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Comparative Study of Dual-Tree Complex Wavelet Transform and Double Density Complex Wavelet Transform for Image Denoising Using Wavelet-Domain

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Abstract- There has been a lot of research work dedicated towards image denoising. However, with the wide spread of image usage in many fields of our lives, it becomes very important to develop new techniques for image denoising. The previous research in image denoising was based on two of the famous techniques in the image denoising named 2-D Dual-tree Complex Wavelet Transform (2D DTCWT) and 2-D Double Density Complex Wavelet Transform (2D DDCWT). In this paper, we introduce a comparative study of applying both Dual-tree Complex Wavelet Transform and Double Density Complex Wavelet Transform techniques to the image denoising. The main goal of this study is to exploit the advantages and disadvantages of using these techniques so as to determine the proper application of both.

Index Terms- Image denoising, Dual-tree complex wavelet transform, and Double Density complex wavelet transform.

I. Introduction

Generally, noise reduction is an essential part of image processing systems. An image is always affected by noise in its capture, acquisition and processing. This denoising is used to improve the quality of corrupted by a lot of noise due to the undesired conditions for image acquisition. Generally, the image quality is measured by the peak signal-to-noise ratio (PSNR) or signal-to noise ratio (SNR).

Traditionally, this is achieved by linear processing such as Wiener filtering. A variety of methods has emerged recently on signal denoising using nonlinear techniques in the case of additive Gaussian noise. To denoise the image we study the Dual Tree Complex Wavelet Transform method first then we study the Double Density Complex Wavelet Transform and compare advantages & disadvantages of the both the methods. Comparison gives a new hybrid model which has the characteristics of both the methods Dual Tree Complex Transform & Double Density Complex Wavelet Transform.

II. DUAL-TREE WAVELET TRANSFORM

We introduce the Dual-Tree Complex Wavelet Transform (DT CWT), which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately with the following properties:

(1) Approximate shift invariance;
(2) Good selectivity and directionality in 2-dimensions (2-D) with Gabor-like filters (also for higher dimensionality).
(3) Phase Information: - Local phase extraction is possible through analytic interpretation of two parallel trees of DT-DWT.
(4) Perfect reconstruction (PR) using short linear phase filters.
(5) Limited Redundancy.

Recently complex-valued wavelet transforms CWT have been proposed to improve upon these DWT deficiencies, with the Dual-Tree CWT (DT-CWT) becoming a preferred approach due to the ease of its implementation. In the DT-CWT, real valued wavelet filters produce the real and imaginary parts of the transform in parallel decomposition trees, permitting exploitation of well established real-valued wavelet implementations and methodologies. A primary advantage of the DT-CWT lies in that it results in decomposition with a much higher degree of directionality than that possessed by the traditional DWT. The real-valued filter coefficients are replaced by Complex-valued coefficients by proper design methodology that satisfies the required conditions for convergence. Then the complex filter can again be decomposed into two real-valued filters. Thus, two real-valued filters that give their respective impulse responses in quadrature will form the Hilbert transform pair. The combined pair of two such filters is termed as an analytic filter. The formulation and interpretation of the analytic filter is figure1.

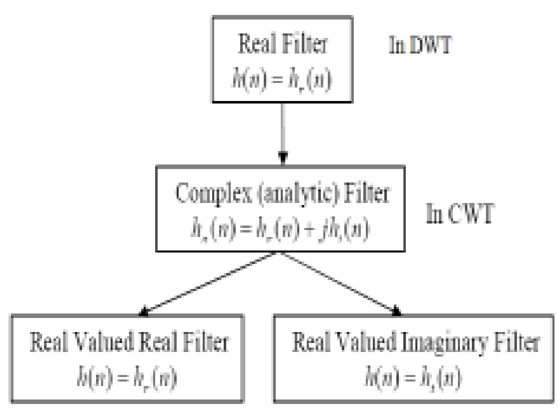

Figure 1: Analytic Filter

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The implementation of 2-D DTCWT consists of two steps. Firstly**,** an input image is decomposed up to a desired level by two separable 2D DWT branches, branch *a* and branch *b*, whose filters are specifically designed to meet the Hilbert pair requirement. Then six high-pass sub bands are generated at each level. HLa, LHa, HHa, HLb, LHb and HHb. ***Secondly*****,** every two corresponding sub bands which have the same pass-bands are linearly combined by either averaging or differencing. As a result, sub bands of 2D DT-CWT at each level are obtained as

(LHa + LHb) /√2, (LHa – LHb) /√2,
(HLa + HLb) /√2, (HLa – HLb) /√2,
(HHa + HHb) /√2, (HHa – HHb) /√2

The six wavelets defined by oriented shown above have the sum/difference operation is orthonormal, which constitutes a perfect reconstruction wavelet transform. The imaginary part of 2D DT-CWT has similar basis function as the real part. The 2D DT-CWT structure has an extension of conjugate filtering in 2D case. The filter bank structure of 2D dual-tree is shown in figure 2.

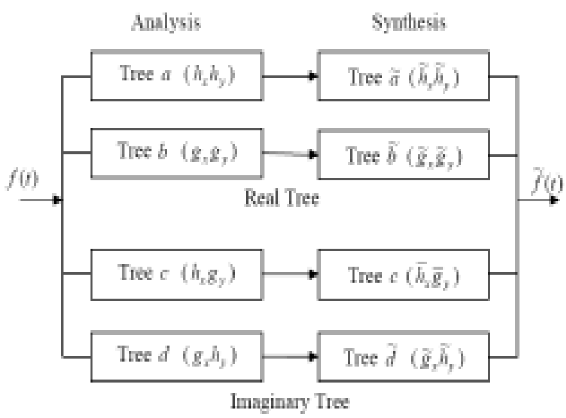

Figure 2: Filter bank structure for 2D DT-CWT

2D structure needs four trees for analysis as well as for synthesis. The pairs of conjugate Filters are applied to two dimensions (x and y) directions, which can be expressed as:

$$(h_x + jg_x)(\bar{h}_y + jg_y) = (h_x h_y - g_x g_y) + j(h_x g_y + g_x h_y)$$

The filter bank structure of *tree a*, similar to standard 2D DWT spanned over 3 level, is shown in figure 3.

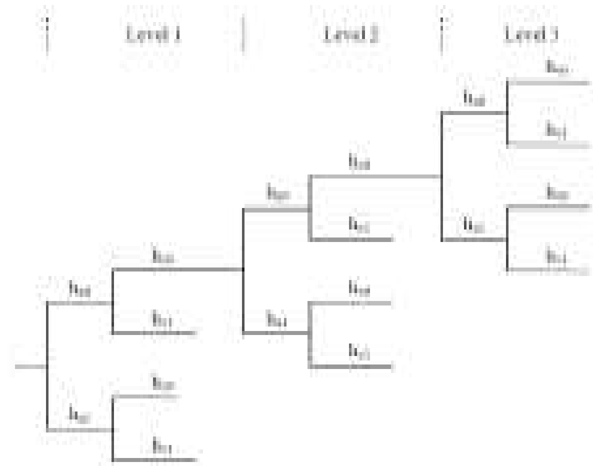

Figure 3: Filter bank structure of tree-a of figure 2

All other *trees-(b, c & d)* have similar structures with the appropriate combinations of filters for row- and column-filtering. The overall 2D dual-tree structure is 4-times redundant (expensive) than the standard 2D DWT. The *tree-a* and *tree-b* form the real pair, while the *tree-c* and *tree-d* form the imaginary pair of the analysis filter bank. Trees-(*a*~, *b*~) and trees- (c~, *d*~) are the real and imaginary pairs respectively in the synthesis filter bank similar to their corresponding analysis pairs.

Using the same principle for the design of shift invariant filter decomposition, Kingsbury suggested to apply a 'dual-tree' of two parallel filter banks are constructed and their band-pass outputs are combined. The structure of a resulting analysis filter bank is shown in Figure: 4, where index *a* stands for the original filter bank and the index *b* is for the additional one. The dual-tree complex DWT of a signal x(n) is implemented using two critically-sampled DWTs in parallel on the same data.

In one dimension, the so-called dual-tree complex wavelet transform provides a representation of a signal x(n) in terms of complex wavelets, composed of real and imaginary parts which are in turn wavelets themselves. In fact, these real and imaginary parts essentially form a quadrature pair.

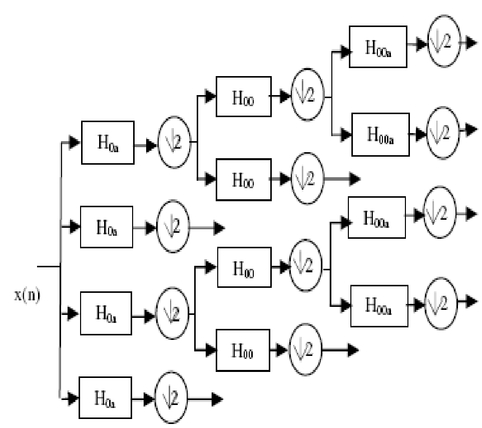

Figure 4: Three level Complex dual tree

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Table 1: First Level DWT Coefficients

H_{0a}	H_{1a}	H_{0b}	H_{1b}
0	0	0.01122679	0
-0.08838834	-0.01122679	0.01122679	0
0.08838834	0.01122679	-0.08838834	-0.08838834
0.69587998	0.08838834	0.08838834	-0.08838834
0.69587998	0.08838834	0.69587998	0.69587998
0.08838834	-0.69587998	0.69587998	-0.69587998
-0.08838834	0.69587998	0.08838834	0.08838834
0.01122679	-0.08838834	-0.08838834	0.08838834
0.01122679	-0.08838834	0	0.01122679
0	0	0	-0.01122679

The dual-tree CDWT uses length-10 filters, the table of coefficients of the analyzing filters in the first stage is shown in Table 1 and the remaining levels are shown in Table 2. The reconstruction filters are obtained by simply reversing the alternate coefficients of the analysis filters.

To extend the transform to higher-dimensional signals, a filter bank is usually applied separably in all dimensions. To compute the 2D CWT of images these two trees are applied to the rows and then the columns of the image as in the basic DWT.

Table2: Remaining Level DWT Coefficients

Tree a		Tree b	
H_{00a}	H_{01a}	H_{00b}	H_{01b}
0.03516384	0	0	-0.03516384
0	0	0	0
-0.08832942	-0.11430184	-0.11430184	0.08832942
0.23389032	0	0	0.23389032
0.76027237	0.58751830	0.58751830	-0.76027237
0.58751830	-0.76027237	0.76027237	0.58751830
0	0.23389032	0.23389032	0
-0.11430184	0.08832942	-0.08832942	-0.11430184
0	0	0	0
0	-0.03516384	0.03516384	0

This operation results in six complex high-pass sub-bands at each level and two complex low-pass sub-bands on which subsequent stages iterate in contrast to three real high-pass and one real low-pass sub-band for the real 2D transform. This shows that the complex transform has a coefficient redundancy of 4:1 or 2m:1 in m dimensions. In case of real 2D filter banks the three high-pass filters have orientations of 0^{θ}, 45^{θ} and 90^{θ} for the complex filters the six sub-band filters are oriented at angles $\pm 15^{\theta}, \pm 45^{\theta}, \pm 75^{\theta}$.

The CDWT decomposes an image into a pyramid of complex sub images, with each level containing six oriented sub images resulting from evenly spaced directional filtering and subsampling, such directional filters are not obtainable by a separable DWT using a real filter pair but complex coefficients makes this selectivity possible.

The Dual-Tree Complex Discrete Wavelet Transform has been developed to incorporate the good properties of the Fourier transform in the wavelet transform. As the name implies, two wavelet trees are used, one generating the real part of the complex wavelet coefficients tree and the other generating the imaginary part tree.

The shift invariance and directionality of the CWT may be applied in many areas of image processing like denoising, feature extraction, object segmentation and image classification. Here we shall consider the denoising example. The Complex Discrete Wavelet Transform is used for denoising as an important signal/image processing application. Complex Discrete Wavelet Transform technique is a powerful tool in removing signal/image noise. For denoising a soft thresholding method is used rather than hard-thresholding in terms of MSE and SNR.

III. DOUBLE DENSITY DISCRETE WAVELET TRANSFORM

The new version of DWT, known as Double Density DWT (DDDWT) is recently developed. It has the following important additional proprieties: -

(1) It employs one scaling function and two distinct wavelets which are designed to be offset from one another by one half.
(2) The double density DWT is over complete by a factor of two.
(3) It is nearly shift-invariant where complex wavelets with real and imaginary parts approximating Hilbert pairs are proposed for denoising signal.

The double density Discrete Wavelet Transform is constructed with analysis and synthesis filter bank and it is shown in the Figure 5.

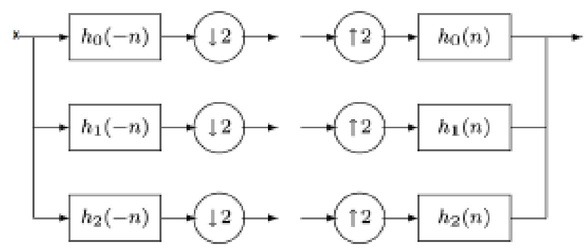


Figure 5: Oversampled analysis and synthesis filter bank

In two dimensions, this transform outperforms the standard DWT in terms of enhancement; however, there is need of improvement because not all of the wavelets are directional. That is, although the double-density DWT utilizes more wavelets, some lack a dominant spatial orientation, which prevents them from being able to isolate those directions.

A solution to this problem is provided by the double-density complex DWT, which combines the characteristics of the double-density DWT and the dual-tree DWT. The double-density complex DWT is based on two scaling functions and four distinct wavelets, each of which is specifically designed such that the two wavelets of the first pair are offset from one other by one half, and the other pair of wavelets form an approximate Hilbert transform pair.

By ensuring these two properties, the double-density complex DWT possesses improved directional selectivity and can be used to implement complex and directional wavelet transforms in multiple dimensions. We construct the filter bank structures for both the double-density DWT and the double-density complex DWT using finite impulse response (FIR) perfect reconstruction filter banks. These filter banks are then applied recursively to the low pass subband, using the analysis filters for the forward transform and the synthesis filters for the inverse transform. By doing this, it is then possible to evaluate each transforms performance in several applications including signal and image enhancement.

A. 1-D Double-Density DWT

The double-density DWT is implemented by recursively applying the 3-channel analysis filter bank to the low pass subband. This process is illustrated in Figure 6.

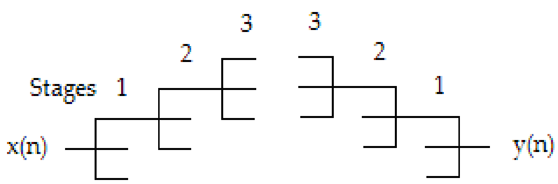


Figure 6: Three stage recursion of the 1-D double-density DWT

Conversely, the inverse double-density DWT is obtained by iteratively applying the synthesis filter bank.

B. 2-D Double-Density DWT

To use the double-density discrete wavelet transform for 2-D signal and image processing, we must implement a two-dimensional analysis and synthesis filter bank structure. This can simply be done by alternatively applying the transform first to the rows, then to the columns of an image. This gives rise to nine 2-D subbands, one of which is the 2-D low pass scaling filter, and the other eight of which make up the eight 2-D wavelet filters, as shown in Figure 7.

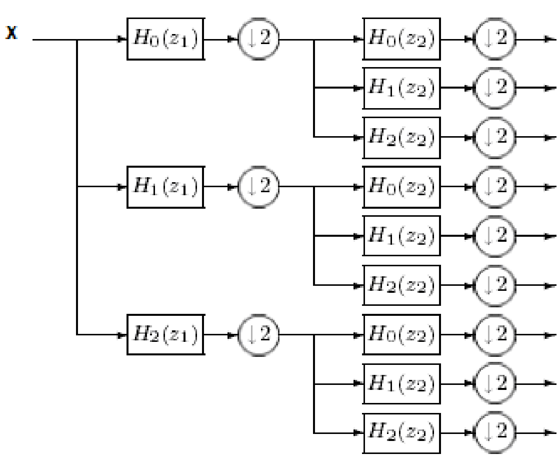


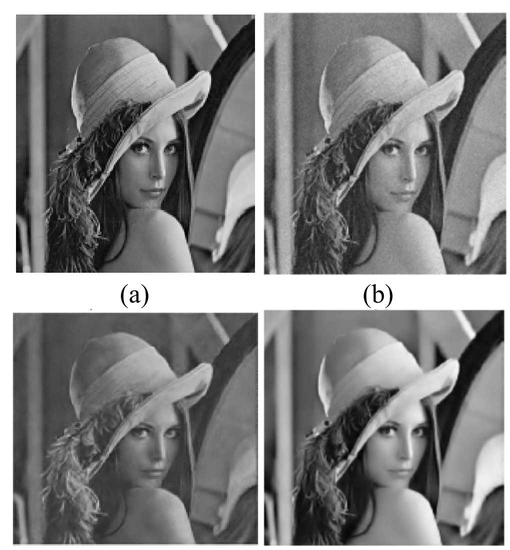
Figure 7: An Oversampled Filter Bank for 2-D Images

The differences between the double-density DWT and the dual-tree DWT can be clarified with the following comparisons.

1) In the dual-tree DWT, the two wavelets form an approximate Hilbert transform pair, whereas in the double-density DWT, the two wavelets are offset by one half.
2) For the dual-tree DWT, there are fewer degrees of freedom for design (achieving the Hilbert pair property adds constraints), whereas for the double-density DWT, there are more degrees of freedom for design.
3) Different filter bank structures are used to implement the dual-tree and double-density DWTs.
4) The dual-tree DWT can be interpreted as a complex-valued wavelet transform, which is useful for signal modeling and denoising (the double-density DWT cannot be interpreted as such).
5) The dual-tree DWT can be used to implement 2-D transforms with directional Gabor-like wavelets, which is highly desirable for image processing (the double-density DWT cannot be, although it can be used in conjunction with specialized post-filters to implement a complex wavelet transform with low-redundancy, as developed.

IV. EXPERIMENTAL RESULTS

Experiments are carried out on grey scale image to compare the performances of DT-CWT and DD-CWT methods. To evaluate the performance of the Dual-Tree and Double-Density DWT are performed. The performance of denoising method (indexed by the PSNR) is confirmed by the visual quality as shown in Figure 8.



(a) (b) (c) (d)

Figure 8: (a) Input Image, (b) Gaussian noisy image for σ=10, (c) Denoised with Dual Tree CWT (d) Denoised images with Double Density DWT

V. CONCLUSIONS

This paper highlighted wavelet based enhancement of gray scale digital images corrupted by additive Gaussian noise. In this study we have evaluated and compared the performances of complex wavelet transforms.

The DT CWT is shift invariant and forms directionally selective diagonal filters. These properties are important for many applications in image processing including denoising, deblurring, segmentation and classification. In this paper we have illustrated the example of the application of complex wavelets for the denoising of Lena images. To obtain further improvements, it is also necessary to develop principled statistical models for the behavior of features under addition of noise, and their relationship to the uncorrupted wavelet coefficients.

In terms of image enhancement, the double-density complex wavelet transform performed much better at suppressing noise over the double-density wavelet transform. However, to improve the performance further it is necessary to use a different threshold for each subband because for this transform the wavelets associated with different subbands have different norms. The simulation results indicate that the complex double density dual tree discrete wavelet transform performances better than others wavelet transform.

The proposed one is a hybrid model of Complex Dual-Tree DWT and Complex Double-Density DWT. The complex double density dual tree discrete wavelet transform (CDDDTDWT) outperforms in comparison with others wavelet transform in the highly corrupted images.

The double-density dual-tree DWT, which is an overcomplete discrete wavelet transform (DWT) designed to simultaneously possess the properties of the double-density DWT and the dual-tree complex DWT. The double-density DWT and the dual-tree complex DWT are similar in several respects (they are both overcomplete by a factor of two, they are both nearly shift-invariant, and they are both based on FIR perfect reconstruction filter banks), but they are quite different from one another in other important respects. Both wavelet transforms can outperform the critically sampled DWT for several signal processing applications, but they do so for different reasons. It is therefore natural to investigate the possibility of a single wavelet transform that has the characteristics of both the double-density DWT and dual-tree complex DWT.

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