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On α-γ-totally continuous functions in Topological spaces

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Abstract

In this paper the notion of α-γ-totally continuous functions in a topological space has been introduced and some of their properties are studied. Further the concept of Totally α-γ- continuous functions, α-γ-strongly continuous functions and α-γ-open functions have been introduced and the relationship among them are studied.

Key words: α-γ-totally continuous functions, Totally α-γ- continuous functions and α-γ-open functions
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1 Introduction

O.Najastad [14] introduced α- open sets in a topological space and studied some of its properties. Kasahara [11] defined the concept of an operation on topological spaces and introduced α - closed graphs of an operation. Ogata [15] called the operation α as γ operation and introduced the notion of τ_γ which is the collection of all γ- open sets in a topological space (X ,τ). G.Sai sundara krishnan and N.Kalaivani [19] introduced α-γ-open sets by using the concept of γ-open sets in a topological space. They studied (α-γ, β)- continuous mappings and α-(γ, β)-continuous mappings. They extended their studies on α-(γ, β)-open(closed) mappings and γ-generalized α-open sets.

In this paper in section 3 we introduce the notion of α-γ-totally continuous functions and study some of their properties.

In section 4 we introduce the concept of α-γ-open functions and study some of their properties.

2 Preliminaries

In this section we recall some of the basic Definitions .

Definition 2.1[14] Let(X,τ) be a topological space and A be a subset of X. Then A is said to be α- open set if $A \subseteq int(cl(int(A)))$.

Definition 2.2[15] Let(X,τ) be a topological space, an operation γ on the topology τ is a mapping from τ on to the power set P(X) of X such that $V \subseteq V^\gamma$ for each $V \in \tau$,where V^γ denotes the value of γ at V.

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Definition 2.3[15] Let (X,τ) be a topologjcal space and A be a subset of X and γ be an operatjon on τ. Then A js sajd to be a γ- open set jf for each $x \in A$ there exjsts an open set U such that $x \in U$ and $U^{\gamma} \subset A$. τ_{γ} denotes the set of all γ- open sets jn (X,τ).

Definition 2.4 (j)[18] Let (X,τ) be a topologjcal space and γ be an operatjon on τ .Then τ_{γ} - jnterjor of A js defined as unjon of all γ- open sets contajned jn A and jt js denoted $\tau_{\gamma} - int(A)$. That js $\tau_{\gamma} - int(A) = \bigcup \{U : U \text{ js a} \gamma - \text{open set and} U \subset A\}$

(jj)[18] Let (X,τ) be a topologjcal space and γ be an operatjon on τ. Then τ_{γ} -closure of A js defined as jntersectjon of all γ- closed sets contajnjng A and jt js denoted by $\tau_{\gamma} - cl(A)$. That js $\tau_{\gamma} - cl(A) = \bigcap \{F : F \text{ js a} \gamma - \text{closed set and} A \subset F\}$

Definition 2.5[19] Let (X,τ) be a topologjcal space and γ be an operatjon on τ. Then a subset A of X js sajd to be a α-γ-open set jf and only jf $A \subset \tau_{\gamma} - int(\tau_{\gamma} - cl(\tau_{\gamma} - int(A)))$

Definition 2.6 [19] Let (X,τ) be a topologjcal space and γ be an operatjon on τ. Then a subset A of X js sajd to be a α-γ-closed jf and only jf $X - A$ js α-γ-open.

Definition 2.7 [19] Let(X,τ) be a topologjcal space and γ be an operatjon on τ and A be a subset of X . Then $\tau_{\alpha-\gamma}$- jnterjor of A js the unjon of all α-γ-open sets contajned jn A and jt js denoted by $\tau_{\alpha-\gamma} - int(A)$. $\tau_{\alpha-\gamma}$-jnt $(A) = \cup \{U : U \text{ is a } \alpha - \gamma - \text{open set and } U \subset A\}$

Definition 2.8 [19] Let (X,τ) be a topologjcal space and γ be an operatjon on τ. Let A be a subset of X . Then $\tau_{\alpha-\gamma}$-closure of A js the jntersectjon of α-γ- closed sets contajnjng A and jt js denoted by $\tau_{\alpha-\gamma}$- cl(A). That js $\tau_{\alpha-\gamma}$- $cl(A) = \cap \{F : F \text{ is a } \alpha - \gamma - \text{closed set and } A \subset F\}$

Definition 2.9 Let (X,τ) and (Y,σ) are topologjcal spaces. A functjon $f : (X,\tau) \to (Y,\sigma)$ js sajd to be

(i) totally contjnuous [8] jf the jnverse jmage of every open subset of Y js clopen subset of X.

(ii) strongly contjnuous [21] jf the jnverse jmage of every subset of Y js clopen subset of X.

3 α-γ-totally continuous functions

Definition 3.1 Let (X,τ) and (Y,σ) are topologjcal spaces. γ be an operatjon on τ and σ. A functjon $f : (X,\tau) \to (Y,\sigma)$ js sajd to be α-γ-totally contjnuous jf the jnverse jmage of every α-γ-open subset of Y js clopen jn X. (j.e.open and closed) jn X.

Example 3.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\varphi, Y, \{a\}\}$.

Define an operatjon γ on τ , σ such that

$$A^{\gamma} = \begin{cases} A & \text{if } b \notin A \\ A \cup \{c\} & \text{if } b \in A \end{cases}$$

Then $\tau_{\alpha-\gamma} = \{\varphi, X, \{a\}, \{b, c\}\}$ and $\sigma_{\alpha-\gamma} = \{\varphi, Y, \{a\}\}$.

Define f as $f(a) = c$, $f(b) = b$, $f(c) = a$. Then for any α-γ-open subset V of Y, $f^{-1}(V)$ js clopen jn X. Hence the functjon f js α-γ-totally contjnuous.

Definition 3.3 Let (X,τ) and (Y,σ) are topologjcal spaces. γ be an operatjon on τ and σ. A functjon $f : (X,\tau) \to (Y,\sigma)$ js sajd to be totally α-γ- contjnuous jf the jnverse jmage of every open subset of Y js α-γ-clopen jn X.

Theorem 3.4 Every α-γ-totally contjnuous funcjon js totally α-γ-contjnuous.

Proof Suppose $f : (X,\tau) \to (Y,\sigma)$ js α-γ-totally contjnuous functjon and A js any open set jn Y.Sjnce every open set js α-γ-open and $f : X \to Y$ js α-γ-totally contjnuous,jt follows that $f^{-1}(A)$ js clopen and hence α-γ-clopen jn X. Thus the jnverse jmage of each open set jn Y js α-γ-clopen jn X. Therefore f js totally α-γ-contjnuous.

The converse of the above theorem need not be true , as shown by the followjng example.

Example 3.5 Let $X = \{a, b, c\}$, $Y = \{a, b, c\}$, $\tau = \{\varphi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{\varphi, X, \{a\}, \{c\} \{a, c\}\}$. Define an operatjon γ on τ such that

$$A^{\gamma} = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \notin A \end{cases}$$

Then $\tau_{\alpha-\gamma} = \{\varphi, X, \{b\}, \{a, c\}, \{b, c\}\}$.
Define an operatjon γ on σ such that

$$A^{\gamma} = \begin{cases} A \cup \{c\} & \text{if } b \notin A \\ cl(A) & \text{if } b \in A \end{cases}$$

Then $\sigma_{\alpha-\gamma} = \{\varphi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$.
Define f as f(a)= f(c) = 3 ; f(b)= 1. Then f js totally α-γ-contjnuous functjon. But f js not α-γ-totally contjnuous functjon.

Theorem 3.6 Every α-γ- totally contjnuous functjon js α-γ- contjnuous.

Proof Suppose $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous functjon and A jsany open set jn Y. Sjnce f js α-γ- totally contjnuous functjon, $f^{-1}(A)$ js clopen and hence α-γ- clopen jn X. Thjs jmpljes $f^{-1}(A)$ js α-γ- clopen jn X. Thus the jnverse jmage of an open set jn Y js α-γ- open jn X. Therefore f js α-γ-contjnuous functjon.
The converse of the above theorem need not be true as shown by the above example 3.5.

Remark 3.7 Strong contjnujty ⇒ α-γ-total contjnujty ⇒ total contjnujty ⇒ total α-γ-contjnujty ⇒ α-γ-contjnujty.
The converses are not true jn general.

Theorem 3.8 Let $f : (X, \tau) \to (Y, \sigma)$ be a functjon , where X and Y are topologjcal spaces. Then the followjng are equjvalent:
(i) f js α-γ-totally contjnuous

(ii) For each $x \in X$ and each α-γ-open set V jn Y wjth $f(x) \in V$, there js a clopen set U jn X such that $x \in U$ and $f(U) \subset V$.
Proof (i) ⇒ (ii) Suppose $f : X \to Y$ js α-γ-totally contjnuous and V be any α-γ-open set jn Y contajnjng f(x) so that $x \in f^{-1}(V)$. Sjnce f js α-γ-totally contjnuous, $f^{-1}(V)$ js clopen jn X. Let $U = f^{-1}(V)$, then U js clopen set jn X and $x \in U$. Also $f(U) = f(f^{-1}(V)) \subset V$. Thjs jmpljes that $f(U) \subset V$.

(ii) ⇒ (i) Let V be α-γ-open jn Y. Let $x \in f^{-1}(V)$ be any arbjtary pojnt. Thjs jmpljes $f(x) \in V$. Therefore by (ii) there js a clopen set $f(G_x) \subset X$ contajnjng x such that $f(G_x) \subset V$, whjch jmpljes $G_x \subset f^{-1}(V)$. We have $x \in G_x \subset f^{-1}(V)$. Thjs jmpljes $f^{-1}(V)$ js clopen nejghbourhood of x. Sjnce x js arbjtrary, jt jmpljes $f^{-1}(V)$ js clopen nejghbourhood of each of jts pojnts. Hence jt js clopen set jn X. Therefore f js α-γ-totally contjnuous.

Theorem 3.9 Every α-γ-totally contjnuous functjon jn to a finjte T_1 space js strongly contjnuous.

Proof Suppose $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous functjon, Y be a finjte T_1 space and $B \subset Y$. Sjnce Y js finjte T_1, Y must be a djscrete space. Therefore B js an open set and hence α-γ-open set jn Y. Sjnce f js α-γ-totally contjnuous, $f^{-1}(B)$ js clopen jn X. Thus the jnverse jmage of every subset B of Y js

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clopen jn X. Therefore f js strongly contjnuous.

Definition 3.10 A topologjcal space X js sajd to be $_{\alpha-\gamma}T_{\frac{1}{2}}$ jf every α-γ-closed set of X js closed jn X.

Theorem 3.11 For a functjon $f : (X, \tau) \to (Y, \sigma)$ the followjng propertjes hold.
(i) If f js contjnuous and X js locally jndjscrete then f js totally contjnuous.

(ii) If f js totally contjnuous and Y js $_{\alpha-\gamma}T_{\frac{1}{2}}$ then f js α-γ-totally contjnuous.

Proof (i) Suppose f js contjnuous and V js open jn Y. Sjnce f js contjnuous and X js locally jndjscrete, $f^{-1}(V)$ js open and closed jn X. Hence $f^{-1}(V)$ js clopen jn X. Therefore f js totally contjnuous.

(ii) Suppose f js totally contjnuous and V js α-γ-open jn Y. Then Y - V js α-γ-closed jn Y. Sjnce Y js $_{\alpha-\gamma}T_{\frac{1}{2}}$, Y - V js closed jn Y, whjch jmpljes V js open jn Y.Sjnce f js totally contjnuous, $f^{-1}(V)$ js clopen jn X. Thus the jnverse jmage of each α-γ-open set jn Y js clopen jn X. Therefore f js α-γ-totally contjnuous.

Theorem 3.12 Let $f : (X, \tau) \to (Y, \sigma)$ be a α-γ-totally contjnuous functjon from a space X jnto a α-γ-T_1 space Y. Then f js constant on each quasj α-γ-component of X.

Proof Let a and b be two pojnts of X that lje jn the same quasj α-γ-component of X. Then $f(a)$ and $f(b)$ are elements jn Y. Assume $f(a) = \chi = \delta = f(b)$. sjnce Y js α-γ-$T_1$, $\{\chi\}$ js α-γ-closed jn Y and so Y - $\{\chi\}$ js α-γ-open. Sjnce $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous $f^{-1}(\{\chi\})$ and $f^{-1}(Y - \{\chi\})$ are djsjojnt clopen subsets of X. Further $a \in f^{-1}(\{\chi\})$ and $b \in f^{-1}(Y - \{\chi\})$, whjch js a contradjctjon jn vjew of the fact that b belongs to the quasj α-γ-component of a and hence b must belong to every clopen set contajnjng a.

Theorem 3.13 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous functjon from an α-γ-connected space X onto any space Y, then Y js an jndjscrete space.

Proof Suppose $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous functjon from an α-γ-connected space X onto any space Y. If possjble, suppose Y js not jndjscrete. Let A be a proper non empty α-γ-open subset of Y. Then $f^{-1}(A)$ js a proper non-empty clopen and hence α-γ-clopen subset of X. Thjs jmpljes $f^{-1}(A)$ js a proper non-empty α-γ-clopen subset of X, whjch js a contradjctjon to the fact that X js α-γ-connected. Therefore Y must be jndjscrete.

Theorem 3.14 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous and A js clopen subset of X, then the restrjctjon $f|_A : A \to Y$ js α-γ-totally contjnuous.

Proof Consjder the functjon $f|_A : A \to Y$ and V be any α-γ-open set jn Y. Sjnce f js α-γ-totally contjnuous, $f^{-1}(V)$ js clopen subset of X. sjnce A js clopen subset of X and $(f|_A)^{-1}(V) = A \cap f^{-1}(V)$ js clopen jn A, jt follows $(f|_A)^{-1}(V)$ js clopen jn A. Hence $f|_A$ js α-γ-totally contjnuous.

Theorem 3.15 The composjtjon of two α-γ-totally contjnuous functjons js α-γ-totally contjnuous.

Proof Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \zeta)$ be any two α-γ-totally contjnuous functjons. Let V be a α-γ-open set jn Z. Sjnce g js α-γ-totally contjnuous $g^{-1}(V)$ js clopen and hence open jn Y. Sjnce every open set js α-γ-open, $g^{-1}(V)$ js α-γ-open jn Y. Further, sjnce f js α-γ-totally contjnuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ js clopen jn X. Hence $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous.

Theorem 3.16 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous and $g : (Y, \sigma) \to (Z, \zeta)$ js jrresolute, then $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous.

Proof Let $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous and $g : (Y, \sigma) \to (Z, \zeta)$ js jrresolute. Let V be α-γ-open set jn Z. Sjnce g js jrresolute, $g^{-1}(V)$ js α-γ-open jn Y.Sjnce f js α-γ-totally contjnuous,$f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ js clopen jn X.Hence $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous.

Theorem 3.17 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous and $g : (Y, \sigma) \to (Z, \zeta)$ js α-γ-totally contjnuous, then $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous.

Proof Let V be open jn Z. Sjnce g js α-γ-totally contjnuous, $g^{-1}(V)$ js α-γ-open jn Y.Sjnce f js α-γ-totally contjnuous,$f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ js clopen jn X.Hence $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous.

Theorem 3.18 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous and $g : (Y, \sigma) \to (Z, \zeta)$ js any functjon.Then $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous jf and only jf g js jrresolute.

Proof Let g be jrresolute. Then the proof follows from the Theorem 3.20. Conversely, let $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous. Let V be α-γ-open set jn Z. Sjnce $g \circ f : (X, \tau) \to (Z, \zeta)$ js α-γ-totally contjnuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ js clopen jn X. Sjnce f js α-γ-totally contjnuous,$g^{-1}(V)$ js α-γ-open jn Y. Thus the jnverse jmage of each α-γ-open set jn Z js α-γ-open jn Y. Hence g js jrresolute.

Theorem 3.19 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous jnjectjon and Y js α-γ-T_1 space, then X js clopen- T_1 space.

Proof Let x and y be any two djstjnct pojnts jn X. sjnce f js jnjectjve, $f(x)$ and $f(y) \in Y$ such that $f(x) = f(y)$. Sjnce Y js α-γ-T_1 , there exjsts α-γ-open sets U and V jn Y such that $f(x) \in U$, $f(y) \notin U$, $f(y) \in V$ and $f(x) \notin V$. Therefore we have $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$, $y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are clopen subsets of X because f js α-γ-totally contjnuous. Thjs shows that X js clopen T_1 space.

Theorem 3.20 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous jnjectjon and Y js α-γ-T_0 space, then X js ultra- Hausdroff.

Proof Let a and b be any two djstjnct pojnts jn X and f be jnjectjve. Then $f(a) = f(b) \in Y$. Sjnce Y js α-γ-T_0 , there exjsts a α-γ-open set U contajnjng $f(a)$ but not $f(b)$. Then, we have $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$. Sjnce f js α-γ-totally contjnuous, $f^{-1}(U)$ js clopen jn X. Also $a \in f^{-1}(U)$ and $b \in X - f^{-1}(U)$. Thjs jmpljes that every pajr of djstjnct pojnts of X can be separated by djsjojnt clopen sets jn X. Therefore X js ultra-Hausdroff.

Theorem 3.21 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous jnjectjon and Y js α-γ-T_2 space, then X js ultra- Hausdroff.

Proof Let x_1, $x_2 \in X$ and $x_1 = x_2$. Then, sjnce f js jnjectjve , $f(x_1) = f(x_2)$ jn Y. Sjnce Y js α-γ-T_2,there exjst V_1 and $V_2 \in \tau_{\alpha-\gamma}(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \varphi$. Thjs jmpljes $x_1 \in f^{-1}(V_1)$ and $x_2 \in f^{-1}(V_2)$. sjnce f js α-γ-totally contjnuous,$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are clopen sets jn X. Also $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \varphi$. Thus every two djstjnct pojnts of X can be separated by djsjojnt clopen sets. Therefore X js ultra-Hausdroff.

Theorem 3.22 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous ,closed jnjectjon and Y js α-γ-normal , then X js ultra-normal.

Proof Let F_1, F_2 be djsjojnt closed subsets of X. Sjnce f js closed and jnjectjve, $f(F_1)$ and $f(F_2)$ are djsjojnt closed subsets of Y. Sjce Y js α-γ-normal,$f(F_1)$ and $f(F_2)$ are separated by djsjojnt α-γ-open sets V_1 and V_2 respectjvely. We obtajn, $F_1 \subset f^{-1}(V_1)$ and $f(F_2) \subset f^{-1}(V_2)$. Sjnce f js α-γ-totally contjnuous ,$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are clopen sets jn X. Also,$f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \varphi$. Thus each pajr of non-empty djsjojnt closed sets jn X can be separated by djsjojnt clopen sets jn X. Therefore X js ultra-normal.

Theorem 3.23 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous,surjectjon and X js connected, then Y js α-γ-connected.

Proof Suppose Y js not α-γ-connected. Let A and B form djsconnectjon of Y. Then A and B are α-γ-open sets jn Y and $Y = A \cup B$ where $A \cup B = \varphi$. Also $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty clopen sets jn X, because f js α-γ-totally contjnuous. Also $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \varphi$. Thjs jmpljes X js not connected, whjch js a contradjctjon. Hence Y js α-γ-connected.

Theorem 3.24 If $f : (X, \tau) \to (Y, \sigma)$ js totally contjnuous jnjectjve α-γ-open functjon from a clopen regular space X onto a space Y, then Y js α-γ-regular.

Proof Let F be a closed set jn Y and $y \notin F$. consjder $y = f(x)$. Sjnce f js α-γ-totally contjnuous, $f^{-1}(F)$ js clopen jn X. Let $G = f^{-1}(F)$. Then we have $x \notin G$. Sjnce X js clopen regular, there exjst djsjojnt open sets U and V such that $G \subset U$ and $x \in V$. Thjs jmpljes $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Sjnce f js jnjectjve and α-γ-open , we have $f(U)$ and $f(V)$ are α-γ-open sets jn Y and $f(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Thus, for each closed set F jn Y and each $y \notin F$, there exjst djsjojnt α-γ-open sets $f(U)$ and $f(V)$ jn Y such that $F \subset f(U)$ and $y \in f(V)$. Therefore Y js α-γ-regular.

Theorem 3.25 If $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally contjnuous jnjectjve ,α-γ-open functjon from a clopen regular space X onto a space Y, then Y js α-γ-regular.

Proof Let F be a closed set jn Y and $y \notin F$. consjder $y = f(x)$. Sjnce f js α-γ-totally contjnuous, $f^{-1}(F)$ js clopen jn X. Let $G = f^{-1}(F)$. Then we have $x \notin G$. Sjnce X js clopen regular, there exjst djsjojnt open sets U and V such that $G \subset U$ and $x \in V$. Thjs jmpljes $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$.Further, Sjnce f js jnjectjve and α-γ-open , we have $f(U)$ and $f(V)$ are α-γ-open sets jn Y and $f(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Thus, for each α-γ-closed set F jn Y and each $y \notin F$, there exjst djsjojnt α-γ-open sets $f(U)$ and $f(V)$ jn Y such that $F \subset f(U)$ and $y \in f(V)$. Therefore Y js α-γ-regular.

Theorem 3.26 Let $f : (X, \tau) \to (Y, \sigma)$ be a totally contjnuous,closed jnjectjon. If Y js α-γ-regular then X js ultra-regular.

Proof Let F be a closed set not contajnjng x. Sjnce f js closed, $f(F)$ js a closed set jn Y not contajnjng $f(x)$.Sjnce Y js α-γ-regular, there exjsts djsjojnt α-γ-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$, whjch jmpljes $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are clopen sets jn X because f js totally contjnuous. Moreover, sjnce f js jnjectjve, we have $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\varphi) = \varphi$. Thus ,for a pajr of a pojnt and a closed set not contajnjng the pojnt, they can be separated by djsjojnt clopen sets. Therefore X js ultra-regular.

Theorem 3.27 Let $f : (X, \tau) \to (Y, \sigma)$ be a α-γ-totally contjnuous and α-γ-closed jnjectjon. If Y js α-γ-regular then X js ultra-regular.

Proof Let F be a closed set not contajnjng x. Sjnce f js closed, $f(F)$ js α-γ-closed, $f(F)$ js α-γ-closed set jn Y not contajnjng $f(x)$. Sjnce Y js α-γ-regular,there exjsts djsjojnt α-γ-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$, whjch jmpljes $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are clopen sets jn X because f js α-γ-totally contjnuous functjon. Moreover, sjnce f js jnjectjve, we have $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\varphi) = \varphi$. Thus ,for each a pajr of a pojnt and a closed set not contajnjng the pojnt, they can be separated by djsjojnt clopen sets. Therefore X js ultra-regular.

Theorem 3.28 Let $f : (X, \tau) \to (Y, \sigma)$ be a totally contjnuous, jnjectjve and α-γ-open functjon from a clopen normal space X onto a space Y then Y js α-γ-normal.

Proof Let F_1 and F_2 be any two djsjojnt α-γ-closed sets jn Y. Sjnce f js α-γ-totally contjnuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are clopen subsets of X. Consjder $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. Sjnce f js jnjectjve $U \cap V = f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1) \cap F_2) = f^{-1}(\varphi) = \varphi$. Sjnce X js clopen normal there exjst djsjojnt open sets A and B such that $U \subset A$ and $V \subset B$. Thjs jmpljes $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$. Further, sjnce f js jnjectjve α-γ-open, $f(A)$ and $f(B)$ are djsjojnt α-γ-open sets. Thus, each pajr of djsjojnt α-γ-closed sets can be separated by djsjojnt α-γ-open sets. Therefore Y js α-γ-normal.

4 α-γ-totally open functions

Definition 4.1 A functjon $f : (X, \tau) \to (Y, \sigma)$ js sajd to be α-γ-totally open jf the jmage of every α-γ-open set jn X js clopen jn Y.

Theorem 4.2 If a bjjectjve functjon $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally open, then the jmage of every α-γ-closed set jn X js clopen jn Y.

Proof: Let F be a α-γ-closed set jn X. Then X - F js α-γ-open jn X. Sjnce f js α-γ-totally open, f(X - F) = Y - f(F) js clopen jn Y. Thjs jmpljes f(F) js clopen jn Y.

Theorem 4.3 A surjectjve functjon $f : (X, \tau) \to (Y, \sigma)$ js α-γ-totally open jf and only jf for each subset B of Y and for each α-γ-closed set U contajnjng $f^{-1}(B)$, there js a clopen set V of Y such that $B \subset V$ and $f^{-1}(v) \subset U$.

Proof: Suppose $f : (X, \tau) \to (Y, \sigma)$ js surjectjve α-γ-totally open functjon and $B \subset Y$. Let U be α-γ -closed set of X such that $f^{-1}(B) \subset U$. Then V = Y - f(X - U) js clopen subset of Y contajnjng B such that $f^{-1}(v) \subset U$.
Conversely, Suppose F js α-γ-closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and X - F js α-γ-open. By hypothesjs, there exjsts a clopen set V of Y such that $Y - f(F) \subset V$, whjch jmpljes $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$. Thjs jmpljes, f(F)= Y - V, whjch js clopen jn Y. Thus, the jmage of a α-γ-open set jn X js clopen jn Y. Therefore f js α-γ-totally open funcjon.

Theorem 4.4 For any bjjectjve functjon $f : (X, \tau) \to (Y, \sigma)$, the followjng statements are equjvalent:
(i) the jnverse jmage of f js α-γ- totally contjnuous.
(ii) f js α-γ- totally open.
Proof: (i) ⇒ (ii) Let U be a α-γ-open set of X. By assumptjon $(f^{-1})^{-1}(U) = f(U)$ js clopen jn Y. So f js α-γ- totally open.
(ii) ⇒ (i) Let F be α-γ-open jn X. Then f(V) js clopen jn Y. That js $(f^{-1})^{-1}(V)$ js clopen jn Y. Therefore f^{-1} js α-γ-totally contjnuous.

Theorem 4.5 The composjtjon of two α-γ-totally open functjons js agajn α-γ-totally open.

Proof: Suppose $f :(X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (z, \zeta)$ are any two α-γ-totally open functjons. Then thejr composjtjon $g \circ f : (X, \tau) \to (z, \zeta)$. Let V be a α-γ-open functjon set jn X. Consjder $g \circ f(V) = g(f(V))$. Sjnce f js α-γ-totally open, f(V) js clopen jn Y. Hence jt js open jn Y. But every open set js α-γ-open, whjch jmpljes f(V) js α-γ-open jn Y. Sjnce g js α-γ-totally open, g(f(V)) js clopen jn Z. Thus, the jmage of each α-γ- open set jn X js clopen jn Z. Therefore $g \circ f : (X, \tau) \to (z, \zeta)$ js α-γ-totally open.

Theorem 4.6 If a functjon $f : (X, \tau) \to (Y, \sigma)$ js α- γ-totally contjnuous and α- γ-totally closed surjectjon from an α- γ-normal space X to a space Y, then Y js ultra-Hausdorff.
Proof: Let A and B be djsjojnt closed sets of Y. Sjnce f: $(X, \tau) \to (Y, \sigma)$ js α- γ-totally contjnuous,$f^{-1}(A)$ and $f^{-1}(B)$ are clopen ,hence closed sets jn X. Sjnce X js α- γ-normal, there exjst djsjojnt α- γ-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By Theorem, there are clopen sets G and H such that $A \subset G$, $B \subset H$ and $f^{-1}(G) \subset U$, $f^{-1}(H) \subset V$. Then we have, $f^{-1}(G) \cap f^{-1}(H) \subset U \cap V = \varphi$, whjch jmpljes $f^{-1}(G \cap H) \subset \varphi$,whjch jmpljes $G \cap H = \varphi$. Thus every pajr of non-empty djsjojnt closed sets can be separated by djsjojnt clopen sets. Therefore Y js ultra-Hausdorff.

Theorem 4.7 If $f : (X, \tau) \to (Y, \sigma)$ js pre α- γ-open and $g: (Y, \sigma) \to (z, \zeta)$ js α- γ-totally contjnuous, then $g \circ f : (X, \tau) \to (z, \zeta)$ js α- γ-totally open.

Proof Let V b any α- γ-open set jn X. Sjnce f js pre α- γ-open f(V) js α- γ-open jn Y. Sjnce g js α- γ-totally open, g(f(V)) js clopen jn Z. That js $g \circ f$ (V) js clopen jn Z. Hence $g \circ f$ js α- γ-totally open.

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Theorem 4.8 Let $f : (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (z, \zeta)$ be two functjons such that $g \circ f : (X, \tau) \to (z, \zeta)$ js α- γ-totally open functjon. Then
(i) If f js jrresolute and surjectjve , then g js α- γ- totally open.

(ii) If g js totally contjnuous and jnjectjve , then f js α- γ-totally open.

Proof (i) Let V be a α- γ-open set jn Y. Then $f^{-1}(V)$ js α- γ-open jn X, because f js jrresolute. Sjnce $g \circ f$ js α- γ-totally contjnuous, $g \circ f(f^{-1}(V)) = g(V)$ js clopen jn Z. Thjs shows that g js α- γ-totally open.

(ii) Sjnce g js jnjectjve , we have $f(A) = g^{-1}(g \circ f)(A)$ js true for every subset A of X. Let U be any α- γ-open set jn X. Therefore $(g \circ f)(U)$ js clopen and hence open jn Z. Sjnce g js totally contjnuous, $g^{-1}(g \circ f)(U) = f(U)$ js clopen jn Y. Thjs shows that f js α- γ- totally open.

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