim of the model is to study the effects of dissipation in the dynamics of a particle that can hop between two different positions. This Reduced Hilbert space allows the problem to be described in terms of $1/2$ spin operators. The resulting Hamiltonian is also referred in the literature as the Spin-Boson model, reading:

$$H = \Delta S_x + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) + S_z \sum_i C_i q_i ,$$

where $S_i = \frac{\sigma_i}{2}, i = x, y, z$ are proportional to the Pauli matrices σ_i, and Δ is the probability of hopping between the two possible positions. Notice that in this model the counter-term is no longer needed, as the coupling to S_z gives already homogeneous dissipation.

The model has many applications. In quantum dissipation it is used as a simple model to study the dynamics of a dissipative particle confined in a double-well potential. In the context of Quantum Computation it represents a qubit coupled to an environment, which can produce decoherence. In the study of amorphous solids it provides the basis of the standard theory to describe their thermodynamic properties.

The dissipative two-level systems represent also a paradigm in the study of quantum phase transitions. For a critical value of the coupling to the bath it shows a phase transition from a regime in which the particle is delocalized among the two positions to another in which it is localized in only one of them. The transition is of Kosterlitz-Thouless kind, as can be seen by deriving the Renormalization group flow equations for the hopping term.

Dissipation desired

Novel concept for universal quantum computers exploits dissipative processes.

Classical computers are not powerful enough to describe even simple quantum systems. All the more it is difficult to understand complex many body systems. Quantum computers which use quantum particles instead of classical bits may help to overcome this problem. Up to now complete isolation of the quantum system from the environment has been considered to be a precondition for the realization of a universal quantum computer – a high challenge for experimental physics. A new concept, developed by Prof. Ignacio Cirac, turns these ideas upside down. As the scientists report in Nature Physics (AOP 20 July 2009, DOI 10.1038/NPHYS1342), quantum systems that are coupled to the environment by dissipative processes can be used for efficient universal quantum computation as well as the preparation of exotic quantum states. Furthermore, these systems exhibit some inherent robustness. Though still being a proof-of-principle demonstration the concept can in principle is verified with systems such as atomic gases in optical lattices or trapped ions.

Standard quantum computation is based on a system of quantum particles such as atoms or ions that serve at storing and encoding information. It exploits the unique property of these particles to take on not only states like '1' or '0' but also all kinds of superposition of these states. Manipulations acting on these qubits are always reversible, dubbed ‚unitary'. Standard circuits consist of quantum gates that entangle two qubits at a time. However, this concept faces a strong adversary: once the system starts leaking information to the environment the quantum effects that give rise to the power of computing, cryptography and simulation – superposition and entanglement of states – get destroyed. Therefore the system has to be extremely well isolated from the environment.

On the contrary, the new concept of Cirac, Verstraete and Wolf makes use of these dissipative processes to perform efficient quantum computation and state engineering. In order to do so the dissipation dynamics has to be engineered such that it drives the system towards a steady state. This steady state can then represent the ground state of the system, it could be a particular exotic state, or it could encode the result of the computation. An advantage is the fact that, given the dissipative nature of the process, the system is driven towards its steady state independently of the initial state and hence of eventual perturbation along the way. That's why 'Dissipative Quantum Computation' (DQC) exhibits an inherent robustness.

Though neither state preparation nor unitary dynamics are required DQC turns out to obtain a computational power that is equivalent to that of standard quantum circuits. Furthermore, this method is particularly suited for preparing interesting quantum states: for example, topological systems give rise to exotic states that play an important role in novel quantum effects like the fractional quantum Hall-effect.

Right now this concept is a proof-of-principle demonstration that dissipation provides an alternative way of carrying out quantum computations or state engineering. It aims however at being adapted in experiments with systems that use atomic gases in optical lattices or trapped ions. “This way of performing quantum computation defies most of the requirements that were thought to be necessary to build such a device”, Prof. Cirac points out. “This may lead to different kinds of realizations of quantum computers that are either most robust or easy to implement. But what is more important, it gives a completely different perspective to the way a quantum computer may work in practice.” [OM]

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LITERATURE REVIEW STUDY:
The following studies bear important implications in so far our analysis of “noise” and Quantum Computation is concerned. It also accentuates and corroborates the study of system of “Measurement of Quantum Gates” and “Collapse of Quantum States” (See equations given in the annexure. In the next paper we shall analyse the correlated model which has wide ranging ramifications on the study of “Measurement of Quantum Gates) and “Collapse of Quantum States”

Quantum Register Based on Individual Electronic and Nuclear Spin Qubits in Diamond (See for details Gurudev dutt etal,),

The key challenge in experimental quantum information science is to identify isolated quantum mechanical systems with long coherence times that can be manipulated and coupled together in a scalable fashion. Authors describe the coherent manipulation of an individual electron spin and nearby individual nuclear spins to create a controllable quantum register. Using optical and microwave radiation to control an electron spin associated with the nitrogen vacancy (NV) color center in diamond, it has been demonstrated robust initialization of electron and nuclear spin quantum bits (qubits) and transfer of arbitrary quantum states between them at room temperature. Moreover, nuclear spin qubits could be well isolated from the electron spin, even during optical polarization and measurement of the electronic state. Finally, coherent interactions between individual nuclear spin qubits were observed and their excellent coherence properties were demonstrated. These registers can be used as a basis for scalable, optically coupled quantum information systems

Efficient measurement of quantum gate error by interleaved randomized benchmarking

Easwar Magesan, etal. have studied the problem of efficient measurement of quantum gate error by interleaved randomized benchmarking .They presented a model for scalable experimental protocol for obtaining estimates of the error rate of individual quantum computational gates. This protocol, in which random Clifford gates are interleaved between gates of interest, provides a bounded estimate of the average error of the gate under test so long as the average variation of the noise affecting the full set of Clifford gates is small. This technique takes into account both state preparation and measurement errors and is scalable in the number of qubits. Protocol to a superconducting qubit system and find gate errors that compare favorably with the gate errors extracted via quantum process tomography

Characterizing measurement-based quantum gates in quantum many-body systems using correlation functions (See for details Thomas Chung, Stephen D. Bartlett, and Andrew C. Doherty)

In measurement-based quantum computation (MBQC), local adaptive measurements are performed on the quantum state of a lattice of qubits. Quantum gates are associated with a particular measurement sequence, and one way of viewing MBQC is that such a measurement sequence prepares a resource state suitable for `gate teleportation'. It is demonstrated how to quantify the performance of quantum gates in MBQC by using correlation functions on the pre-measurement resource state.

Coupled quantum dots as quantum gates (See for details Guido Burkard Daniel Loss David P. DiVincenzo)

Authors' consider a quantum-gate mechanism based on electron spins in coupled semiconductor quantum dots. Such gates provide a general source of spin entanglement and can be used for quantum computers. Authors determine the exchange coupling J in the effective Heisenberg model as a function of magnetic (B) and electric fields, and of the interdot distance a within the Heitler-London approximation of molecular physics. This result is refined by using sp hybridization, and by the Hund-Mulliken molecular-orbit approach, which leads to an extended Hubbard description for the two-dot system that shows a remarkable dependence on B and adue to the long-range Coulomb interaction. We find that the exchange J changes sign at a finite field (leading to a pronounced jump in the magnetization) and then decays exponentially. The magnetization and the spin susceptibilities of the coupled dots are calculated. Authors show that the dephasing due to nuclear spins in GaAs can be strongly suppressed by dynamical nuclear-spin polarization and/or by magnetic fields.

Demonstration of controlled-NOT quantum gates on a pair of superconducting quantum bits (See for details J. H. Plantenberg, P. C. de Groot, C. J. P. M. Harmans & J. E. Mooij)
Quantum computation requires quantum logic gates that use the interaction within pairs of quantum bits (qubits) to perform conditional operations. Superconducting qubits may offer an attractive route towards scalable quantum computing. In previous experiments on coupled superconducting qubits, conditional gate behaviour and entanglement were demonstrated. Here authors demonstrate selective execution of the complete set of four different controlled-NOT (CNOT) quantum logic gates, by applying microwave pulses of appropriate frequency to a single pair of coupled flux qubits. All two-qubit computational basis states and their superpositions are used as input, while two independent single-shot SQUID detectors measure the output state, including qubit–qubit correlations. Authors determine the gate's truth table by directly measuring the state transfer amplitudes and by acquiring the relevant quantum phase shift using a Ramsey-like interference experiment. The four conditional gates result from the symmetry of the qubits in the pair: either qubit can assume the role of control or target, and the gate action can be conditioned on either the 0-state or the 1-state. These gates are now sufficiently characterized to be used in quantum algorithms, and together form an efficient set of versatile building blocks.

.QUANTUM GATE AND MEASUREMENT EMULATOR:

QGAME (Quantum Gate and Measurement Emulator) is a system that allows a user to run quantum computing algorithms on an ordinary digital computer. Because quantum computers have complexity advantages over classical computers, any classical emulator will necessarily be less efficient than the quantum computer that it is emulating. QGAME nonetheless allows the user to find out what outputs the quantum program would produce, and with what probabilities (since quantum computation is in general not deterministic).
QGAME is based on the "quantum gate array" model of quantum computation, in which quantum "gates" (represented as square matrices) are applied to a register of qubits (via tensor product formation and matrix multiplication). QGAME always starts with all qubits having the value zero (in the state |00...0>), applies a sequence of gates, and returns values about the resulting state. Measurement gates cause the system to branch, following one execution path (with the appropriate quantum state collapse) for each possible value. Final measurements are made across the end-states of all of the resulting branches.
Conditional Quantum Dynamics and Logic Gates(See for details Adriano Barenco, David Deutsch, and Artur Ekert)
Quantum logic gates provide fundamental examples of conditional quantum dynamics. They could form the building blocks of general quantum information processing systems which have recently been shown to have many interesting nonclassical properties. Authors describe a simple quantum logic gate, the quantum controlled-NOT, and analyze some of its applications. They also discuss two possible physical realizations of the gate, one based on Ramsey atomic interferometry and the other on the selective driving of optical resonances of two subsystems undergoing a dipole-dipole interaction.
Realizable Universal Quantum Logic Gates (See for details Tycho Sleator and Harald Weinfurter)
Authors identify a 2-bit quantum gate that is sufficient to build any quantum logic network. The existence of such a 2-bit universal gate considerably simplifies the search for physical realizations of quantum computational networks. Authors propose an explicit construction of this gate, which is based on cavity QED techniques and may be realizable with current technology.

NONLOCALITY:

Quantum mysteries such as entanglement and nonlocality were historically considered a philosophical quibble, the words of mountain missionary honing Vedic wisdom, until physicists discovered that these mysteries might be harnessed to devise new efficient algorithms. But while the technology for isolating 5 or even 7 qubits (the basic unit of information in the quantum computer) is now within reach (Schrader et al. 2004, Knill et al. 2000), only a handful of quantum algorithms exist, and the question whether these can solve classically intractable computational problems is still open. It may turn out that the technological capabilities that allow us to isolate quantum systems by shielding them from the effects of decoherence for a period of time long enough to manipulate them will also allow us to make progress in some fundamental problems in the foundations of quantum theory itself.
Quantum gate, quantum computation,. Quantum logic and propositional logic.:
In quantum computing and specifically the quantum circuit model of computation, a quantum gate (or quantum logic gate) is a basic quantum circuit operating on a small number of qubits. They are the building blocks of quantum circuits, like classical logic gates are for conventional digital circuits.
Unlike many classical logic gates, quantum logic gates are reversible. However, classical computing can be performed

using only reversible gates. For example, the reversibleToffoli gate can implement all Boolean functions. This gate has a direct quantum equivalent, showing that quantum circuits can perform all operations performed by classical circuits.

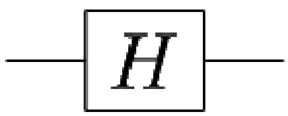
Quantum logic gates are represented by unitary matrices. The most common quantum gates operate on spaces of one or two qubits, just like the common classical logic gates operate on one or two bits. This means that as matrices, quantum gates can be described by 2 × 2 or 4 × 4 unitary matrices.

Commonly used gates

Quantum gates are usually represented as matrices. A gate which acts on k qubits is represented by a 2k x 2k unitary matrix. The number of qubits in the input and output of the gate have to be equal. The action of the quantum gate is found by multiplying the matrix representing the gate with the vector which represents the quantum state.

Hadamard gate

The Hadamard gate acts on a single qubit. It maps the basis state $|0\rangle$ to $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ and represents a rotation of π about the x- and z-axes. It is represented by the Hadamard matrix:



Circuit representation of Hadamard gate

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Since the rows of the matrix are orthogonal, H is indeed a unitary matrix.

Pauli-X gate

The Pauli-X gate acts on a single qubit. It is the quantum equivalent of a NOT gate. It equates to a rotation of the Bloch Sphere around the X-axis by π radians. It maps $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. It is represented by the Pauli matrix:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Pauli-Y gate

The Pauli-Y gate acts on a single qubit. It equates to a rotation around the Y-axis of the Bloch Sphere by π radians. It maps $|0\rangle$ to $i|1\rangle$ and $|1\rangle$ to $-i|0\rangle$. It is represented by the Pauli Y matrix:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$$

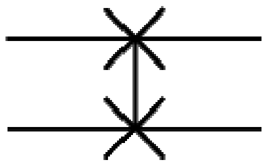
Phase shift gates

This is a family of single-qubit gates that leave the basis state $|0\rangle$ unchanged and map $|1\rangle$ to $e^{i\theta}|1\rangle$. The probability of measuring a $|0\rangle$ or $|1\rangle$ is unchanged after applying this gate; however it modifies the phase of the quantum state. This is equivalent to tracing a horizontal circle (a line of latitude) on the Bloch Sphere by θ radians.

$$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

Where θ is the phase shift. Some common examples are the $\frac{\pi}{8}$ gate where θ = $\frac{\pi}{4}$, the phase gate where θ = $\frac{\pi}{2}$ and the Pauli-Z gate where θ = π.

Swap gate

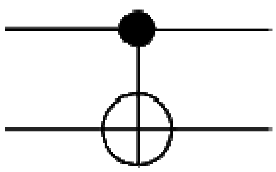


Circuit representation of SWAP gate

The swap gate swaps two qubits. It is represented by the matrix:

$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Controlled gates



Circuit representation of controlled NOT gate

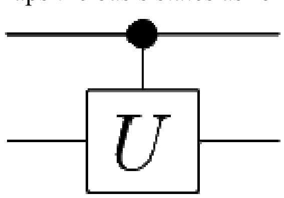
Controlled gates act on 2 or more qubits, where one or more qubits act as a control for some operation. For example, the controlled NOT gate (or CNOT) acts on 2 qubits, and performs the NOT operation on the second qubit only when the first qubit is $|1\rangle$, and otherwise leaves it unchanged. It is represented by the matrix

$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

More generally if U is a gate that operates on single qubits with matrix representation

$$U = \begin{bmatrix} x_{00} & x_{01} \\ x_{10} & x_{11} \end{bmatrix},$$

Then the controlled-U gate is a gate that operates on two qubits in such a way that the first qubit serves as a control. It maps the basis states as follows.



Circuit representation of controlled-U gate

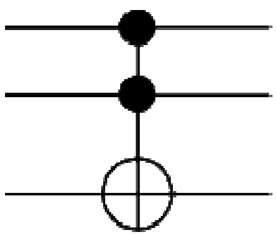
$$|00\rangle \mapsto |00\rangle$$
$$|01\rangle \mapsto |01\rangle$$
$$|10\rangle \mapsto |1\rangle U|0\rangle = |1\rangle \left(x_{00}|0\rangle + x_{10}|1\rangle\right)$$
$$|11\rangle \mapsto |1\rangle U|1\rangle = |1\rangle \left(x_{01}|0\rangle + x_{11}|1\rangle\right)$$

The matrix representing the controlled U is

$$C(U) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & x_{00} & x_{01} \\ 0 & 0 & x_{10} & x_{11} \end{bmatrix}.$$

When U is one of the Pauli matrices, σx, σy, or σz, the respective terms "controlled-X", "controlled-Y", or "controlled-Z" are sometimes used.

Toffoli gate



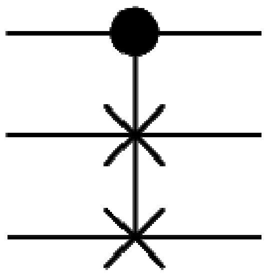
Circuit representation of Toffoli gate

The Toffoli gate, also CCNOT gate, is a 3-bit gate, which is universal for classical computation. The quantum Toffoli gate is the same gate, defined for 3 qubits. If the first two bits are in the state $|1\rangle$, it applies a Pauli-X on the third bit, else it does nothing. It is an example of a controlled gate. Since it is the quantum analog of a classical gate, it is completely specified by its truth table.

INPUT			OUTPUT		
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

It can be also described as the gate which maps $|a, b, c\rangle$ to $|a, b, c \oplus ab\rangle$.

Fredkin gate



Circuit representation of Fredkin gate

The Fredkin gate (also CSWAP gate) is a 3-bit gate that performs a controlled swap. It is universal for classical computation. It has the useful property that the numbers of 0s and 1s are conserved throughout, which in the billiard ball model means the same number of balls are output as input. This corresponds nicely to the conservation of mass in physics, and helps to show that the model is not wasteful.

INPUT			OUTPUT		
C	I1	I2	C	O1	O2
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

Universal quantum gates

Informally, a set of universal quantum gates is any set of gates to which any operation possible on a quantum computer can be reduced, that is, any other unitary operation can be expressed as a finite sequence of gates from the set. Technically, this is impossible since the number of possible quantum gates is uncountable, whereas the number of finite sequences from a finite set is countable. To solve this problem, we only require that any quantum operation can be approximated by a sequence of gates from this finite set. Moreover, for the specific case of single qubit unitaries the Solovay–Kitaev theorem guarantees that this can be done efficiently.

One simple set of two-qubit universal quantum gates is the Hadamard gate (H), the $\pi/8$ gate $R(\pi/4)$, and the controlled NOT gate.

A single-gate set of universal quantum gates can also be formulated using the three-qubit Deutsch gate $D(\theta)$, which performs the transformation

$$|a,b,c\rangle \mapsto \begin{cases} i\cos(\theta)|a,b,c\rangle + \sin(\theta)|a,b,1-c\rangle & \text{for } a=b=1 \\ |a,b,c\rangle & \text{otherwise.} \end{cases}$$

The universal classical logic gate, the Toffoli gate, is reducible to the Deutsch gate, $D(\frac{\pi}{2})$, thus showing that all classical logic operations can be performed on a universal quantum computer.

PHYSICAL COMPLEXITIES:

(1). A Turing machine consists of an unbounded tape, a head that is capable of reading from the tape and of writing onto it and can occupy an infinite number of possible states, and an instruction table (a transition function).

(2)This table, given the head's initial state and the input it reads from the tape in that state, determines

(a) The symbol that the head will write on the tape

, (b) the internal state it will occupy, and

(c) The displacement of the head on the tape

. In 1936 Turing showed that since one can encode the instruction table of a Turing machine T and express it as a binary number # (T), there exists a universal Turing machine U that can simulate the instruction table of any Turing machine on any given input with at most a polynomial slowdown (i.e., the number of computational steps required by U to execute the original program T on the original input will be polynomially bounded in # (T)). That the Turing machine model (what we now days call “an algorithm”) captures the concept of computability in its entirety is the essence of the Church-Turing thesis, according to which any effectively calculable function can be computed using a Turing machine. Admittedly, no counterexample to this thesis (which is the result of convergent ideas of Turing, Post, Kleene and Church) has yet been found. But since it identifies the class of computable functions with the class of those functions which are computable using a Turing machine, this thesis involves both a precise mathematical notion, predicational anteriority, character constitution, ontological consonance, primordial exactitude, coextensive representations and an informal and intuitive notion, hence cannot be proved or disproved. Simple cardinality considerations show, however, that not all functions are Turing-computable (the set of all Turing machines is countable, while the set of all functions from the natural numbers to the natural numbers is not), and the discovery of this fact came as a complete surprise in the 1930s (Davis 1958).

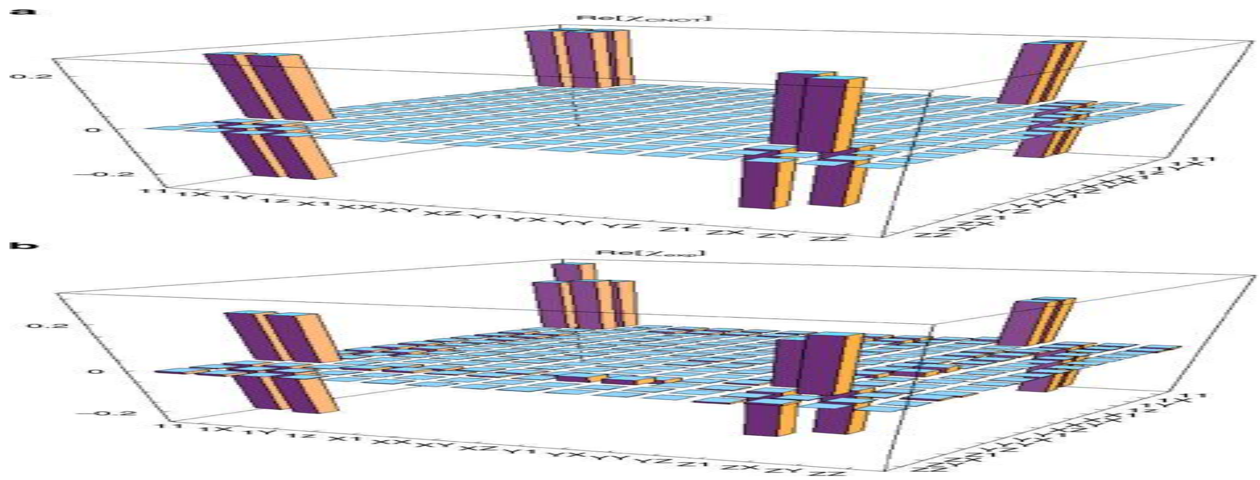
COMPUTATIONAL COMPLEXITY:

The cost of computing a function is also of great importance, and this cost, also known as computational complexity, is measured naturally in the physical resources (e.g., time, space, energy) invested in order to solve the computational problem at hand. Computer scientists classify computational problems according to the way their cost function behaves as a function of their input size, n, (the number of bits required to store the input) and in particular, whether it increases exponentially or polynomially with n. Tractable problems are those which can be solved in polynomial cost, while intractable problems are those which can only be solved in an exponential cost (the former solutions are commonly regarded as efficient although an exponential-time algorithm could turn out to be more efficient than a polynomial-time algorithm for some range of input sizes). If we further relax the requirement that a solution to a computational problem be always correct, and allow probabilistic algorithms with a negligible probability of error, we can dramatically reduce the computational cost. Probabilistic algorithms are non-deterministic Turing machines whose transition function can randomly change the head's configuration in one of several possible ways. The most famous example of an algorithm of this kind is the probabilistic algorithm that decides whether a given natural number is prime in a polynomial number of steps (Rabin 1976).

QUANTUM YIELD:

The Swiss Federal Institute of Technology EPFL has developed a concept of an integrating sphere for the absolutes measures of the quantum efficiency for small quantities of solid samples or solutions (60 µL).

Quantum yield is frequently used to characterize luminescent material. It corresponds to the ratio between the number of emitted photons at a given wavelength, usually in the UV-visible region, and the number of absorbed photons. Using an integrating sphere is a classical approach to that measurement. However, greatest care must be given to the signal to noise ratio, to retro diffusion of the excitation light, to possible contamination of the sphere when manipulating the sample and to the critical calibration process.



Wolfram (1985) claims that any physical system can be simulated (to any degree of approximation) by a universal Turing machine, and that complexity bounds on Turing machine simulations have physical significance. For example, if the computation of the minimum energy of some system of n particles requires (e) at least an exponentially increasing number of steps in n, then the actual relaxation of this system to its minimum energy state will also take an exponential time. Aharonov (1998) strengthens this thesis (in the context of showing its putative incompatibility with quantum mechanics) when she says that a probabilistic Turing machine can simulate any reasonable physical device at polynomial cost. Further examples for this thesis can be found in Copeland (1996). In order for the physical Church-Turing thesis to make sense we have to relate the space and time parameters of physics to their computational counterparts: memory capacity and number of computation steps, respectively. There are various ways to do that, leading to different formulations of the thesis (Pitowsky 1990). For example, one can encode the set of instructions of a universal Turing machine and the state of its infinite tape in the binary development of the position coordinates of a single particle. Consequently, one can physically 'realize' a universal Turing machine as a billiard ball with hyperbolic mirrors (Moore 1990, Pitowsky 1996). For the most intuitive connection between abstract Turing machines and physical devices see the pioneering work of Gandy (1980), simplified later by Sieg and Byrnes (1999). It should be stressed that there is no relation between the original Church-Turing thesis and its physical version (Pitowsky and Shagrir 2003), and while the former concerns the concept of computation that is relevant to logic (since it is strongly tied to the notion of proof which requires validation), it does not analytically entail that all computations should be subject to validation. Indeed, there is a long historical tradition of analog computations which use continuous physical processes (Dewdney 1984), and the output of these computations is validated either by repetitive "runs" or by validating the physical theory that presumably governs the behavior of the analog computer.

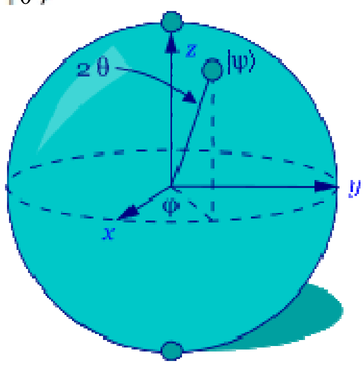
Physical "Short-cuts" of Computation.

Setting aside the hype ,hoopla ,hullabaloo, and brouhaha around "hyper computers", even if we restrict ourselves only to Turing-computable functions and focus on computational complexity, we can still find many physical models that purport to display "short-cuts" in computational resources. Consider, e.g., the DNA model of computation that was claimed (Adleman 1994, Lipton 1995) to solve NP-complete problems in polynomial time. A closer inspection shows that the cost of the computation in this model is still exponential since the number of molecules in the physical system grows exponentially with the size of the problem. Or take an allegedly instantaneous solution to another NP-complete problem using a construction of rods and balls (Vergis et.al. 1986) that unfortunately ignores the accumulating time-delays in the rigid rods that result in an exponential overall slowdown. Another example is the physical simulation of the factorization of numbers into primes that uses only polynomial resources in time and space, but requires an exponentially increasing precision

The Qubit

The qubit is the quantum analogue of the bit, the classical fundamental unit of information. It is a mathematical object with specific properties that can be realized physically in many different ways as an actual physical system. Just as the classical bit has a state (either 0 or 1), a qubit also has a state. Yet contrary to the classical bit, $|0\rangle$ and $|1\rangle$ are but two possible states of the qubit, and any linear combination (superposition) thereof is also physically possible. In general, thus, the physical state of a qubit is the superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (where α and β are complex numbers). The state of a qubit can be described as a vector in a two-dimensional Hilbert space, a complex vector space (see the entry on quantum mechanics). The special states $|0\rangle$ and $|1\rangle$ are known as the computational basis states, and form an orthonormal basis for this vector space. According to quantum theory, when we try to measure the qubit in this basis in order to determine its state, we get either $|0\rangle$ with probability $|\alpha|^2$ or $|1\rangle$ with probability $|\beta|^2$. Since $|\alpha|^2 + |\beta|^2 = 1$ (i.e., the qubit is a unit vector in the aforementioned two-dimensional Hilbert state), we may (ignoring the overall phase factor) effectively write its state as $|\psi\rangle$ = cos (θ) $|0\rangle$ + eiφsin(θ) $|1\rangle$, where the numbers θ and φ define a point on the unit three-dimensional sphere, as shown here. This sphere is often called the Bloch sphere, and it provides a useful means to visualize the state of a single qubit.

$|0\rangle$



$|1\rangle$

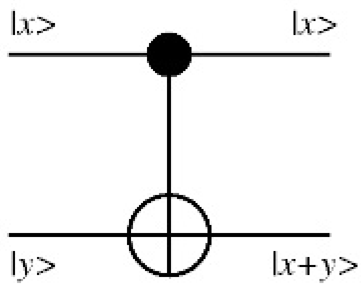
The Bloch Sphere

Theoretically, a single qubit can store an infinite amount of information, yet when measured it yields only the classical result (0 or 1) with certain probabilities that are specified by the quantum state. In other words, the measurement changes the state of the qubit, "collapsing" it from the superposition to one of its terms. The crucial point is that unless the qubit is measured, the amount of "hidden" information it stores is conserved under the dynamic evolution (namely, Schrödinger's equation). This feature of quantum mechanics allows one to manipulate the information stored in unmeasured qubits with quantum gates, and is one of the sources for the putative power of quantum computers.

Quantum Gates

Classical computational gates are Boolean logic gates that perform manipulations of the information stored in the bits. In quantum computing these gates are represented by matrices, and can be visualized as rotations of the quantum state on the Bloch sphere. This visualization represents the fact that quantum gates are unitary operators, i.e., they preserve the norm of the quantum state (if U is a matrix describing a single qubit gate, then U†U=I, where U† is the adjoint of U, obtained by transposing and then complex-conjugating U). As in the case of classical computing, where there exists a universal gate (the combinations of which can be used to compute any computable function), namely, the NAND gate which results from performing an AND gate and then a NOT gate, in quantum computing it was shown (Barenco et al., 1995) that any multiple qubit logic gate may be composed from a quantum CNOT gate (which operates on a multiple qubit by flipping or preserving the target bit given the state of the control bit, an operation analogous to the classical XOR, i.e., the exclusive OR gate) and single qubit gates. One feature of quantum gates that distinguishes them from classical gates is that they are reversible: the inverse of a unitary matrix is also a unitary matrix, and thus a quantum gate can always be inverted by another quantum gate.

$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$



The CNOT Gate

Quantum register

A quantum register (also known as a qregister) is the quantum mechanical analogue of a classical processor register. A mathematical description of a quantum register is achieved by using a tensor product of qubit bra or ket vectors. For example, an n qubit quantum register is described by an element $|\Psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \ldots \otimes |\psi\rangle_n$ in the tensor product Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_n$

The Quantum Memory Register

We have considered a two state quantum system, a qubit. However a quantum system is by no means constrained to be a two state system. Much of the above discussion for a 2 state quantum system is applicable to a general n state quantum system.

In a n state system our Hilbert Space has n perpendicular axes, or eigenstates, which represent the possible states the system can be measured in. As with the two state systems, when we measure our n state quantum system, we will always find it to be in exactly one of the n states, and not a superposition of the n states. The system is still allowed to exist in any superposition of the n states while it is not being measured.

Mathematically if two state quantum system with coordinate axes x_0, x_1 can be fully described by:

$$|X\rangle = w_0 * |x_0\rangle + w_1 * |x_1\rangle \equiv (w_0, w_1)$$

Then an n state quantum system with coordinate axes $x_0, x_1, \ldots, x_{n-1}$ can be fully described by:

$$|X\rangle = \sum_{k=0}^{n-1} w_k * |x_k\rangle$$

In general a quantum system with n base states can be represented by the n complex numbers w_0 to w_{n-1}. When this is done the state may be written as:

$$|X\rangle = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_{n-1} \end{pmatrix}$$

Where it is understood that w_k refers to the complex weighting factor for the k'th eigenstate.

Using this information we can construct a quantum memory register out of the qubits . We may store any number x in our quantum memory register as long as we have enough qubits, just as we may store any number x in a classical register as long as we have enough classical bits to represent that number. The state of the quantum register with n states is give by the formula above. Note that in general a quantum register composed of m qubits requires 2^m complex numbers to completely describe its state: A m qubit register can be measured to be in one of 2^m states, and each state requires one complex number to represent the projection of that total state onto that state. In contrast a classical register composed of m bits requires only m integers to fully describe its state.

This means that is some sense one can store an exponentially greater amount of information in a quantum register than in a classical memory register of the same number of (q) bits. Here we see some of the first hints that a quantum computer can be exponentially more powerful than a classical computer in some respects. Recall that from our discussion of complexity that problems whose best known algorithms yield a solution in polynomial time are generally thought of as being tractable, and that problems whose best known algorithms take exponential time are thought of as intractable. ome tractable! This is a large part of the motivation for the study of quantum computing.

UNITARY GATES:

Unitary gates manipulate the information stored in the quantum register, and in this sense ordinary (unitary) quantum evolution can be regarded as computation (DiVicenzo 1995 showed how a small set of single-qubit gates and a two-qubit gate is universal, in the sense that a circuit combined from this set can approximate to arbitrary accuracy any unitary transformation of n qubits). In order to read the result of this computation, however, the quantum register must be measured. The measurement gate is a non-unitary gate that “collapses” the quantum superposition in the register onto one of its terms with the corresponding probability. Usually this measurement is done in the computational basis, but since quantum mechanics allows one to express an arbitrary state as a linear combination of basis states, provided that the states are orthonormal (a condition that ensures normalization) one can in principle measure the register in any arbitrary orthonormal basis. This, however, doesn't mean that measurements in different bases are efficiently equivalent. Indeed, one of the difficulties in constructing efficient quantum algorithms stems exactly from the fact that measurement collapses the state, and some measurements are much more complicated than others.

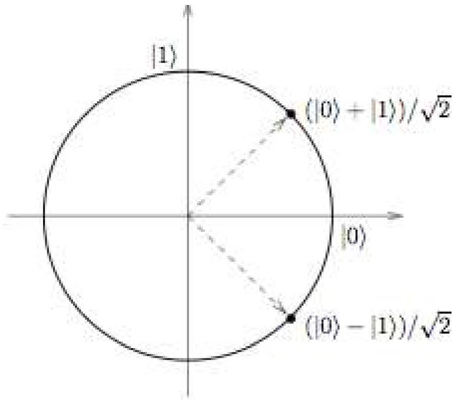
Quantum Circuits

Quantum circuits are similar to classical computer circuits in that they consist of wires and logical gates. The wires are used to carry the information, while the gates manipulate it (note that the wires do not correspond to physical wires; they may correspond to a physical particle, a photon, moving from one location to another in space, or even to time-evolution). Conventionally, the input of the quantum circuit is assumed to be a computational basis state, usually the state consisting of all $|0\rangle$. The output state of the circuit is then measured in the computational basis, or in any other arbitrary orthonormal basis. The first quantum algorithms (i.e. Deutsch-Jozsa, Simon, Shor and Grover) were constructed in this paradigm. Additional paradigms for quantum computing exist today that differ from the quantum circuit model in many interesting ways. So far, however, they all have been demonstrated to be computationally equivalent to the circuit model (see below), in the sense that any computational problem that can be solved by the circuit model can be solved by these new models with only a polynomial overhead in computational resources.

Quantum Algorithms

Algorithm design is a highly complicated task, and in quantum computing it becomes even more complicated due to the attempts to harness quantum mechanical features to reduce the complexity of computational problems and to “speed-up” computation. Before attacking this problem, we should first convince ourselves that quantum computers can be harnessed to perform standard, classical, computation without any “speed-up”. In some sense this is obvious, given the belief in the universal character of quantum mechanics, and the observation that any quantum computation that is diagonal in the computational basis, i.e., involves no interference between the qubits, is effectively classical. Yet the demonstration that quantum circuits can be used to simulate classical circuits is not straightforward (recall that the former are reversible while the latter use gates which are inherently irreversible). Indeed, quantum circuits cannot be used directly to simulate classical computation, but the latter can still be simulated on a quantum computer using an intermediate gate, namely the Toffoli gate. This gate has three input bits and three output bits, two of which are control bits, unaffected by the action of the gate. The third bit is a target bit that is flipped if both control bits are set to 1, and otherwise is left alone. This gate is reversible (its inverse is itself), and can be used to simulate all the elements of the classical irreversible circuit with a reversible one. Consequently, using the quantum version of the Toffoli gate (which by definition permutes the computational basis states similarly to the classical Toffoli gate) one can simulate, although rather tediously, irreversible classical logic gates with quantum reversible ones. Quantum computers are thus capable of performing any computation which a classical deterministic computer can do.

What about non-deterministic computation? Not surprisingly, a quantum computer can simulate also this type of computation by using another famous quantum gate, namely the Hadamard gate, which receives as an input the state $|0\rangle$ and produces the state $(|0\rangle + |1\rangle)/\sqrt{2}$. Measuring this output state yields $|0\rangle$ or $|1\rangle$ with 50/50 probability, which can be used to simulate a fair coin toss.



$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The Hadamard Gate

Obviously, if quantum algorithms could be used only to simulate classical algorithms, then the technological advancement in information storage and manipulation, encapsulated in "Moore's law", would have only trivial consequences on computational complexity theory, leaving the latter unaffected by the physical world. But while some computational problems will always resist quantum "speed-up" (in these problems the computation time depends on the input, and this feature will lead to a violation of unitarity hence to an effectively classical computation even on a quantum computer—see Myers 1997 and Linden and Popescu 1998), the hope is, nonetheless, that quantum algorithms may not only simulate classical ones, but that they will actually outperform the latter in some cases, and in so doing help to re-define the abstract notions of tractability and intractability and violate the physical Church-Turing thesis, at least as far as computational complexity is concerned.

Quantum-Circuit-Based Algorithms

The first quantum algorithms were designed to exploit the adequacy of quantum computation to computational problems which involve oracles. Oracles are devices which are used to answer questions with a simple yes or no. The questions may be as elaborate as one can make them, the procedure that answers the questions may be lengthy and a lot of auxiliary data may get generated while the question is being answered. Yet all that comes out of the oracle is just yes or no. The oracle architecture is very suitable for quantum computers. The reason for this is that, as stressed above, the read-out of a quantum system is probabilistic. Therefore if one poses a question the answer to which is given in the form of a quantum state, one will have to carry out the computation on an ensemble of quantum computers to get anywhere. On the other hand if the computation can be designed in such a way that one does get yes or no in a single measurement (and some data reduction may be required to accomplish this), then a single quantum computer and a single quantum computation run may suffice.

The Deutsch Oracle

This oracle (Deutsch 1989) answers the following question. Suppose we have a function f: $\{0,1\} \rightarrow \{0,1\}$, which can be either constant or balanced. In this case, the function is constant if $f(0) = f(1)$ and it is balanced if $f(0) \neq f(1)$. Classically it would take two evaluations of the function to tell whether it is one or the other. Quantum, we can answer this question in one evaluation. The reason for this complexity reduction is, again, the superposition principle. To see why consider the following quantum algorithm. One can prepare the input qubits of the Deutsch oracle as the superposition $(|0\rangle + |1\rangle)/\sqrt{2}$ (using the Hadamard gate on $|0\rangle$) and the superposition $(|0\rangle - |1\rangle)/\sqrt{2}$ (using the Hadamard gate on $|1\rangle$). The oracle is implemented with a quantum circuit which takes inputs like $|x,y\rangle$ to $|x, y\oplus f(x)\rangle$, where $\oplus$ is addition modulo two, which is exactly what a XOR gate does. The first qubit of the output of this oracle is then fed into a Hadamard gate, and the final output of the algorithm is the state

$\pm |f(0)\oplus f(1)\rangle([|0\rangle - |1\rangle]/\sqrt{2})$.

Since $f(0)\oplus f(1)$ is 0 if the function is constant and 1 if the function balanced, a single measurement of the first qubit of the output suffices to retrieve the answer to the question whether the function is constant or balanced. In other words, we can distinguish in one run of the algorithm between the two quantum disjunctions without finding out the truth values of the disjoints themselves in the computation.

The Deutsch-Jozsa Oracle

This oracle (Deutsch and Jozsa 1992) generalizes the Deutsch oracle to a function f: {0, 1} n → {0,1}. We ask the same question: is the function constant or balanced. Here balanced means that the function is 0 on half of its arguments and 1 on the other half. Of course in this case the function may be neither constant nor balanced. In this case the oracle doesn't work: It may say yes or no but the answer will be meaningless. Also here the algorithm allows one to evaluate a global property of the function in one measurement because the output state is a superposition of balanced and constant states such that the balanced states all lie in a subspace orthogonal to the constant states and can therefore be distinguished from the latter in a single measurement. In contrast, the best deterministic classical algorithm would require 2n/2+1 queries to the oracle in order to solve this problem.

The Simon Oracle

Suppose we have a Boolean function f: {0, 1} n → {0,1}n. The function is supposed to be 2-to-1, i.e., for every value of f there are always two x1 and x2 such that f(x1) = f(x2). The function is also supposed to be periodic, meaning that there is a binary vector a such that f(xa) = f(x), where ⊕ designates addition modulo 2, i.e., 1 ⊕ 1 = 0. The Simon oracle returns the period a in a number of measurements linear in n, which is exponentially faster than any classical algorithm (Simon 1994). Simon's oracle reduces to Deutsch XOR oracle when n=2, and can indeed be regarded as an extension of the latter, in the sense that a global property of a function, in this case its period, can be evaluated in an efficient number of measurements, given that the output state of the algorithm is decomposed into orthogonal subspaces, only one of which contains the solution to the problem, hence repeated measurements in the computational basis will be sufficient to determine this subspace. In other words, Simon's oracle is yet another example where a quantum algorithm can evaluate a disjunction without determining the truth value of the disjoints. For more on the logical analysis of these first quantum-circuit-based algorithms see Bub (2006b).

Shor's Algorithm

The three oracles just described, although demonstrating the potential superiority of quantum computers over their classical counterparts, nevertheless deal with apparently unimportant computational problems. Indeed, it is doubtful whether the research field of quantum computing would have attracted so much attention and would have evolved to its current status if its merit could be demonstrated only with these problems. But in 1994, after realizing that Simon's oracle can be harnessed to solve a much more interesting and crucial problem, namely factoring, which lies at the heart of current cryptographic protocols such as the RSA (Rivest 1978), Peter Shor has turned quantum computing into one of the most exciting research domains in quantum mechanics.

Shor's algorithm (1994) exploits the ingenious number theoretic argument that two prime factors p,q of a positive integer N=pq can be found by determining the period of a function f(x) = yx mod N, for any y < N which has no common factors with N other than 1 (Nielsen and Chuang 2000, App. 4). The period r of f(x) depends on y andN. Once one knows the period, one can factor N if r is even and yr/2 ≠ −1 mod N, which will be jointly the case with probability greater than 1/2 for any y chosen randomly (if not, one chooses another value of y and tries again). The factors of N are the greatest common divisors of yr/2 ± 1 and N, which can be found in polynomial time using the well known Euclidean algorithm. In other words, Shor's remarkable result rests on the discovery that the problem of factoring reduces to the problem of finding the period of a certain periodic function f: Zn → ZN, where Zn is the additive group of integers mod n (Note that f(x) = yx mod N so that f(x+r) = f(x) if x+r ≤n. The function is periodic if r divides n exactly, otherwise it is almost periodic). That this problem can be solved efficiently by a quantum computer is demonstrated with Simon's oracle.

Shor's result is the most dramatic example so far of quantum "speed-up" of computation, notwithstanding the fact that factoring is believed to be only in NP and not inNP-complete. To verify whether n is prime takes a number of steps which is a polynomial in log2n (the binary encoding of a natural number n requires log2n resources). But nobody knows how to factor numbers into primes in polynomial time, not even on a probabilistic Turing machine, and the best classical algorithms we have for this problem are sub-exponential. This is yet another open problem in the theory of computational complexity. Modern cryptography and Internet security protocols such public key and electronic signatures are based on these facts (Giblin 1993): It is easy to find large prime numbers fast, and it is hard to factor large composite numbers in any reasonable amount of time. The discovery that quantum computers can solve factoring in polynomial time has had, therefore, a dramatic effect. The implementation of the algorithm on a physical machine would have economic, as well as scientific consequences.

Grover's Algorithm

Suppose you have met someone who kept her name secret, but revealed her telephone number to you. Can you find out her name using her number and a phone directory? In the worst case, if there are n entries in the directory, the computational resources required will be linear in n. Grover (1996) showed how this task, namely, searching an unstructured database, could be done with a quantum algorithm with complexity of the order √n. Agreed, this "speed-up" is more modest than Shor's since searching an unstructured database belongs to the class P, but contrary to Shor's case, where the classical complexity of factoring is still unknown, here the superiority of the quantum algorithm, however modest, is definitely provable. That this quadratic "speed-up" is also the optimal quantum "speed-up" possible for this problem was proved by Bennett, Bernstein, Brassard and Vazirani (1997).

Although- the purpose of Grover's algorithm is usually described as "searching a database", it may be more accurate to describe it as "inverting a function". Roughly speaking, if we have a function y=f(x) that can be evaluated on a quantum computer, Grover's algorithm allows us to calculate x when given y. Inverting a function is related to searching a database because we could come up with a function that produces a particular value of y if x matches a desired entry in a database, and another value of y for other values of x. The applications of this algorithm are far-reaching (over and above finding the name of the mystery 'date' above). For example, it can be used to determine efficiently the number of solutions to an N-item search problem, hence to perform exhaustive searches on a class of solutions to an NP-complete problem and substantially reduce the computational resources required for solving it.

Adiabatic Algorithms

More than a decade has passed since the discovery of the first quantum algorithm, but so far little progress has been made with respect to the "Holy Grail" of solving anNP-complete problem with a quantum-circuit model. As stressed above, Shor's algorithm stands alone in its exponential "speed-up", yet while no efficient classical algorithm for factoring is known to exist, there is also no proof that such an algorithm doesn't or cannot exist. In 2000 a group of physicists from MIT and Northeastern University (Farhi et al. 2000) proposed a novel paradigm for quantum computing that differs from the circuit model in several interesting ways. Their goal was to try to solve with this algorithm an instance of satisfiability—deciding whether a proposition in the propositional calculus has a satisfying truth assignment—which is one of the most famous NP-complete problems (Cook 1971).

According to the adiabatic theorem (e.g., Messiah 1961) and given certain specific conditions, a quantum system remains in its lowest energy state, known as the ground state, along an adiabatic transformation in which the system is deformed slowly and smoothly from an initial Hamiltonian to a final Hamiltonian (as an illustration, think of moving a baby who is sleeping in a cradle from the living room to the bedroom. If the transition is done slowly and smoothly enough, and if the baby is a sound sleeper, then it will remain asleep during the whole transition). The most important condition in this theorem is the energy gap between the ground state and the next excited state (in our analogy, this gap reflects how sound asleep the baby is). Being inversely proportional to the evolution time T, this gap controls the latter. If this gap exists during the entire evolution (i.e., there is no level crossing between the energy states of the system), the theorem dictates that in the adiabatic limit (when $T \rightarrow \infty$) the system will remain in its ground state. In practice, of course, T is always finite, but the longer it is, the less likely it is that the system will deviate from its ground state during the time evolution.

The crux of the quantum adiabatic algorithm which rests on the adiabatic theorem lies in the possibility of encoding a specific instance of a given decision problem in a certain Hamiltonian (this can be done by capitalizing on the well-known fact that any decision problem can be derived from an optimization problem by incorporating into it a numerical bound as an additional parameter). One then starts the system in a ground state of another Hamiltonian which is easy to construct, and slowly evolves the system in time, deforming it towards the desired Hamiltonian. According to the quantum adiabatic theorem and given the gap condition, the result of such a physical process is another energy ground state that encodes the solution to the desired decision problem. The adiabatic algorithm is thus a rather 'laid back' algorithm: one needs only to start the system in its ground state, deform it adiabatically, and measure its final ground state in order to retrieve the desired result. But whether or not this algorithm yields the desired "speed-up" depends crucially on the behavior of the energy gap as the number of degrees of freedom in the system increases. If this gap decreases exponentially with the size of the input, then the evolution time of the algorithm will increase exponentially; if the gap decreases polynomially, the decision problem so encoded could be solved efficiently in polynomial time. Although physicists have been studying spectral gaps for almost a century, they have never done so with quantum computing in mind. How this gap behaves in general remains thus far an open empirical question.

The quantum adiabatic algorithm holds much promise (Farhi et al. 2001), and recently it was shown (Aharonov et al. 2004) to be polynomially equivalent to the circuit model (that is, each model can simulate the other with only polynomial, i.e., modest, overhead of resources, namely, number of qubits and computational steps), but the caveat that is sometimes left unmentioned is that its application to an intractable computational problem may sometimes require solving another, as intractable a task (this general worry was first raised by a philosopher; see Pitowsky 1990). Indeed, Reichardt (2004) has shown that there are simple problems for which the algorithm will get stuck in a local minimum, in which there are exponentially many eigenvalues all exponentially close to the ground state energy, so applying the adiabatic theorem, even for these simple problems, will take exponential time, and we are back to square one.

Measurement-Based Algorithms

Measurement-based algorithms differ from the circuit model in that instead of applying unitary evolution as the basic

mechanism for the manipulation of information, these algorithms use only non-unitary measurements as their computational steps. These models are especially interesting from a foundational perspective because they have no evident classical analogues and because they offer a new insight on the role of entanglement in quantum computing (Jozsa 2005). They may also have interesting consequences for experimental considerations, suggesting a different kind of computer architecture which is more faults tolerant (Nielsen and Dawson 2004).
The measurement-based algorithms fall into two categories. The first is teleportation quantum computing (based on an idea of Gottesman and Chuang 1999, and developed into a computational model by Nielsen 2003 and Leung 2003). The second is the “one way quantum computer”, known also as the “cluster state” model (Raussendorf and Briegel 2000). The interesting feature of these models is that they are able to represent arbitrary quantum dynamics, including unitary dynamics, with basic non-unitary measurements. The measurements are performed on a pool of highly entangled states (the amount of entanglement needed is still under dispute), and are adaptive, i.e., each measurement is done in a different basis which is calculated classically, given the result of the earlier measurement (the first model uses measurements of 2 or more qubits, while the second uses only single qubit measurements; in the first model only bi-partite entanglement is used, while in the second one has multi-partite entanglement across all qubits). Such exotic models might seem redundant, especially when they have been shown to be polynomially equivalent to the standard circuit model in terms of computational complexity (Raussendorf et al. 2003). Their merit, however, lies in the foundational lessons they drive home: with these models the separation between the classical part (i.e., the calculation of the next measurement-basis) and the quantum parts (i.e., the measurement and the entangled states) of the computation becomes evident, hence it may be easier to pinpoint the quantum resources that are responsible for the putative “speed-up”.

Topological-Quantum-Field-Theory (TQFT) Algorithms

Another exotic model for quantum computing which is attracting a lot of attention lately, especially from Microsoft inc. (Freedman 1998), is the Topological Quantum Field Theory model. In contrast to the straightforward and standard circuit model, this model resides in the most abstract reaches of theoretical physics. The exotic physical systems TQFT describes are topological states of matter. That the formalism of TQFT can be applied to computational problems was shown by Witten (1989) and the idea was later developed by others. Also here the model was proved to be efficiently simulated on a standard quantum computer (Freedman, Kitaev, Wang 2000, Aharonov et al. 2005), but its merit lies in its high tolerance to errors resulting from any possible realization of a large scale quantum computer (see below). Topology is especially helpful here because many global topological properties are, by definition, invariant under deformation, and given that most errors are local, information encoded in topological properties is robust against them.

Realizations

The quantum computer might be the theoretician's dream, but as far as experimentalists are concerned, its realization is a nightmare. The problem is that while some prototypes of the simplest elements needed to build a quantum computer have already been implemented in the laboratory, it is still an open question how to combine these elements into scalable systems. Shor's algorithm may break the RSA code, but it will remain an anecdote if the largest number that it can factor is 15. In the circuit-based model the problem is to achieve a scalable quantum system that at the same time will allow one to
(1) Robustly represent quantum information,
(2) perform a universal family of unitary transformation,
(3) Prepare a fiducially initial state, and
(4) Measure the output result.
Alternative paradigms may trade some of these requirements with others, but the gist will remain the same, i.e., one would have to achieve control of one's quantum system in such a way that the system will remain “quantum” albeit macroscopic or even mesoscopic in its dimensions.
In order to deal with these requirements, several ingenious solutions were devised, including quantum error correction codes (Shor 1995) and fault tolerant computation (Shor and DiVicenzo 1996, Aharonov and Ben-Or 1997) that dramatically reduce the spread of errors during a ‘noisy’ quantum computation. The problem with these active error correction schemes is that they were devised for a very unrealistic noise model which treats the computer as quantum and the environment as classical (Alicki, Lidar & Zanardi 2006) Once a more realistic noise model is allowed, the feasibility of large scale, fault tolerant and computationally superior quantum computers is less clear (Hagar 2009).
Another scheme to reduce errors in the implementation of quantum algorithms on large scale quantum computers is to encode information in noiseless subsystems, or decoherence free subspaces (Lidar, Chuang & Whaley 1998). This strategy seems more promising from a physical point of view, yet also here the question of how those noiseless subspaces scale with the size of the computer remains open. If one hopes to solve intractable problems efficiently with a scalable quantum computer, then the construction of the theoretical operator that measures a quantum state which

encodes a solution to an NP-hard problem should not require an exponential time, or solving yet another NP-hard problem.
Finally, as the implementation of Shor's algorithm on a large scale quantum computer seems still beyond our reach, quantum information scientists turn to the original goal of using quantum computers to simulate quantum systems. While Feynman's conjecture is still unproven, complexity theorists attempt to narrow the gap between what they believe is true about quantum mechanics, namely, that it's exponentially-hard to simulate on a classical computer, and what experimentalists can currently demonstrate (e.g., Aaronson & Arkhipov 2010).
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What is Quantum in Quantum Computing?

Notwithstanding the excitement around the discovery of Shor's algorithm, and apart from the almost insurmountable problem of practically realizing and implementing a large scale quantum computer, a crucial theoretical question remains open, namely, what physical resources are responsible for the putative power of quantum computing? Put another way, what are the essential features of quantum mechanics that allow one to solve problems or simulate certain systems far more efficiently than on a classical computer? Remarkable is also the fact that the relevance of features commonly thought essential to the superiority of quantum computers, e.g., entanglement and interference (Josza 1997), has recently been questioned (Linden and Popescu 1999, Biham 2004). Moreover, even if these features do play an essential role in the putative quantum "speed-up", it is still unclear how they do so (Fortnow 2003).
Theoretical as it may seem, the question "what is quantum in quantum computing?" has an enormous practical consequence. One of the embarrassments of quantum computing is the fact that, so far, only one algorithm has been discovered, namely Shor's, for which a quantum computer is significantly faster than any known classical one. It is almost certain that one of the reasons for this scarcity of quantum algorithms is related to the lack of our understanding of what makes a quantum computer quantum (see also Preskill 1998 and Shor 2004). As an ultimate answer to this question one would like to have something similar to Bell's (1964) famous theorem, i.e., a succinct crispy statement of the fundamental difference between quantum and classical systems, encapsulated in the non-commutative character of observables. Quantum computers, unfortunately, do not seem to allow such simple characterization. Observables—in the quantum circuit model there are only two, the preparation of the initial state and the observation of the final state, in the same basis, and of the same variable, at the end of the computation—are not as important here as in Bell's case since any measurement commutes with itself. The non-commutativity in quantum computing lies much deeper and it is still unclear how to cash it into useful currency. Quantum computing skeptics (Levin 2003) happily capitalize on this puzzle: If no one knows why quantum computers are superior to classical ones, how can we be sure that they are, indeed, superior?
The elusive character of the physical resource responsible for the quantum "speed-up" can be nicely demonstrated with the following example. Consider a solution of a decision problem, say satisfiability, with a quantum algorithm based on the circuit model. What we are given here as input is a proposition in the propositional calculus and we have to decide whether it has a satisfying truth assignment. As Pitowsky (2002) shows, the quantum algorithm appears to solve this problem by testing all 2nassignments "at once", yet this quantum 'miracle' helps us very little since any measurement performed on the output state collapses it, and if there is one possible truth assignment that solves this decision problem, the probability of retrieving it is 2−n, just as in the case of a classical probabilistic Turing machine which guesses the solution and then checks it. Pitowsky's conclusion is that in order to enhance computation with quantum mechanics we must construct 'clever' superpositions that increase the probability of successfully retrieving the result far more than that of a pure guess. Shor's algorithm and the class of algorithms that evaluate a global property of a function (this class is known as the hidden subgroup class of algorithms) are (so far) a unique example of both a construction of such 'clever' superposition and a retrieval of the solution in polynomial time. The quantum adiabatic algorithm may give us similar results, contingent upon the existence of an energy gap that decreases polynomially with the input.
This question also raises important issues about how to measure the complexity of a given quantum algorithm. The answer differs, of course, according to the particular model at hand. In the adiabatic model, for example, one needs only to estimate the energy gap behavior and its relation to the input size (encoded in the number of degrees of freedom of the Hamiltonian of the system). In the measurement-based model, one counts the number of measurements needed to reveal the solution that is hidden in the input cluster state (since the preparation of the cluster state is a polynomial process, it does not add to the complexity of the computation). But in the circuit model things are not as straightforward. After all, the whole of the quantum-circuit-based computation can be be simply represented as a single unitary transformation from the input state to the output state.
This feature of the quantum circuit model supports the conjecture that the power of quantum computers, if any, lies

not in quantum dynamics (i.e., in the Schrödinger equation), but rather in the quantum state, or the wave function. Another argument in favor of this conjecture is that the Hilbert subspace "visited" during a quantum computational process is, at any moment, a linear space spanned by all of the vectors in the total Hilbert space which have been created by the computational process up to that moment. But this Hilbert subspace is thus a subspace spanned by a polynomial number of vectors and is thus at most a polynomial subspace of the total Hilbert space. A classical simulation of the quantum evolution on a Hilbert space with polynomial number of dimensions (that is, a Hilbert space spanned by a number of basis vectors which is polynomial in the number of qubits involved in the computation), however, can be carried out in a polynomial number of classical computations. Were quantum dynamics the sole ingredient responsible to the efficiency of quantum computing, the latter could be mimicked in a polynomial number of steps with a classical computer (see, e.g., Vidal 2003).

The key point, of course, is that one does not end a quantum computation with an arbitrary superposition, but aims for a very special, 'clever' state—to use Pitowsky's term. Quantum computations may not always be mimicked with a classical computer because the characterization of the computational subspace of certain quantum states is difficult, and it seems that these special, 'clever', quantum states cannot be classically represented as vectors derivable via a quantum computation in an optimal basis, or at least that one cannot do so in such way that would allow one to calculate the outcome of the final measurement made on these states.

Consequently, in the quantum circuit model one should count the number of computational steps in the computation not by counting the number of transformations of the state, but by counting the number of one- or two-qubit local transformations that are required to create the 'clever' superposition that ensures the desired "speed-up". (Note that Shor's algorithm, for example, involves three major steps in this context: First, one creates the 'clever' entangled state with a set of unitary transformations. The result of the computation—a global property of a function—is now 'hidden' in this state; second, in order to retrieve this result, one projects it on a subspace of the Hilbert space, and finally one performs another set of unitary transformations in order to make the result measurable in the original computational basis. All these steps count as computational steps as far as the efficiency of the algorithm is concerned. See also Bub 2006a.) The trick is to perform these local one- or two-qubit transformations in polynomial time, and it is likely that it is here where the physical power of quantum computing may be found.

A POT POURRI AND OLIO PODRADA OF CONSCIOUSNESS AND QUANTUM GRAVITY?

The quantum information revolution has prompted several physicists and philosophers to claim that new insights can be gained from the rising new science into conceptual problems in the foundations of quantum mechanics (Fuchs 2002, Bub 2005). Yet while one of the most famous foundational problems in quantum mechanics, namely the quantum measurement problem, remains unsolved even within quantum information theory (see Hagar 2003 and Hagar and Hemmo 2006 for a critique of the quantum information theoretic approach to the foundations of quantum mechanics and the role of the quantum measurement problem in this context), some quantum information theorists dismiss it as a philosophical quibble (Fuchs 2002). Indeed, in quantum information theory the concept of "measurement" is taken as a primitive, a "black box" which remains unanalyzed. The measurement problem itself, furthermore, is regarded as a misunderstanding of quantum theory. But recent advances in the realization of a large scale quantum computer may eventually prove quantum information theorists wrong: Rather than supporting the dismissal of the quantum measurement problem, these advances may surprisingly lead to its empirical solution.

The speculative idea is the following. As it turns out, collapse theories— one form of alternatives to quantum theory which aim to solve the measurement problem—modify Schrödinger's equation and give different predictions from quantum theory in certain specific circumstances. These circumstances may be realized, moreover, decoherence effects could be suppressed (Bassi et al. 2005). Now one of the most difficult obstacles that await the construction of a large scale quantum computer is its robustness against decoherence effects (Unruh 1995). It thus appears that the technological capabilities required for the realization of a large scale quantum computer are exactly those upon which the distinction between "true" and "false" collapse (Pearle 1998), i.e., between collapse theories and environmentally induced decoherence, is contingent. Consequently, while quantum computing may elucidate the essential distinction between quantum and classical physics, its physical realization would shed light also on one of the long standing conceptual problems in the foundations of the theory, and would serve as yet another example of experimental metaphysics (the term was coined by Abner Shimony to designate the chain of events that led from the EPR argument via Bell's theorem to Aspect's experiments).

Subject-Object problem raising its ugly head again?

Another philosophical implication of the realization of a large scale quantum computer regards the long-standing debate in the philosophy of mind on the autonomy of computational theories of the mind (Fodor 1974). In the shift from strong to weak artificial intelligence, the advocates of this view tried to impose constraints on computer

programs before they could qualify as theories of cognitive science (Pylyshyn 1984). These constraints include, for example, the nature of physical realizations of symbols and the relations between abstract symbolic computations and the physical causal processes that execute them. The search for the computational feature of these theories, i.e., for what makes them computational theories of the mind, involved isolating some features of the computer as such. In other words, the advocates of weak AI were looking for computational properties, or kinds, that would be machine independent, at least in the sense that they would not be associated with the physical constitution of the computer, nor with the specific machine model that was being used. These features were thought to be instrumental in debates within cognitive science, e.g., the debate between functionalism and connectionism (Fodor and Pylyshyn 1988).
Note, however, that once the physical **Church-Turing thesis is violated, some computational notions cease to be autonomous. In other words, given that quantum computers may be able to efficiently solve classically** intractable problems, hence re-describe the abstract space of computational complexity, computational concepts and even computational kinds such as 'an efficient algorithm' or 'the class NP', will become machine-dependent, and recourse to 'hardware' will become inevitable in any analysis thereof.
Advances in quantum computing may thus militate, ENCUMBER, CHECKMATE, against the functionalist view about the unphysical character of the types and properties that are used in computer science. In fact, these types and categories may become physical as a result of this natural development in physics (e.g., quantum computing, chaos theory). Consequently, efficient quantum algorithms may also serve as counterexamples to a-priori arguments against reductionism (Pitowsky 1996).

"DISCORDANCE" OR "NOISE"
ASSUMPTIONS:

a) NOISE classified into three categories;
   1) Category 1 OF NOISE OR DISCORDANCE
   2) Category 2 OF NOISE OR DISCORDANCE
   3) Category 3 OF NOISE OR SICORDANCE

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the "NOISE" AND "DISCORDANT NOTE" CLASSIFICATION would be in the fitness of things. For category 3. "Over and above" nomenclature could be used to encompass a wider range of consumption due to cellular respiration. Similarly, a "less than" scale for category 1 can be used.

b) The speed of growth of "NOISE" OR "DISCORDANCE" UNDER category 1 is proportional to the "NOISE" UNDER CATEGORY TWO ESSENTIALLY, the accentuation coefficient in the model is representative of the constant of proportionality between under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable. The dissipation of "Noise" or "Discordance"

NOTATION :

G_{13} : CATEGORY ONE OF "NOISE" OR "DISCORDANCE"
G_{14} : CATEGORY TWO OF "NOISE" OR DISCORDANCE"
G_{15} :CATEGORY THREE OF "NOISE " OR "DISCORDANCE"

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}$: Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

(a) The growth speed in category 1 is the sum of a accentuation term $(a_{13})^{(1)}G_{14}$ and a dissipation term $-(a'_{13})^{(1)}G_{13}$, the amount of dissipation taken to be proportional to the CATEGORY ONE

(b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(1)}G_{13}$ and $-(a'_{14})^{(1)}G_{14}$ the inflow from the category 1 dependent on the category. ONE.

(c) The growth speed in category 3 is equivalent to $(a_{15})^{(1)}G_{14}$ and $-\left(a'_{15}\right)^{(1)}G_{15}$ dissipation ascribed only to depletion phenomenon.

GOVERNING EQUATIONS:

The differential equations governing the above system can be written in the following form

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left(a'_{15}\right)^{(1)}G_{15} \quad 3$$

$$(a_i)^{(1)} > 0 \quad , \quad i = 13,14,15 \quad 4$$

$$\left(a'_i\right)^{(1)} > 0 \quad , \quad i = 13,14,15 \quad 5$$

$$(a_{14})^{(1)} < (a'_{13})^{(1)} \quad 6$$

$$(a_{15})^{(1)} < (a'_{14})^{(1)} \quad 7$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - \left(a'_{13}\right)^{(1)}G_{13}} = dt \quad 8$$

$$\frac{dG_{14}}{(a_{14})^{(1)}G_{13} - \left(a'_{14}\right)^{(1)}G_{14}} = dt \quad 9$$

Or we write a single equation as

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - \left(a'_{13}\right)^{(1)}G_{13}} = \frac{dG_{14}}{(a_{14})^{(1)}G_{13} - \left(a'_{14}\right)^{(1)}G_{14}} = \frac{dG_{15}}{(a_{15})^{(1)}G_{14} - \left(a'_{15}\right)^{(1)}G_{15}} = dt \quad 10$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α ,β ,γ all positive we can write equation (10) as

$$\frac{\alpha dG_{13}}{\alpha\left((a_{13})^{(1)}G_{14} - \left(a'_{13}\right)^{(1)}G_{13}\right)} = \frac{\beta dG_{14}}{\beta\left((a_{14})^{(1)}G_{13} - \left(a'_{14}\right)^{(1)}G_{14}\right)} = \frac{\gamma dG_{15}}{\gamma\left((a_{15})^{(1)}G_{14} - \left(a'_{15}\right)^{(1)}G_{15}\right)} = dt \quad 11$$

The general solution of the SYSTEM can be written in the form

$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$ Where $i = 13,14,15$ and C_{13}, C_{14}, C_{15} are arbitrary constant coefficients.

STABILITY ANALYSIS :

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} > 0$ all the components of the solution tend to zero, and the solution is stable with respect to the initial data.

2. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $\left(\lambda_{14} + (a'_{13})^{(1)}\right)G_{13}^0 - (a_{13})^{(1)}G_{14}^0 \neq 0, (\lambda_{14} < 0)$, the first two components of the solution tend to infinity as $t \to \infty$, and $G_{15} \to 0$, ie. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tends to zero.

3. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $\left(\lambda_{14} + (a'_{13})^{(1)}\right)G_{13}^0 - (a_{13})^{(1)}G_{14}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

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From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of oxygen consumption converges to equilibrium.
2.The approach to equilibrium is a steady one , and there exists progressively diminishing oscillations around the equilibrium point
3.Conditions 1 and 2 are independent of the size and direction of initial disturbance
4.The actual shape of the time path of the system is determined by efficiency parameter , the strength of the response of the portfolio in question, and the initial disturbance
5.Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold
6.Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question,
7. Some authors Nober F J, Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

QUANTUM COMPUTATION PORTFOLIO:

Assumptions:

(1) 'Quantum Computation' IS classified into three categories analogous to the stratification that was resorted to in 'NOISE' OR 'DISCORDANCE' sector. Correspondingly, the transference process also takes place in the similar fashion.

(2) Category 2 of quantum computation.

(3) Category 3 of Quantum Computation

a) The speed of growth of Quantum Computation in category 1 is a linear function of the Quantum Computation classified under in category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between balance in category 1 and category 2.

b) The dissipation coefficient in the growth model is attributable to Dissipation and Quantum Computation which is delineated and disseminated in extensively in the above

c) Inflow into category 2 is only from category 1 in the form of transfer from the category 1.This is evident from the parametric classification scheme. As a result, the speed of growth of category 2 is dependent upon inflow, which is a function of the quantum computation of.

d) The Quantum Computation in sector in category 3 is because of transfer from category 2. It is dependent on the Quantum Computation in sector 2

NOTATION :

T_{13} : **CATEGORY ONE OF QUANTUM COMPUTATION**
$\boldsymbol{T_{14}}$ **: CATEGORY TWO OF QUANTUM COMPUTATION**
$\boldsymbol{T_{15}}$ **: CATEGORY THREE OF QUANTUM COMPUTATION**

$(b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$: Accentuation coefficients

$(b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

Under the above assumptions, we derive the following :

a) The growth speed in category 1 is the sum of two parts:
   1. A term $+(b_{13})^{(1)}T_{14}$ proportional to the QUANTUM COMPUTATION IN category 2
   2. A term $-(b'_{13})^{(1)}T_{13}$ representing the quantum of balance dissipated from category 1.

b) The growth speed in category 2 is the sum of two parts:
   1. A term $+(b_{14})^{(1)}T_{13}$ constitutive of the amount of inflow from the category 1
   2. A term $-(b'_{14})^{(1)}T_{14}$ the dissipation factor.

c) The growth speed under category 3 is attributable to inflow from category 2

GOVERNING EQUATIONS: SYSTEM : QUANTUM COMPUTATION AND "NOISE" OR "DISCORDANCE"

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13}$$ 12

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14}$$ 13

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - (b'_{15})^{(1)}T_{15}$$ 14

$$(b_i)^{(1)} > 0 \quad , \quad i = 13,14,15$$ 15

$(b_i')^{(1)} > 0 \quad , \qquad i = 13,14,15$ 16

$(b_{14})^{(1)} < (b_{13}')^{(1)}$ 17

$(b_{15})^{(1)} < (b_{14}')^{(1)}$ 18

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha_i'T_i + \beta_i'T_i + \gamma_i'T_i = C_i'e_i^{\lambda' _i t}$, $i = 13,14,15$ where $C_{13}', C_{14}', C_{15}'$ are arbitrary constant coefficients and $\alpha_{13}', \alpha_{14}', \alpha_{15}', \gamma_{13}', \gamma_{14}', \gamma_{15}'$ corresponding multipliers to the characteristic roots of the system

QUANTUM COMPUTATION AND "NOISE" OR "DISCORDANCE"-DUAL SYSTEM ANALYSIS

We will denote

1) By $T_i(t), i = 13,14,15$, the three parts of the QUANTUM COMPUTATION system analogously to the G_i of the "NOISE" OR "DISCORDANCE" SYSTEM
2) By $(a_i'')^{(1)}(T_{14}, t)$ $(T_{14} \geq 0, t \geq 0)$,the ACCENTUATION COEFFCIENT
3) By $(-b_i'')^{(1)}(G_{13}, G_{14}, G_{15}, t) = -(b_i'')^{(1)}(G, t)$, the DISSIPATION COEFFCIENT DETRITION

QUANTUM COMPUTATION AND "NOISE" OR "DISCORDANCE" SYSTEM GOVERNING EQUATIONS:

The differential system of this model is now

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t)\right]G_{13}$ 19

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}, t)\right]G_{14}$ 20

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}, t)\right]G_{15}$ 21

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G, t)\right]T_{13}$ 22

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G, t)\right]T_{14}$ 23

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G, t)\right]T_{15}$ 24

$+(a_{13}'')^{(1)}(T_{14}, t) =$ **First augmentation factor**
$-(b_{13}'')^{(1)}(G, t) =$ **First detritions factor**

Where we suppose

(A) $(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0,$
$i, j = 13,14,15$

(B) The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(1)}, \; (r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq \; (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

25

26

(C) $lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 27
$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = \; (r_i)^{(1)}$ 28

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \; (r_i)^{(1)}$ are positive constants and $i = 13,14,15$

They satisfy Lipschitz condition:

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

29

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, T)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

30

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) And (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the **first augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

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Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and \; (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \; (r_i)^{(1)}, i = 13,14,15,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}\left[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}\right] < 1$$ 32

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Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 34

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 35

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 36

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$ 37

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$ 38

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$ 39

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$ 40

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$ 41

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$ 42

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} = \left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right) \quad 43$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)} \right] \quad 44$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{13})^{(1)} \quad 45$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)} \quad 46$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) = \sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\} \quad 47$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$:

$$(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} + \int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} + (a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} + G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)} \quad 48$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses on 25,26,27,28 and 29 it follows

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$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$ 50

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$. 51

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(\textit{and not on } t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$ 52

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(1)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating 53

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 54

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and ε_1 : 55

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying 56

$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$:

(b) By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations 57
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$
and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ and 58

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 59

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the
roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ 60
and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 61

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ \boldsymbol{if}\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, \boldsymbol{if}\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$ 62
and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ 63

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$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously

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$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ \boldsymbol{if}\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\ , \boldsymbol{if}\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$ 66

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \boldsymbol{if}\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined by 59 and 61 respectively 67

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$ 68

where $(p_i)^{(1)}$ is defined by equation 25

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$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$

$\Big(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t} \right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})} [e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}] + G_{15}^0 e^{-(a'_{15})^{(1)} t})$ 70

$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$ 71

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$

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$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le$

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})} \left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t} \right] + T_{15}^0 e^{-(R_2)^{(1)} t}$

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Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)} (m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$ 74

$(R_1)^{(1)} = (b_{13})^{(1)} (\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$ 75

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \Big((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)} (T_{14}, t) \Big) - (a''_{14})^{(1)} (T_{14}, t) \nu^{(1)} - (a_{14})^{(1)} \nu^{(1)}$$

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Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

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It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

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From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$ 79

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

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In the same manner , we get

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}} \qquad 82$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

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(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

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(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$ 85

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain 86

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

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Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ **this also defines** $(\nu_0)^{(1)}$ **for the special case** .

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, **and definition of** $(u_0)^{(1)}$**.**

4. STATIONARY SOLUTIONS AND STABILITY

Stationary value :

In all the cases $G = G_0$, $G < G_0$, $G > G_0$ the condition that the rate of change of oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of (G) is that the rate of change of "NOISE" OR "DISCORDANCE" function at $G_0$is zero.
2. A sufficient condition for the stationary value at G_0 , to be maximum is that the acceleration of the "NOISE" OR "DISCORDANCE"is less than zero.
3. A sufficient condition for the stationary value at G_0, be minimum is that acceleration of "NOISE" OR "DISCORDANCE" is greater than zero.
4. With the rate of change of G namely "NOISE" OR "DISCORDANCE" defined as the accentuation term and the dissipation term, we are sure that the rate of change of "NOISE" is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms

of stationeries.

We can prove the following

Theorem 3: If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions (with the notations 25,26,27,28)

$(a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a_{14}')^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13}')^{(1)}(r_{14})^{(1)} - (b_{14}')^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation 25 are satisfied , then the system

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$(a_{13})^{(1)}G_{14} - \left[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14})\right]G_{13} = 0$ 89

$(a_{14})^{(1)}G_{13} - \left[(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14})\right]G_{14} = 0$ 90

$(a_{15})^{(1)}G_{14} - \left[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14})\right]G_{15} = 0$ 91

$(b_{13})^{(1)}T_{14} - [(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G)\ ]T_{13} = 0$ 92

$(b_{14})^{(1)}T_{13} - [(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G)\ ]T_{14} = 0$ 93

$(b_{15})^{(1)}T_{14} - [(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G)\ ]T_{15} = 0$ 94

has a unique positive solution , which is an equilibrium solution for the system (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}')^{(1)}(a_{14}'')^{(1)}(T_{14}) + (a_{14}')^{(1)}(a_{13}'')^{(1)}(T_{14}) + (a_{13}'')^{(1)}(T_{14})(a_{14}'')^{(1)}(T_{14}) = 0$$

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Definition and uniqueness of T_{14}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)} G_{14}}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)} G_{14}}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$$

(b) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 96

$$\varphi(G) = (b_{13}')^{(1)}(b_{14}')^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \left[(b_{13}')^{(1)}(b_{14}'')^{(1)}(G) + (b_{14}')^{(1)}(b_{13}'')^{(1)}(G)\right] + (b_{13}'')^{(1)}(G)(b_{14}'')^{(1)}(G) = 0$$

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Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G^*)]}$$ 98

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24

ASYMPTOTIC STABILITY ANALYSIS 99

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$$ 100

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$$ 101

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T^*_{13} \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T^*_{14} \mathbb{G}_j\right)$$ 102

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T^*_{15} \mathbb{G}_j\right)$$ 103

The characteristic equation of this system is

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$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$
$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)} G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)} G^*_{13}\right)\right]$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)} T^*_{14} + (b_{14})^{(1)} s_{(13),(14)} T^*_{14}\right)$$ 105
$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)} G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)} G^*_{14}\right)$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)} T^*_{14} + (b_{14})^{(1)} s_{(13),(13)} T^*_{13}\right)$$ 106
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right) (\lambda)^{(1)}\right)$$
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right) (\lambda)^{(1)}\right)$$ 107
$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right) (\lambda)^{(1)}\right)(q_{15})^{(1)} G_{15}$$
$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)} G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)} G^*_{13}\right)$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)} T^*_{14} + (b_{14})^{(1)} s_{(13),(15)} T^*_{13}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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GOVERNING EQUATIONS

NOISE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - (a'_{13})^{(1)} G_{13}$ 1a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - (a'_{14})^{(1)} G_{14}$ 2a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - (a'_{15})^{(1)} G_{15}$ 3a

QUANTUM COMPUTATION

$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - (b'_{13})^{(1)} T_{13}$ 4a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - (b'_{14})^{(1)} T_{14}$ 5a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - (b'_{15})^{(1)} T_{15}$ 6a

MEASUREMENT OF QUANTUM GATES

$\frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - (a'_{16})^{(2)} G_{16}$ 7a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - (a'_{17})^{(2)} G_{17}$ 8a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - (a'_{18})^{(2)} G_{18}$ 9a

COLLAPSE OF QUANTUM STATES

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - (b'_{16})^{(2)}T_{16}$ 10a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - (b'_{17})^{(2)}T_{17}$ 11a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - (b'_{18})^{(2)}T_{18}$ 12a

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS

QUANTUM COMPUTATION AND "NOISE" OR "DISCORDANCE" SYSTEM

$(-b''_i)^{(1)}(G_{13}, G_{14}, G_{15}, t) = -(b''_i)^{(1)}(G, t)$, i = 13,14,15 the contribution of the consumption of oxygen due to cellular respiration to the dissipation coefficient of the terrestrial organisms

NOISE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14}, t)}\right] G_{13}$ 13a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14}, t)}\right] G_{14}$ 14a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14}, t)}\right] G_{15}$ 15a

Where $\boxed{+(a''_{13})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3

QUABNTUM COMPUTATION

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G, t)}\right] T_{13}$ 16a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G, t)}\right] T_{14}$ 17a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G, t)}\right] T_{15}$ 18a

Where $\boxed{-(b''_{13})^{(1)}(G, t)}$, $\boxed{-(b''_{14})^{(1)}(G, t)}$, $\boxed{-(b''_{15})^{(1)}(G, t)}$ are first **detrition** coefficients for category 1, 2 and 3

MEASUREMENT OF QUANTUM GATES AND COLLAPSE OF QUANTUM STATES SYSTEM

$(-b''_i)^{(2)}(G_{16}, G_{17}, G_{18}, t) = -(b''_i)^{(2)}(G_{19}, t)$, i = 16,17,18 the factor

MEASUREMENT OF QUANTUM GATES

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17}, t)}\right] G_{16}$ 19a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17}, t)}\right] G_{17}$ 20a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17}, t)}\right] G_{18}$ 21a

Where $\boxed{+(a''_{16})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17}, t)}$ are first **augmentation** coefficients for category 1, 2 and 3

COLLAPSE OF QUATUM STATES

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19}, t)}\right] T_{16}$ 22a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19}, t)}\right] T_{17}$ 23a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19}, t)}\right] T_{18}$ 24a

Where $\boxed{-(b''_{16})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19}, t)}$ are first **detritions** coefficients

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

QUANTUM COMPUTATION AND MEASUREMENT OF QUATUM GATES

MEASUREMENT OF QUANTUM GATES

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{-(a''_{13})^{(1,1)}(T_{14},t)}\right] G_{16}$ 25a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{-(a''_{14})^{(1,1)}(T_{14},t)}\right] G_{17}$ 26a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{-(a''_{15})^{(1,1)}(T_{14},t)}\right] G_{18}$ 27a

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first **augmentation** coefficients for category 1, 2 and 3

$\boxed{-(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{-(a''_{15})^{(1,1)}(T_{14},t)}$ are second **detrition** coefficients for category 1, 2 and 3

QUANTUM COMPUTATION

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{+(b''_{16})^{(2,2)}(G_{19},t)}\right] T_{13}$ 28a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{+(b''_{17})^{(2,2)}(G_{19},t)}\right] T_{14}$ 29a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{+(b''_{18})^{(2,2)}(G_{19},t)}\right] T_{15}$ 30a

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first **detritions** coefficients for category 1, 2 and 3

$\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}$ are second **augmentation** coefficients for category 1, 2 and 3

NOISE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)}\right] G_{13}$ 31a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)}\right] G_{14}$ 32a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)}\right] G_{15}$ 33a

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are first **augmentation** coefficients for category 1, 2 and 3

MEASUREMENT OF QUANTUM GATES

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)}\right] T_{16}$ 34a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)}\right] T_{17}$ 35a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)}\right] T_{18}$ 36a

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first **detrition** coefficients for category 1, 2 and 3 of decomposer organism due to disintegration of dead organic matter by decomposer organism

GOVERNING EQUATIONS OF
THE "NOISE" OR" DISCORDANCE" AND QUANTUM COMPUTATION SYSTEM

COLLAPSE OF QUANTUM STATES

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1)}(G,t)}\right] T_{16}$ 37a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1)}(G,t)}\right] T_{17}$ 38a

$\frac{dT_{6}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1)}(G,t)}\right] T_{18}$ 39a

Where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are **first detritions** coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are **second detrition** coefficients for category 1, 2 and 3

NOISE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\right] G_{13}$ 40a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\right] G_{14}$ 41a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2)}(T_{17},t)}\right] G_{15}$ 42a

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are **first augmentation** coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are **second augmentation** coefficients for category 1, 2 and 3

MEASUREMENT OF QUANTUM GATES

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)}\right] G_{16}$ 43a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)}\right] G_{17}$ 44a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)}\right] G_{18}$ 45a

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are **first augmentation** coefficients for category 1, 2 and 3

QUANTUM COMPUTATION

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)}\right] T_{13}$ 46a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)}\right] T_{14}$ 47a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)}\right] T_{15}$ 48a

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are **first detrition** coefficients for category 1, 2 and 3

GOVERNING EQUATIONS OF THE SYSTEM

QUANTUM COMPUTATION-NOISE AND COLLAPSE OF QUANTUM STATES AND MEASUREMENT OF QUANTUM GATES

MEASUREMENT OF QUANTUM GATES

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\right] G_{16}$ 49a

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\right] G_{17}$ 50a

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}\right] G_{18}$ 51a

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are **first augmentation** coefficients for category 1, 2 and 3 *nd*

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}$ are **second augmentation** coefficient for category 1, 2 and 3

QUANTUM COMPUTATION

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} \boxed{-(b''_{13})^{(1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\right] T_{13}$ 52a

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\right] T_{14}$ 53a

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\right] T_{15}$ 54a

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are **first detrition** coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ are **second detrition** coefficient for category 1, 2 and 3

NOISE OR DISCORDANCE

$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} \right] G_{13}$ 55a

$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} \right] G_{14}$ 56a

$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} \right] G_{15}$ 57a

Where $\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1)}(T_{14},t)}$ are **first augmentation** coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are **second augmentation** coefficient for category 1, 2 and 3

COLLAPSE OF QUANTUM STATES

$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[(b'_{16})^{(2)} \boxed{-(b''_{16})^{(2)}(G_{19},t)} \boxed{-(b''_{13})^{(1,1,1)}(G,t)} \right] T_{16}$ 58a

$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[(b'_{17})^{(2)} \boxed{-(b''_{17})^{(2)}(G_{19},t)} \boxed{-(b''_{14})^{(1,1,1)}(G,t)} \right] T_{17}$ 59a

$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[(b'_{18})^{(2)} \boxed{-(b''_{18})^{(2)}(G_{19},t)} \boxed{-(b''_{15})^{(1,1,1)}(G,t)} \right] T_{18}$ 60a

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are **first detrition** coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1)}(G,t)}$ are **second detritions** coefficients for category 1,2 and 3

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environment **explains w**hy collapse happens where it does" and saying "the environment explains why collapse seems to happen even though it doesn't really happen

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