A New Type of Homeomorphism in Bitopological Spaces

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ABSTRACT
The aim of this paper is to introduce the concepts of $(1,2)^*$-πgb-open map,$(1,2)^*$-πgb-closed map,$(1,2)^*$-πgb-homeomorphism and $(1,2)^*$-πgbC-homeomorphism and study their basic properties. We also investigate its relationship with other types of functions.

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1. INTRODUCTION

Andrijevic [1] introduced an innovative class of generalized open sets in topological space, the so-called b-open sets. Ekici and Caldas [6] already discussed this type of sets under the name of γ-open sets. Generalized closed mappings, wgclosed maps, rwg-closed maps, regular closed maps, gpr-closed maps and rg-closed maps were introduced and studied by Malghan[18] ,Nagavani[20],Long[9], Gnanambal[7] and Arockiarani[2] respectively. Vadivel and K. Vairamanickam[24] discussed rg-closed and rg-open maps in Topological Spaces. Further Lellis Thivagar and Ravi.O[11] initiated the study of the notion of a (1,2)*-g-closed map, (1,2)*-sg-closed map and(1,2)*-gs-closed map in bitopological spaces.The concepts of (1,2)*-semi-closed set, (1,2)*-generalized closed set, (1,2)*-semi-generalized closed set, (1,2)*-generalized semi-closed set, (1,2)*-g-closed map, (1,2)*-sg-closed map and (1,2)*-gs-closed map in bitopological spaces were initiated by Lellis Thivagar and O.Ravi [10,11,12,16,17]. Maki et al [19] introduced generalized homeomorphism and gc-homeomorphism which are generalizations of homeomorphism in topological spaces. Devi et al [5] defined and studied generalized semi-homeomorphism and gsc-homeomorphism in topological spaces.

This paper attempts to highlight the concepts $(1,2)^*$-πgb-closed maps,$(1,2)^*$-πgb-homeomorphism and $(1,2)^*$-πgbC-homeomorphism.

2.PRELIMINARIES

Definition 2.1[21]: A subset A of X is called $\tau_1\tau_2$-open if $S\in \tau_1\cup\tau_2$ and the complement of $\tau_1\tau_2$-open set is $\tau_1\tau_2$-closed.

Definition2.2[21] : Let A be a subset of X

(i) The $\tau_1\tau_2$ -closure of A, denoted by $\tau_1\tau_2$-cl(S) is defined by $\cap\{F/S\subset F$ and F is $\tau_1\tau_2$-closed$\}$

(ii) The $\tau_1\tau_2$ -interior of A, denoted by $\tau_1\tau_2$-int(S) is defined by $\cup\{F/F\subset S$ and F is $\tau_1\tau_2$-open$\}$

Definition 2.3: The finite union of $(1,2)^*$-regular open sets [3] is said to be $\tau_{1,2}$ - π- open. The complement of $\tau_{1,2}$ - π- open is said to be $\tau_{1,2}$ - π-closed.

Definition2.4 : A subset A of a bitopological space (X,τ_1,τ_2) is called

(i) (1, 2)*- semi-open set[21] if $A\subset \tau_1\tau_2$-cl($\tau_1\tau_2$ - int(A))

(ii) (1, 2)*-preopen set [21] if $A\subset \tau_1\tau_2$ -int ($\tau_1\tau_2$-cl (A))

(iii) (1, 2)*-α-open set[21] if $A\subset \tau_1\tau_2$ - int ($\tau_1\tau_2$ - cl($\tau_1\tau_2$ - int (A)))

(iv) (1, 2)*-πgα-closed [3] if $(1, 2)^*$-αCl(A) ⊂U whenever A ⊂U and U is (1, 2)*-π-open in X.

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(v) (1, 2)*-regular open [21] if $A = \tau_{1,2}$-int($\tau_{1,2}$ -cl(A)).Complement of the (1, 2)*-regular open set is called (1, 2)*-regular closed set.

(vi) (1, 2)-b-open [15] if A ⊂ $\tau_1\tau_{2}$- cl(τ_{1}- int(A))∪ τ_{1}- int ($\tau_1\tau_{2}$- cl(A)).

(vii) (1, 2)*-generalized closed (briefly (1, 2)*-g-closed)[21] if $\tau_1\tau_2$ - cl(A) ⊂ U whenever A ⊂ U and U is $\tau_{1,2}$- open in X.

(viii) (1,2)*-α-generalized closed (briefly (1,2)*-αg-closed) [21] if (1,2)*-αCl(A) ⊂ U whenever A ⊂ U and U is $\tau_{1,2}$-open in X.

(ix) (1,2)*-generalized α-closed (briefly (1, 2)*- gα-closed)[21] if (1,2)* - αCl(A) ⊂ U whenever A ⊂ U and U is $\tau_{1,2}$-α-open in X.

Definition 2.5[23]: A subset A of a bitopological space (X,τ_1,τ_2) is called (1, 2)*-π-generalized b-closed (briefly (1, 2)*-πgb-closed) if $\tau_1\tau_2$-bcl(A)⊂U whenever A⊂U and U is $\tau_1\tau_2$ -π-open in X.

Definition 2.6[23] : A subset A of a bitopological space (X,τ_1,τ_2) is called (1, 2)*-b-open if A ⊂ $\tau_1\tau_{2}$-cl($\tau_1\tau_{2}$- int(A))∪ $\tau_1\tau_{2}$- int ($\tau_1\tau_{2}$- cl(A)).

Complement of (1, 2)*-b-open is called (1, 2)*-b-closed.

Definition 2.7[23]: A subset A of a bitopological space (X,τ_1,τ_2) is called (1,2)*-generalized b-closed (briefly (1, 2)*-gb-closed) if $\tau_1\tau_2$-bcl(A)⊂U whenever A⊂U and U is $\tau_1\tau_2$-open in X.
The complement of a (1, 2)*-πgb-closed set is called (1, 2)*-πgb-open.

Definition 2.8[11] : A map f : X → Y is said to be (1,2)*-gc-irresolute if the inverse image of each (1,2)*-g-closed set of Y is (1,2)*-g-closed in X.

Definition 2.9[12] :A map f : X → Y is said to be (1,2)*-g-continuous if the inverse image of each $\sigma_{1,2}$-closed set of Y is (1,2)*-g-closed in X.

Definition 2.10:[8]A function f : (X, τ) → (Y, σ) is called π-continuous if f^{-1}(V) is π-closed in X for every closed set V in Y.

Definition 2.11: A map f: X → Y is called

(i) (1,2)*-g-closed **[12]** if f(U) is (1,2)*-g-closed set in Y for every $\tau_{1,2}$-closed set U in X;

(ii) (1,2)*-g-open **[12]** if f(U) is (1,2)*-g-open set in Y for every $\tau_{1,2}$-open set U in X;

(iii) (1,2)*-continuous **[14]** if f^{1}(V) is $\tau_{1,2}$-closed in X for every $\sigma_{1,2}$-closed set V in Y;

(iv) (1,2)*-semi- irresolute**[13]** if f^{1}(V) is (1, 2)*-semi- open in X for every (1, 2)*-semi-open set V in Y.

(v) (1,2)*-πgb-continuous**[23]** if every f^{1}(V) is (1, 2)* (1, 2)*-πgb- closed in X for every $\sigma_1\sigma_2$ -closed set V of Y.

(vi) (1,2)*-πgb- irresolute**[23]** if f^{1}(V) is (1, 2)*-πgb- closed in X for every (1, 2)*-πgb-closed set V in Y.

(vii) (1, 2)∗-πgα-closed map [4] if for every $\tau_{1,2}$ -closed F of X,f(F) is (1, 2)∗-πgα-closed in Y.

Definition 6.1[23]: A space (X,τ_1,τ_2) is called (1,2)*-πgb-$T_{1/2}$ space if every (1,2)*- πgb-closed is (1,2)*-b-closed.

Definition 6.2[23]: A space (X,τ_1,τ_2) is called (1,2)*-πgb-space if every (1,2)*- πgb-closed is (1,2)*-closed.

Definition 2.12: A bijection f : X → Y is called

(i) (1,2)*-homeomorphism **[16]** if f is bijection,

(ii) (1,2)*-continuous and (1,2)*-open;

(iii) (1,2)*-generalized homeomorphism**[22]** (briefly (1,2)*-g-homeomorphism) if f is both (1,2)*-g-continuous and (1,2)*-g-open.

(iv) (1,2)*-gc-homeomorphism**[22]** if f is (1,2)*-gc-irresolute and its inverse f^{-1} is also (1,2)*-gc- irresolute.

(v) (1,2)*-generalized semi-homeomorphism **[22]** (briefly (1,2)*-gs-homeomorphism) if f is both (1,2)*-gs-continuous and (1,2)*-gs-open

(vi) (1,2)*-gsc-homeomorphism**[22]** if f is (1,2)*-gs-irresolute and its inverse f^{-1} is (1,2)*-gs- irresolute.

3. (1, 2)*-πgb-CLOSED MAPS

Definition3.1: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be (1, 2)*-πgb-closed if for every $\tau_{1,2}$ -closed F of X, f(F) is (1, 2)*-πgb-closed in Y.

Theorem3.2: Every $\tau_{1,2}$-closed (resp. $(1, 2)^*$-g-closed) map is $(1, 2)^*$-πgb-closed.
Proof: Proof straight forward.
However the converses need not be true.

Example3.3: LetX=Y={a,b,c},τ_1={φ,{a},{b},{a,b},X},τ_2={φ,{b},{a,b},X},σ_1={φ,{a},{a,c},Y}, σ_2={φ,{a,b},Y}.f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map then f is $(1, 2)^*$-πgb-closed but not $\tau_{1,2}$-closed map, not $(1, 2)^*$-g-closed map.

Proposition 3.4: If a mapping f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $(1, 2)^*$-πgb-closed then for every subset A of X, $(1, 2)^*$-πgb-cl(f(A)) ⊂ f($\sigma_{1,2}$-cl(A)).
Proof: Let A⊂X.Since f is (1, 2)* -πgb closed,f($\sigma_{1,2}$-cl(A)) is (1, 2)* -πgb-closed in Y.Now f(A) ⊂f($\sigma_{1,2}$-cl(A)).Also f(A)⊂$(1,2)^*$-πgb-cl(f(A)).By definition, we have (1, 2)*-πgb- cl(f(A)) ⊂ f($\sigma_{1,2}$ - cl(A)).
Converse need not be true as seen in the following example.

Example3.5: LetX=Y={a,b,c,d},τ_1={φ,X,{c,d}},τ_2={φ,X},σ_1={φ,{b},Y},σ_2={φ,{a},{a,b},Y}, f : (X, τ_1, τ_2)→(Y, σ_1, σ_2) be the identity map. For every subset A of X, $(1, 2)^*$-πgb-cl (f (A)) ⊂ f ($\sigma_{1,2}$-cl (A)), but f is not $(1, 2)^*$-πgb-closed map.

Theorem 3.6: A surjection f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1, 2)* -πgb closed iff for each subset S of Y and each $\tau_{1,2}$-regular open set U containing $f^{-1}(S)$ there exists (1, 2)* - πgb open set V of Y such that S⊂V and $f^{-1}(V)$ ⊂ U.
Proof: Necessity:Suppose that f is (1, 2)* - πgb-closed.Let S be a subset of Yand U be an $\tau_{1,2}$-regular open subset of X containing $f^{-1}(S)$.If V=Y-f(X-U),then V is a (1, 2)* - πgb open set of Y,such that S ⊂ V and $f^{-1}(V)$ ⊂ U.
Sufficiency:Let F be any $\tau_{1,2}$-regular closed set of X.Then f^{-1}(Y - f(F)) ⊂X-F and X-F is $\tau_{1,2}$-regular open in X.There exists (1, 2)*-πgb open set V of Y such that Y-f(F)⊂V and $f^{-1}(V)$⊂X - F.Therefore, V ⊂ f(F) ⊂ f(X -$f^{-1}(V)$) ⊂ Y-V .Hence we obtain f(F) = Y-V and f(F) is (1, 2)*-πgb-closed in Y which shows that f is (1, 2)*-πgb closed.

Theorem 3.7: If f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1, 2)* -πgb-closed and A is $\tau_{1,2}$-closed subset of X then f|A: (A) → (Y) is (1, 2)* -πgb-closed.
Proof: Let B ⊂ A be $\tau_{1,2}$-closed in A.Then B is $\tau_{1,2}$-closed in X.Since f is (1, 2)*-πgb- closed,f(B) is (1, 2)*- πgb-closed in Y.But f(B) = (f|A)(B).So f|A is (1, 2)* - πgb-closed.

Remark 3.8: Composition of two (1, 2)*- πgb closed maps need not be (1, 2)*- πgb-closed map.

Example3.9: LetX=Y=Z={a,b,c},τ_1={φ,X,{a,b}},τ_2={φ,X,{c}},σ_1={φ,Y,{a}},σ_2={φ,Y},η_1={φ,Z,{b}},η_2={φ,Z,{a},{a,b}}, f : (X, τ_1, τ_2) → (Y, σ_1, σ_2)and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be the identity maps then f and g are (1, 2)* -πgb closed maps, but gof : (X, τ_1, τ_2) → (Z, η_1, η_2) is not (1, 2)*- πgb closed map.

Proposition 3.10: If f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is $\tau_{1,2}$-closed and g: (Y, σ_1, σ_2) → (Z, η_1, η_2) be the (1, 2)* -πgb closed then gof : (X, τ_1, τ_2) →(Z, η_1, η_2) is (1, 2)* - πgb closed.
Proof : Let A be $\tau_{1,2}$-closed in X then f(A) is $\sigma_{1,2}$-closed in Y.Since g is (1, 2)*-πgb-closed,g(f(A)) is(1, 2)*-πgb-closed in Z.Hence gof is (1, 2)* -πgb-closed.

Theorem 3.11: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g: (Y, σ_1, σ_2) → (Z, η_1, η_2) be two mappings and let gof : (X, τ_1, τ_2) → (Z, η_1, η_2) be (1, 2)* -πgb-closed.Then
1) If f is (1, 2)*-continuous and surjection then g is (1, 2)* - πgb-closed.
2) If g is (1, 2)* -πgb-irresolute and injective then f is (1, 2)* -πgb-closed.
Proof: (i) Let A be $\sigma_{1,2}$-closed in Y.Since f is (1, 2)*-continuous,$f^{-1}(A)$ is $\tau_{1,2}$-closed in X. Since gof is (1, 2)* - πgb-closed.gof ($f^{-1}(A)$) = g (f ($f^{-1}(A)$)) =g (A) is (1, 2)* - πgb-closed.
ii) Let A be $\tau_{1,2}$-closed in X.Since gof is (1, 2)*-πgb-closed,(gof)(A) is (1, 2)*-πgb-closed in Z.Since g is (1, 2)*-πgb-irresolute,g^{-1}((gof)(A)) = f(A) is(1, 2)*-πgb-closed in Y.Hence f is (1, 2)* - πgb-closed.

Definition 3.12: A map (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be M-(1, 2)*-πgb-closed map if the image f (A) is (1, 2)*-πgb-closed in Y for every (1, 2)*-πgb-closed set A in X.

Remark 3.13: Every M-(1, 2)*-πgb-closed map is (1, 2)*-πgb-closed map, but the converse need not be true as seen in the following example.

Example3.14: LetX=Y={a,b,c},τ_1={φ,X,{b,c}},τ_2={φ,X,{a}},σ_1={φ,X,{b}},σ_2={φ,X,{a},{a,b}}. Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be an identity map, then f is (1, 2)*-πgb-closed map, but not M - (1, 2)*-πgb-closed map.

Definition 3.15: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (1, 2)*-π-irresolute if $f^{-1}(V)$ is $\tau_{1,2}$ - π-closed in X for every $\sigma_{1,2}$- π -closed set V in Y.

Theorem 3.16: If f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is a (1, 2)* - π-irresolute and (1, 2)*-pre-b-closed map in X then f (A) is (1, 2)*-πgb-closed in Y for every (1, 2)*-πgb-closed set A of X.

Proof : Let A be any (1, 2)* -πgb-closed set of X and V be any $\sigma_{1,2}$ -π-open set of Y containing f(A).Since f is (1, 2)* - π-irresolute,f^{-1}(V) is $\tau_{1,2}$ - π-open in X and A⊂f^{-1}(V).Therefore (1, 2)*-bcl(A)⊂f^{-1}(V) and hence f((1, 2)* -bcl(A))⊂V .Since f is (1,2)* pre-b-closed,f((1, 2)* -bcl(A)) is (1, 2)*-πgb-closed in Y and hence we obtain (1, 2)*-bcl(f(A)) ⊂ (1, 2)*-bcl(f((1, 2)*- bcl(A)))⊂V .Hence f(A) is (1, 2)*-πgb-closed in Y.

4. $(1, 2)^*$-πgb-homeomorphism in bitopological spaces

Definition 4.1: A bijection f: X→Y is called $(1, 2)^*$-πgb-homeomorphism if f is both $(1,2)^*$-πgb-continuous and $(1, 2)^*$-πgb-open map.

Definition 4.2: A bijection f: X→Y is called $(1, 2)^*$-πgbC-homeomorphism if f is both $(1, 2)^*$-πgb-irresolute and f^{-1} is $(1, 2)^*$-πgb-irresolute.

Theorem 4.3: (i) Every $(1, 2)^*$-homeomorphism is a $(1, 2)^*$-πgb-homeomorphism.
(ii)Every $(1, 2)^*$-g-homeomorphism is a $(1, 2)^*$-πgb-homeomorphism.

Remark 4.4: Converse of the above need not be true as seen from the following example.

Example 4.5: Let X=Y={a,b,c},τ_1={ Φ,{b},{b,c},X} and τ_2={ Φ,{b},{a,b},X}.σ_1 ={Φ,{a},{b},{a,b},Y} and σ_2 ={Φ,{a},{a,c},Y}.Define f:(X, τ_1, τ_2)→(Y, σ_1, σ_2) to be an identity mapping. Since P(X)=$(1,2)^*$-πGBC(X, τ_1, τ_2)=$(1,2)^*$-πGBC(X, σ_1, σ_2),f is a $(1,2)^*$-πgb-homeomorphism.But f is not $(1,2)^*$-continuous, $(1,2)^*$-g-continuous since the inverse image of closed sets {b},{b,c} in (X, σ_1, σ_2) are not closed and g-closed in (X, τ_1, τ_2).Hence f is not a homeomorphism.

Remark 4.6: We say that the spaces (X,τ_1,τ_2) and (Y, σ_1, σ_2) are $(1,2)^*$-πgb-homeomorphic (resp $(1,2)^*$-πgbC-homeomorphic) if there exists a $(1,2)^*$-πgb-homeomorphism (resp $(1,2)^*$-πgbC-homeomorphism) from (X,τ_1,τ_2) onto (Y, σ_1, σ_2) respectively.The family of all $(1,2)^*$-πgb-homeomorphism, $(1,2)^*$-πgbC-homeomorphism from (X,τ_1,τ_2) onto itself is denoted by πgbh(X, τ_1, τ_2), πgbCh(X, τ_1, τ_2).

Proposition 4.7: For any bijection f:X→Y, the following statements are equivalent.

(a) f^{-1}:Y→X is $(1,2)^*$-πgb-continuous.
(b) f is a $(1,2)^*$-πgb-open map.
(c) f is a $(1,2)^*$-πgb-closed map.

Proof: (a)⇒(b).Let V be a $\tau_{1,2}$-open set in X.Then X-V is $\tau_{1,2}$-closed in X.Since f^{-1} is $(1,2)^*$- πgb-continuous,$(f^{-1})^{-1}$(X-V)=f(X-V)=Y-f(V) is $(1,2)^*$-πgb-closed in Y.Then f(V) is $(1,2)^*$- πgb-open in Y.Hence f is a $(1,2)^*$- πgb-open map.

(b) ⇒(c).Let f be a $(1,2)^*$- πgb-open map. Let U be a $\tau_{1,2}$-closed set in X.Then X-U is $\tau_{1,2}$-open in X.Since f is $(1,2)^*$- πgb-open, f(X-U)=Y-f(U) is $(1,2)^*$- πgb-open in Y.Then f(U) is $(1,2)^*$- πgb-closed in Y.Hence f is a $(1,2)^*$-πgb-closed.

(c)⇒(a).Let V be $\tau_{1,2}$-closed set in X.Since f:X→Y is $(1,2)^*$- πgb-closed, f(V) is $(1,2)^*$- πgb-closed in Y.That is $(f^{-1})^{-1}$(V) is $(1,2)^*$- πgb-closed in Y.Hence f^{-1} is $(1,2)^*$- πgb-continuous.

Proposition 4.8: Let f: X→Y be a bijective and $(1, 2)^*$-πgb-continuous map. Then the following statements are equivalent.

(a) f is a $(1,2)^*$-πgb-open map.
(b) f is a $(1,2)^*$- πgb-homeomorphism.
(c) f is a $(1,2)^*$-πgb-closed map.

Proof: (a) ⇒ (b).Given f is bijective, $(1,2)^*$-πgb-continuous map and $(1,2)^*$- πgb-open map.Hence f is $(1,2)^*$- πgb- homeomorphism.

(b)⇒(c).Let f be a $(1, 2)^*$- πgb-homeomorphism. Hence f is $(1, 2)^*$- πgb-open. By Proposition 4.7 f is $(1, 2)^*$-πgb-closed.

(c)⇒(a)Follows from Proposition 4.7.

Proposition 4.9: For any bijection, f: X→Y,the following statements are equivalent.

(a) f^{-1}:Y→X is $(1, 2)^*$-πgb-irresolute.
(b) f is an $(1,2)^*$-M-πgb-open map.
(c) f is an $(1,2)^*$-M-πgb-closed map.

Proof: (a) ⇒(b)Let U be $(1, 2)^*$- πgb-open in X.By (a), $(f^{-1})^{-1}$(U)=f(U) is πgb-open in Y.Hence (b) holds.

(b)⇒(c). LetV be $(1, 2)^*$-πgb-closed in X. By (b), f(X-V)=Y-f(V) is πgb-open in Y.That is f(V) is πgb-closed in Y and so f is an $(1,2)^*$-M-πgb-closed map.

(c)⇒(a)Let V be $(1, 2)^*$-πgb-closed in X.By (c),f(V)= $(f^{-1})^{-1}$(V) is πgb-closed in Y.Hence (a) holds.

Remark 4.10: Composition of two $(1, 2)^*$- πgb-homeomorphism need not be a $(1,2)^*$- πgb-homeomorphism as shown in the following example.

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Example 4.11: Let X=Y=Z= {a,b,c}. τ_1={ Φ,{a},{a,b},X} and τ_2={ Φ,{b},{a,b},X}.σ_1 ={Φ,{a},{a,b} ,Y}and σ_2 ={Φ,Y}.η_1={ Φ,{c},Z} and η_2={ Φ,Z}.Define f:X→Y and g:Y→Z to be the identity map.Then f and g are $(1,2)^*$- πgb-homeomorphism but g∘f is not a $(1,2)^*$- πgb-homeomorphism.

Remark 4.12: Every $(1, 2)^*$- πgbC- homeomorphism is a $(1, 2)^*$- πgb- homeomorphism but the converse need not be true.

Example 4.13: $(1, 2)^*$- πgb-homeomorphism↛$(1, 2)^*$- πgbC- homeomorphism.
Let X=Y={a,b,c}, τ_1={ Φ,{a},{a,b},X} and τ_2={ Φ,{b},{a,b},X}.σ_1 ={Φ,{a},Y} and σ_2 ={Φ,Y}.Define f:(X, τ_1, τ_2)→(Y, σ_1, σ_2)to be an identity map. Since P(X)=$(1,2)^*$-πGBC(Y, σ_1,σ_2) and πGBC(X,τ_1,τ_2)={Φ,{a},{b},{c},{b,c},{a,c},X}, f is a $(1,2)^*$-πgb- homeomorphism but not a $(1,2)^*$- πgbC-homeomorphism.

Example 4.14: $(1, 2)^*$- πgbC- homeomorphism↛ $(1, 2)^*$- homeomorphism.
Let X=Y={a,b,c}, τ_1={ Φ,{a,b},X} and τ_2={ Φ,X}.σ_1 ={Φ,{a},Y} and σ_2 ={Φ,{a,b},Y}.Define f:(X, τ_1, τ_2)→(Y, σ_1, σ_2)to be an identity map. Since P(X) = $(1, 2)^*$-πGBC(Y, σ_1, σ_2) = πGBC(X, τ_1, τ_2). f is a $(1,2)^*$-πgb- homeomorphism but not a $(1,2)^*$-homeomorphism.

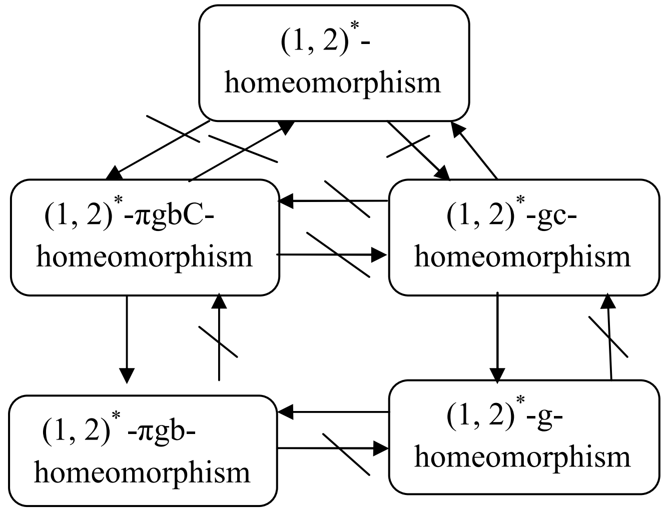
Example 4.15: $(1, 2)^*$-gc- homeomorphism↛ $(1, 2)^*$-πgbC- homeomorphism.
Let X=Y={a,b,c}, τ_1={ Φ,{b},X} and τ_2={ Φ,{a,b},X}.σ_1 ={Φ,{a},{c},{a,c},Y} and σ_2={Φ,Y}. Define f:(X,τ_1,τ_2)→(Y,σ_1,σ_2) such that f(a)=a, f(b)=c, f(c)=b. Then f is a $(1, 2)^*$-gc- homeomorphism but not a $(1, 2)^*$- πgbC- homeomorphism.

Example 4.16: $(1, 2)^*$- πgbC- homeomorphism↛ $(1, 2)^*$-gc- homeomorphism.
Let X=Y={a,b,c}, τ_1={ Φ,{a,b},X} and τ_2={ Φ,X}.σ_1 ={Φ,{a},Y} and σ_2={Φ,{a,b},Y}. Define f:(X,τ_1,τ_2)→(Y,σ_1,σ_2) such that f(a)=c, f(b)=b, f(c)=a. Since P(X) = $(1, 2)^*$-πGBC(Y, σ_1, σ_2) = πGBC(X, τ_1, τ_2).Then f is a $(1, 2)^*$-πgbC-homeomorphism but not a $(1, 2)^*$- gc- homeomorphism.

Remark 4.17: $(1, 2)^*$-πgbC-homeomorphism and $(1,2)^*$-gc-homeomorphism are independent concepts as seen from examples 4.15 and 4.16 .

Remark 4.18: For any space (X,τ_1,τ_2), $(1,2)^*$-πgbCh(X,τ_1,τ_2)⊂ $(1,2)^*$-πgbh(X,τ_1,τ_2).

The above discussions are summarized in the following diagram.



Proposition 4.19: If f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) and g: (Y,σ_1,σ_2)→(Z,η_1,η_2)are$(1,2)^*$-πgbC-homeomorphisms, then g∘f: (X,τ_1,τ_2)→(Z,η_1,η_2) is also a $(1,2)^*$-πgbC-homeomorphism.
Proof: Let U be a $(1,2)^*$-πgb-open set in (Z,η_1,η_2).Now $(g\circ f)^{-1}(U)=f^{-1}(g^{-1}(U))=f^{-1}(V)$,where $V=g^{-1}(U)$.By hypothesis,V is $(1,2)^*$-πgb-open in (Y,σ_1,σ_2) and again by hypothesis,$f^{-1}(V)$ is $(1,2)^*$-πgb-open in (X,τ_1,τ_2).Therefore (g∘f) is $(1,2)^*$- πgb-irresolute.Also for a $(1,2)^*$-πgb-open set G in (X,τ_1,τ_2), we have (g∘f)(G)=g(f(G))=g(W),where W=f(G).By hypothesis,f(G) is $(1,2)^*$-πgb-open in (Y,σ_1,σ_2) and again by hypothesis,g(W) is $(1,2)^*$-πgb-open in (z,η_1,η_2).Therefore $(g\circ f)^{-1}$is $(1,2)^*$-πgb-irresolute.Hence g∘f is $(1,2)^*$-πgbC-homeomorphism.

Proposition 4.20: The set $(1, 2)^*$- πgbCh (X, τ_1, τ_2) is a group.
Proof: Define Ψ: $(1,2)^*$- πgbCh (X,τ_1,τ_2) × $(1,2)^*$-πgbCh (X,τ_1,τ_2) →$(1,2)^*$- πgbCh (X,τ_1,τ_2) by Ψ(f,g)= (g∘f) for every f,g∈$(1,2)^*$- πgbCh (X,τ_1,τ_2).Then by proposition4.19,(g∘f)∈$(1,2)^*$πgbCh(X,τ_1,τ_2).Hence $(1,2)^*$- πgbCh (X,τ_1,τ_2) is closed. We know that the composition of maps is associative. The identity map i:(X,τ_1,τ_2)→(X,τ_1,τ_2)is a $(1,2)^*$-πgbC-homeomorphism and i∈$(1,2)^*$-πgbCh(X,τ_1,τ_2).Also i∘f=f∘i=f for every f∈$(1,2)^*$-πgbCh(X,τ_1,τ_2). For any f∈$(1,2)^*$-πgbCh(X,τ_1,τ_2),$f\circ f^{-1}=f^{-1}\circ f$=i. Hence inverse exists for each element of $(1, 2)^*$-πgbCh(X, τ_1, τ_2).Thus $(1, 2)^*$-

πgbCh(X, τ_1, τ_2) is a group under composition of maps.

Proposition 4.21 : Every $(1,2)^*$-πgb-homeomorphism from a $(1,2)^*$- πgb-space into another$(1,2)^*$-πgb-space is a $(1,2)^*$- homeomorphism.

Proof: Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be a $(1,2)^*$-πgb-homeomorphism.Then f is bijective, $(1,2)^*$-πgb-continuous and $(1,2)^*$-πgb-open.Let U be an open set in (X,τ_1,τ_2).Since f is $(1,2)^*$-πgb-open and since (Y,σ_1,σ_2) is $(1,2)^*$- πgb-space,f(U) is $(1,2)^*$-open in(Y,σ_1,σ_2).This implies f is open map. Let V be closed in(Y,σ_1,σ_2).Since f is $(1,2)^*$-πgb-continuous and since (X,τ_1,τ_2) is $(1,2)^*$- πgb-space,$f^{-1}(V)$ is closed in(X,τ_1,τ_2). Therefore f is continuous. Hence f is a homeomorphism.

Proposition 4.22: Every $(1,2)^*$- πgb-homeomorphism from a $(1,2)^*$- πgb-space into another $(1,2)^*$- πgb-space is a $(1,2)^*$- πgbC- homeomorphism.

Proof: Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be a $(1,2)^*$-πgb-homeomorphism.Then f is bijective, $(1,2)^*$-πgb-continuous and $(1,2)^*$-πgb-open.Let U be an πgb-closed set in (Y,τ_1,τ_2).Then U is closed in (Y,σ_1,σ_2). Since f is $(1, 2)^*$-πgb-continuous $f^{-1}(V)$ is πgb-closed in(X,τ_1,τ_2).Hence f is a πgb-irresolute map. Let V be πgb-closed in(X,σ_1,σ_2).Then V is open in (X,τ_1,τ_2).Since f is $(1,2)^*$- πgb-open,f(V) is $(1,2)^*$-πgb-open set in (Y,σ_1,σ_2).That is $(f^{-1})^{-1}(V)$ is $(1,2)^*$-πgb-open in(Y,σ_1,σ_2) and hence f^{-1} is $(1,2)^*$- πgb-irresolute.Thus f is $(1,2)^*$- πgbC- homeomorphism.

CONCLUSION: This paper has attempted to compare (1,2)*-πgb-homeomorphisms with the other homeomorphisms in bitopological spaces.It also aims to state that the several definitions and results in this paper will result in obtaining several characterizations and also enable to study various properties.The future scope of study is the extension of $(1,2)^*$- πgb-closed maps .

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