ISSN 2250-3153

A Technique for Constructing Even-order Magic Squares using Basic Latin Squares

Tomba I.

Department of Mathematics, Manipur University, Imphal, Manipur (INDIA)
tombairom@gmail.com

Abstract: Tomba (2012) introduced a technique for construction (n x n) magic squares (when n is odd) using basic Latin Squares by fixing the pivot element and arranging other elements in an orderly manner [10]. However, even-order magic squares can't be constructed using the same procedure because of duplication in diagonal elements. In this paper, a technique for constructing (n x n) magic squares (when n is doubly even) using basic Latin square is developed. Doubly even magic squares are made by fixing the column associated with the elements, adjacent to the pivot element and arranging in an orderly manner that generates a magic parametric constant (T, known as Tomba's constant) and sub-magic parametric constants (T_i) and finally derived by minor adjustment on the values of T_i s. The technique can provide weak magic squares for singly even cases. The construction is illustrated with suitable examples.
Key-words: Latin square (basic), singly even and doubly even magic square (normal), weak magic squares, magic parametric constant and sub-magic parametric constants
AMS Classification No: A- 05 and A-22

I. INTRODUCTION

Magic squares are practically important of the properties of equality in the sum of its rows, columns, diagonals. Latin squares and Greco-Latin squares are used in statistical research particularly in agricultural sciences and design of experiments whereas magic squares are used in puzzle games of cubes, pattern recognition and magic carpet constructions, magic square cipher in Cryptology etc.

II. LATIN SQUARES

In a Latin square, Latin letters are seen once in each row and column. In a Latin square, the sums of rows and columns are equal but not the sums of diagonals.

A basic (4 x 4) Latin square can be represented with Latin letters A, B,C and D as:

$$\begin{bmatrix} A & B & C & D \\ B & C & D & A \\ C & D & A & B \\ D & A & B & C \end{bmatrix} \text{ represented as } \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad [1]$$

Where, $\sum_i a_{ij} = \sum_j a_{ij}$ but $\sum_i d_{ij} \neq \sum_j d_{ij}$ with diagonal notation d_{ij}

2.1 Symmetric properties of Basic Latin Squares

Lemma-1: A basic (n x n) Latin square (n is odd) is symmetric and non-duplicated.

Let a (3 x 3) basic Latin square be

$$\begin{bmatrix} A & B & C \\ B & C & A \\ C & A & B \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Here $a_{ij} = a_{ji}$, for all i and j but $a_{11} = A \quad a_{22} = C \quad a_{33} = B$ [2]

The diagonal elements are not equal or repeated

$\Rightarrow$ non-duplicated

Lemma-2: A (n x n) basic Latin square (n is even) is symmetric but duplicated

Again, let a (4 x 4) basic Latin square be

$$\begin{bmatrix} A & B & C & D \\ B & C & D & A \\ C & D & A & B \\ D & A & B & C \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Clearly, $\{a_{ij}\} = \{a_{ji}\}$ for all i and $j \Rightarrow$ Basic Latin squares (of all orders) are symmetric.

but, $a_{11} = A,\ a_{22} = C, a_{33} = C\ a_{44} = C$ [3]

The diagonal elements are equal or repeated $\Rightarrow$ duplicated

Lemma-3: Conversely, a (n x n) square (n is odd), satisfying the symmetric and non duplication properties is a basic Latin square.

Lemma-4: In a basic Latin square (n is odd), one of the sum of diagonal is equal to the sum of rows or columns.

Lemma-5: In a basic Latin square (n is even for singly even or doubly even), the diagonal elements are equal or repeated $\Rightarrow$ duplicated

III. MAGIC SQUARES (normal)

An arrangement of non repeated integers ($n \geq 0$) in an array of equal rows and columns such that the sums of its rows, columns and diagonals are equal.

For a normal magic square, the following properties can be established

(i) Elements or numbers ($n \geq 0$) are consecutive and not repeated

(ii) Sums of the rows, columns and diagonals are equal to the magic sum, S

$$\Rightarrow \sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij} \; i, j = 1, 2, ..., n \qquad [4]$$

(iii) Equality property of the rows, columns and diagonals remain unaltered for rotations and reflections.

3.1: Odd order Magic Squares, derived from basic Latin Squares

For any n (n is odd), a magic square constructed using basic Latin square, the following theorem holds

Theorem: A (n x n) matrix $\{b_{ij}\}$, developed by using basic Latin square format when the pivot element is fixed and rearranging in an orderly manner represents a magic square of order n (n is odd). [For details; See IJSRP, May 2012: Tomba I.

3.2 Other Magic squares

There exists different (n x n) magic square not satisfying these properties. Examples of such magic squares, not satisfying the above properties are: magic squares (special or random, prime numbers etc.)

Examples: (i) MS (special) $\begin{bmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{bmatrix}$

(ii) MS (prime numbers) $\begin{bmatrix} 17 & 39 & 71 \\ 113 & 59 & 5 \\ 47 & 29 & 101 \end{bmatrix}$.

It satisfies; $\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$ however these magic squares are not normal because (i) the elements are repeated and non-consecutive and in (ii) the numbers (prime) are not repeated but non-consecutive.

3.3 Weak magic squares (normal)

A (n x n) array $\{b_{ij}\}$ with diagonal notation d_{ij} satisfying the properties

(i) Elements or numbers (n ≥ 0) are consecutive and not repeated

(ii) Sums of diagonals are equal to the magic sum, S $\Rightarrow$ $\sum_i d_{ij} = \sum_j d_{ij} \; i, j = 1, 2, ..., n$

(iii) Sums of the rows, columns are equal to the magic sum (S), except for some I and j

$$\Rightarrow \sum_i b_{ij} = \sum_j b_{ij} \; i, j = 1, 2, ..., n \qquad [6]$$

(iv) Equality property of the rows, columns and diagonals remain unaltered for rotations and reflections.

Then the matrix $\{b_{ij}\}$ satisfying the above properties can be treated as weak magic squares

Lemma 6: Magic squares are weak magic squares but the converse is not true.

Since the elements are consecutive, non-repeated and diagonal sums are equal in both magic squares and weak magic squares. Sums of the rows and columns are equal (except for some rows and columns) in case of weak magic square, whereas sums of rows and columns are equal for the magic squares, hence the lemma holds.

3.5 More Properties

(i) For the consecutive numbers (n ≥ 1), then magic sum, S $= \frac{1}{2}\{n(n^2+1)\}$

(ii) If the consecutive number (n ≥ 0), then it gives the lowest magic square.

(iii) If the consecutive number starts from s+1 where s ≥ 1, then S = $n\left\{ s + \frac{(n^2+1)}{2} \right\}$

(iv) The magic parametric constant, sub-magic parametric constants can be determined

3.6 Alternate Structures of $(n\,x\,n)$ magic squares

Let $\{a_{ij}\}$ be a magic square satisfying the properties (a) to (d). Equality in the sums of rows, columns and diagonals will remain unchanged for rotations and reflections Hence alternate structures of a magic square can be expressed (if the rotation is clockwise or anticlockwise for $(k\frac{\pi}{2}); \; k = \pm 1, \; \pm 2, \; ... \pm m$) as $\{a_{ij}(k)\}$ where $\{a_{ij}\} = \{a_{ij}(k)\}$ for all $i = 0, 4, 8, ...$

3.7 More properties on alternative structures

Infinite number of magic squares can be generated by multiplying or adding by a number p ≥ 1 to each element of the given magic square.

If the minimum element/number is 0, then $\{a_{ij}\}$ gives the lowest magic square

Sum of two magic squares in the same rotation/reflection gives a magic square

Sum of two magic squares in different rotation are not magic squares.

Product of two magic squares is not a magic square

Magic squares in the same rotation/reflection are additive

IV. METHODOLOGY

For constructing (n x n) magic square
using basic Latin squares when n is even

The technique of constructing doubly even magic square using basic Latin square principle can be expressed as follows:

Let the n^2 matrix $\{a_{ij}\}$; $i, j = 1, 2, ...n$ with the consecutive elements/ numbers be arranged in Basic Latin square format as;

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n-1} & a_{1n} \\ a_{22} & a_{23} & \ldots & a_{2n} & a_{21} \\ \ldots & & & & \\ a_{nn} & a_{n1} & \ldots & a_{nn-2} & a_{nn-1} \end{bmatrix}$$

Where, $\sum_i a_{ij} = S$ for all i, S = $\frac{1}{2}\{n(n^2+1)\}$ [7]

This condition will be true for all n (odd or even) due to basic Latin square property.

The pivot element is unique for n is odd but when n is even, it lies between two elements.

Since the pivot element is not fixed, we select the column, associated with the numbers adjacent to the pivot element and assign it as the diagonal elements and arrange the other elements in an orderly manner to get a new matrix $\{b_{ij}\}; i, j = 1,2,.....n$, satisfying the property of

$\sum_i d_{ij} = \sum_j d_{ij} = S$ for all i and j [8]

Again, retaining the diagonal elements unchanged, make symmetric transformations of $\{b_{ij}\}$to $\{b_{ji}\}$, to find out the extreme corner blocks and central block of (2 x 2) each.

Reverting rows and columns systematically, a magic parametric constant (T) and a set of sub magic parametric constants ($T_1\ T_2$...) can be determined.

By adjusting the values in the parametric constants and sub parametric constants, the required magic squares (even order) can be constructed as

$$\begin{bmatrix} b_{11} & b_{12} & b_{1j} & b_{1n} \\ b_{21} & b_{22} & b_{2j} & b_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ b_{n1} & b_{n2} & b_{nj} & b_{nn} \end{bmatrix}$$

Satisfying $\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$ [9]

$\Rightarrow \{b_{ij}\}; i, j = 1,2,...n$ is a n^2 (doubly-even) magic square.

4.2. Steps for constructing magic square (n is even)

Construction of magic square using basic Latin square is expressed in the following steps:

Step-1: First arrange the consecutive numbers ($a_{11}\ a_{12} ... a_{1n}$), ($a_{21}\ a_{22} ... a_{2n}$), … ($a_{n1}\ a_{n2} ... a_{nn}$) in basic Latin square format.

Step-2: Find the range of $\frac{1}{2}(a_{11} + a_{nn})$ and select the column associated with this range assign it as diagonal elements and arrange other elements in an orderly manner, giving diagonals sums equal $\sum_i d_{ij} = \sum_j d_{ij}$

Step-3: Retaining the diagonal elements unchanged, make transformations of $\{b_{ij}\}$to $\{b_{ji}\}$, to construct the extreme corner blocks of (2 x 2)..

Step-4: Reverting rows and columns in a systematic manner, a magic parametric constant (T) and a set of sub-magic parametric constants ($T_1\ T_2$ ...) can be determined.

The reversion process can be made as follows: Depending upon n

(i) n (singly even), $2 \le \{\frac{n}{2} \pm 2k\} \angle (n-2)$ for k = 0, 1, 2 ...n

(ii) n (doubly even), $2 \le \{\frac{n-2}{2} \pm 2k\} \angle (n-2)$ for k = 0, 1, 2 ....n

Step-5: Main adjustments should be made on the elements corresponding to the magic parametric constant (T), whereas minor adjustments should be made on other elements of sub-magic parametric constants to get the magic square $\{b_{ij}\}; i, j = 1,2,...n$ satisfying the properties of $\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$

The technique can be used for constructing doubly even magic squares. For singly even cases, it can provide weak magic squares only. Illustrations are shown as follows:

V. NUMERICAL EXAMPLES (doubly even cases)

5.1: To construct (4 x 4) magic square using basic Latin Square

Step-1: Let the numbers be (1 to 16) arranged in square format be

1	2	3	4
6	7	8	5
11	12	9	10
16	13	14	15

The arrangement gives the column totals equal, $\sum_i a_{ij} = 34$ for all i

Here, S = $\frac{1}{2}\{n(n^2+1)\}$= 34 for n = 4

and { $\frac{1}{2}(1+16)$ } lies between 8 and 9. [10]

Step-2: Select the row associated with these elements (say 3, 8, 9, 14) and assign it as diagonal elements.

Rearranging the other elements in an orderly manner to get a new (4 x 4) array $\{b_{ij}\}$ satisfying $\sum_i d_{ij} = \sum_j d_{ij}$ = 34

ISSN 2250-3153

15	11	7	3
16	12	8	4
3	9	5	1
4	10	6	2

Step-3: Retaining the diagonal elements un-changed, make transformations of $\{b_{ij}\}$ to $\{b_{ji}\}$ to generate the extreme corner blocks of (2 x 2).

15	6	10	3
1	12	8	13
4	9	5	16
14	7	11	2

The extreme corner blocks: $\begin{bmatrix} 15 & 6 \\ 1 & 12 \end{bmatrix} \begin{bmatrix} 10 & 3 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} 4 & 9 \\ 14 & 7 \end{bmatrix} \begin{bmatrix} 5 & 16 \\ 11 & 2 \end{bmatrix}$

No central block is created

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\} = 0$ for n = 4 and therefore no magic parametric constant is available.

Step-5: No minor adjustment needed and therefore the construction of (4 x 4) magic squares is completed in Step-3.

5.2 To construct (8 x 8) magic square

Step-1: Let the consecutive numbers be (1, 2, 3,... 64), arranged in 8 rows and 8 columns be rearranged in Latin Square format as:

1	2	3	4	5	6	7	8
10	11	12	13	14	15	16	9
19	20	21	22	23	24	17	18
28	29	30	31	32	25	26	27
37	38	39	40	33	34	35	36
46	47	48	41	42	43	44	45
55	56	49	50	51	52	53	54
64	57	58	59	60	61	62	63

Step-2: Select the row associated with the element (32 and 33) and assign this row as diagonal elements. Rearranging the other elements in an orderly manner to get a new matrix giving the diagonal sums equal satisfying

$$\sum_i d_{ij} = \sum_j d_{ij} = S \text{ where S = 260}$$

61	53	45	37	29	21	13	5
62	54	46	38	30	22	14	6
63	55	47	39	31	23	15	7
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
58	50	42	34	26	18	10	2
59	51	43	35	27	19	11	3
60	52	44	36	28	20	12	4

Step-4: Retaining the diagonal elements unchanged, make transformations of $\{b_{ij}\}$ to $\{b_{ji}\}$ to construct the extreme corner blocks and central block of (2 x 2) each

.

61	12	20	28	36	44	52	5
3	54	19	27	35	43	14	59
2	10	47	26	34	23	50	58
1	9	17	33	25	41	49	57
8	16	24	40	32	48	56	64
7	15	42	31	39	18	55	63
6	51	22	30	38	46	11	62
60	13	21	29	37	45	53	4

Extreme corner blocks: $\begin{bmatrix} 61 & 12 \\ 3 & 54 \end{bmatrix} \begin{bmatrix} 52 & 5 \\ 14 & 59 \end{bmatrix} \begin{bmatrix} 6 & 51 \\ 60 & 13 \end{bmatrix} \begin{bmatrix} 11 & 62 \\ 53 & 4 \end{bmatrix}$

Central block: $\begin{bmatrix} 33 & 25 \\ 40 & 32 \end{bmatrix}$

Step-5: Reverting the 3th column and 5th column and 3th row and 5th row, keeping the diagonal elements, extreme corner blocks and central block unchanged, a magic parametric constants (T) can be determined

61	12	21	28	37	44	52	5	**260**
3	54	22	27	38	43	14	59	**260**
58	50	47	26	39	23	10	2	**260**
1	8_{+1}	48	**40**	**32**	41_{-17}	49	56_{+1}	**275**
64	57_{-1}	17	**33**	**25**	24_{+17}	16	9_{-1}	**245**
7	15	42	31	34	18	55	63	**260**
6	51	19	30	35	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**236**	**260**	**260**	**284**	**260**	**260**	**260**

ISSN 2250-3153

Step-6: Here T = 65 (sum of the elements in 4th column and 5th column or 4th row and 5th row).

Sub-magic parametric constants are T_1 = 49 (3th column and 4th column), T_2 = 81 (5th column and 6th column), T_3 = 59 (3th row and 4th row), T_4 = 71 (5th row and 6th row).

Main adjustments on the values of T and then on the values of sub parametric constants

61	12	21	28	37	44	52	5	**260**
3	54	22	27_{+11}	38_{+-11}	43	14	59	**260**
58	50	47	26_{++5}	39	23	10	2	255_{++5}
1	9	48	**40**	**32**	24	49	57	**260**
64	56	17_{+24}	**33**	**25**	41_{-24}	16	8	**260**
7	15	42	31_{-5}	34	18	55	63	265_{-5}
6	51	19	30_{+5}	35_{+-5}	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**260**	244_{++16}	276_{+-16}	**260**	**260**	**260**	**260**

Secondary adjustments are made on the values of the sub-magic parametric constants as shown above and finally the necessary magic square of 8x8 is available as follows;

61	12	21	28	37	44	52	5	**260**
3	54	22	38	27	43	14	59	**260**
58	50	47	31	39	23	10	2	**260**
1	9	48	**40**	**32**	24	49	57	**260**
64	56	41	**33**	**25**	17	16	8	**260**
7	15	42	26	34	18	50	63	**260**
6	51	19	35	30	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**

Alternative approach

Step-5: Suppose the central block is assumed as $\begin{bmatrix} 32 & 40 \\ 25 & 33 \end{bmatrix}$

Reverting the 4th column and 5th column, keeping the diagonal elements and extreme corner blocks unchanged, sub-parametric constants derived are :
T_1 (3rd row + 4th row), T_2 (5th row + 6th row),
T_3 (3rd column + 4th column) and T_4 (5th column + 6th column).

Here, T_1 = 49 , T_2 = 81, T_3= 59, T_4 =71.
And the parametric constant can be determined indirectly as T = 65 (3rd row + crossed 5th row or 4th row + crossed 5th row or 3rd column +crossed 5th column or 4th column + crossed 5th column)

65 = (20 + 45), (19 + 46)....or (29+36), (30+35) .... or (2+63), (10+55)...... or (64+1), (56+9)......

61	12	20	29_{+8}	37_{-8}	44	52	5	**260**
3	54	19	30_{+8}	38_{-8}	43	14	59	**260**
2	10	47	31	39	23	50	58	**260**
57	49	41	32	40	17	9_{+7}	1_{+7}	246_{+14}
64	56	48	25	33	24	16_{-7}	8_{-7}	274_{-14}
7	15	42	26	34	18	55	63	**260**
6	51	22	27_{+8}	35_{-8}	46	11	62	**260**
60	13	21	28_{+8}	36_{-8}	45	53	4	**260**
260	**260**	**260**	228_{+32}	292_{-32}	**260**	**260**	**260**	**260**

Step-6: Since T_1 = 49, T_2 = 81, T_3 = 59, T_4 = 71

Minor adjustments are made on the values of these sub-magic parametric constants as shown above and finally the necessary magic square of (8 x 8) is available as follows;

61	12	20	37	29	44	52	5	**260**
3	54	19	38	30	43	14	59	**260**
2	10	47	31	39	23	50	58	**260**
57	49	41	32	40	17	16	8	**260**
64	56	48	25	33	24	9	1	**260**
7	15	42	26	34	18	55	63	**260**
6	51	22	35	27	46	11	62	**260**
60	13	21	36	28	45	53	4	**260**
260	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**

5.3 To construct a (12 X 12) magic square

Step-1: Let the consecutive numbers be (1, 2, 3,... 144), be arranged in 12 rows and 12 columns in basic Latin square format. Selecting the column associated with 72 and 73 as diagonal element, rearranging in an orderly manner, a new matrix is available which satisfies the property

$$\sum_i d_{ij} = \sum_j d_{ij} = S \text{ where, S = 870}$$

139	127	115	103	91	79	67	55	43	31	19	7
140	128	116	104	92	80	68	56	44	32	20	8
141	129	117	105	93	81	69	57	45	33	21	9
142	130	118	106	94	82	70	58	46	34	22	10
143	131	119	107	95	83	71	59	47	35	23	11
144	132	120	108	96	84	72	60	48	36	24	12
133	121	109	97	85	73	61	49	37	25	13	1
134	122	110	98	86	74	62	50	38	26	14	2
135	123	111	99	87	75	63	51	39	27	15	3
136	124	112	100	88	76	64	52	40	28	16	4
137	125	113	101	89	77	65	53	41	29	17	5
138	126	114	102	90	78	66	54	42	30	18	6

Step-2: Retaining the diagonal elements unchanged make symmetric transformations $\{b_{ij}\}$ to $\{b_{ji}\}$ to construct the extreme corner blocks and central block of (2 x 2) each.

Extreme corner blocks: $\begin{bmatrix} 139 & 18 \\ 5 & 128 \end{bmatrix}$ $\begin{bmatrix} 126 & 7 \\ 20 & 137 \end{bmatrix}$ $\begin{bmatrix} 8 & 125 \\ 138 & 10 \end{bmatrix}$ $\begin{bmatrix} 17 & 140 \\ 127 & 6 \end{bmatrix}$

Central block: $\begin{bmatrix} 33 & 25 \\ 40 & 32 \end{bmatrix}$

139	18	31	42	55	66	79	90	103	114	126	7
5	128	32	41	56	65	80	89	104	113	20	137
4	16	117	40	57	64	81	88	105	33	124	136
3	15	34	106	58	63	82	87	46	111	123	135
2	14	35	38	95	62	83	59	107	110	122	134
1	13	36	37	60	84	72	85	108	109	121	133
12	24	25	48	49	73	61	96	97	120	132	144
11	23	26	47	86	71	74	50	98	119	131	143
10	22	27	99	51	70	75	94	39	118	130	142
9	21	112	45	52	69	76	93	100	28	129	141
8	125	29	44	53	68	77	92	101	116	17	140
138	19	30	43	54	67	78	91	102	115	127	6

Step-3 Reverting 3rd, 5th, 7th and 9th columns, 3rd, 5th, 7th and 9th rows retaining the diagonal elements, extreme corner blocks and central block unchanged, magic parametric constant and sub-magic parametric constants can be determined:

139	18	31	42	55	66	79	90	103	114	126	7	870
5	128	32	41	56	65	80	89	104	113	20	137	870
136	124	117	105	88	81	64+5	57-5	40+5	33	16	4	865/870
3	15	34	106	58	63+19	82-19	87+7	46	111	123	135	863/870
134	122	110	107	95	74-3	71+3	59	38+9	35	14	2	861/870
1	13	25+84	48+60	60+36	84	72	96-36	108-60	109-84	121	133	870
144	132	120	97	85	73	61	49	37	36	24	12	870
11	23	26+1	47-9	86	62	83	50	98	119-1	131	143	879/870
142	130	118+1	99	94-7	75	70	51	39	27-1	22	10	877/870
9	21	112	45-5	52+5	76	69-5	93	100	28	129	141	875/870
8	125	29	44	53	68	77	92	101	116	17	140	870
138	19	30	43	54	67	78	91	102	115	127	6	870
870	870	784/ 870	824/ 870	836/ 870	854/ 870	886/ 870	904/ 870	916/ 870	956/ 870	870	870	870

Step-4: Main adjustments are made on the elements of T (corresponding to the central block) and secondary adjustments are made on other elements as:
Finally the required (12x12) magic square is obtained as :

139	18	31	42	55	66	79	90	103	114	126	7	**870**
5	128	32	41	56	65	80	89	104	113	20	137	**870**
136	124	117	105	88	81	69	52	45	33	16	4	**870**
3	15	34	106	58	82	63	94	46	111	123	135	**870**
134	122	110	107	95	71	74	59	47	35	14	2	**870**
1	13	109	108	96	84	72	60	48	25	121	133	**870**
144	132	120	97	85	73	61	49	37	36	24	12	**870**
11	23	27	38	86	62	83	50	98	118	131	143	**870**
142	130	119	99	87	75	70	51	39	26	22	10	**870**
9	21	112	40	57	76	64	93	100	28	129	141	**870**
8	125	29	44	53	68	77	92	101	116	17	140	**870**
138	19	30	43	54	67	78	91	102	115	127	6	**870**
870	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**	**870**

VI NUMERICAL EXAMPLES (Singly even cases)

For singly even cases, the technique generates weak magic squares only

6.1 To construct a (6 x 6) weak magic square

Step-1: Let the consecutive numbers be (1, 2, ...36) arranged in 6 rows and 6 columns be arranged in Latin square format as;

1	2	3	4	5	6
8	9	10	11	12	7
15	16	17	18	13	14
22	23	24	**19**	**20**	21
29	30	25	**26**	**27**	28
36	31	32	33	34	35

Satisfying $\sum_i a_{ij}$ = S for all i, where, S =111. Since, $\frac{1}{2}(1+16)$ lies between 18 and 19

Step-2: Select the row associated with the element (18, 19) as (4, 11, 18, 19, 26, 33) and assign this row as diagonal elements

Rearrange the other elements in an orderly manner to get a new matrix $\{b_{ij}\}$ to make the diagonal sums equal,

34	28	22	16	10	4
35	29	23	17	11	5
36	30	24	18	12	6
31	25	19	13	7	1
32	26	20	14	8	2
33	27	21	15	9	3

Step-3: Retaining the diagonal elements unchanged, make symmetric transformations of $\{b_{ij}\}$to $\{b_{ji}\}$, to construct the extreme corner and central blocks of (2 x 2) each.

The extreme corner blocks $\begin{bmatrix} 34 & 9 \\ 2 & 29 \end{bmatrix} \begin{bmatrix} 27 & 4 \\ 11 & 32 \end{bmatrix} \begin{bmatrix} 5 & 26 \\ 33 & 10 \end{bmatrix} \begin{bmatrix} 8 & 35 \\ 28 & 3 \end{bmatrix}$

And central block $\begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix}$

34	9	15	21	27	4
2	29	14	20	11	32
1	7	24	18	25	31
6	12	19	**13**	**30**	36
5	26	17	**23**	**8**	35
33	10	16	22	28	3

A magic parametric constant, T (3rd row + 4th row) or (3rd column + 4th column) (except for the central block of 2x2) is determined as T = 37.

Step-4: Reverting the 3rd row and 3rd column, retaining the diagonal elements and central block unchanged, a (6 x 6) matrix is available, where T = 37 (3rd row + 4th row or 3rd column + 4th column) as;

34	9	16	21	27	4	**111**
2	29	17	20_{-6}	11	32	**111**
31	25_{+5}	24	18	7	1	**106**
6	12	19	13	30_{-5}	36	**116**
5	26	14_{+6}	23	8	35	**111**
33	10	15	22	28	3	**111**
111	**111**	**105**	**117**	**111**	**111**	**111**

Here, T = 37 (36+1 or 30+7 or 15+22 or 16 +21 or 20+17 or 14+23, or 24+13 or 19+18)

34	9	16	21	27	4	**111**
2	29	17	14	11	32	**105**
31	30	24	18	7	1	**111**
6	12	19	13	25	36	**111**
5	26	20	23	8	35	**117**
33	10	15	22	28	3	**111**
111	**116**	**111**	**111**	**106**	**111**	**111**

Step-5: After simple adjustments in the value of the magic parametric constants, we get the weak magic square, generated from Latin Squares (taking the central block as $\begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix}$

Adjustment to make row and column sums equal will affect the sum of the diagonals or adjustment to make diagonal sums equal will affect the sum of the rows and columns, generating only weak magic squares.

Depending upon the central block, different forms of weak magic squares can be generated

(a) Central block $\begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix}$,

(i)

34	9	16	21	27	4	**111**
2	29	17	14	11	32	**105**
31	30	24	18	7	1	**111**
6	12	19	13	25	36	**111**
5	26	20	23	8	35	**116**
33	10	15	22	28	3	**111**
111	**116**	**111**	**111**	**106**	**111**	**111**

(ii)

34	9	16	21	27	4	**111**
2	29	23	14	11	32	**111**
31	30	24	18	7	1	**111**
6	12	19	13	25	36	**111**
5	26	20	17	8	35	**105**
33	10	15	22	28	3	**111**
111	**116**	**117**	**105**	**106**	**111**	**111**

(b) Central block $\begin{bmatrix} 13 & 19 \\ 18 & 24 \end{bmatrix}$

(i)

34	9	22	15	27	4	**111**
2	29	17	14	11	32	**105**
36	25	13	19	12	6	**111**
1	7	18	24	30	31	**111**
5	26	20	23	8	35	**117**
33	10	21	16	28	3	**111**
111	**106**	**111**	**111**	**116**	**111**	**111**

(ii)

34	9	22	15	27	4	**111**
2	29	23	14	11	32	**111**
36	25	13	19	12	6	**111**
1	7	18	24	30	31	**111**
5	26	20	17	8	35	**111**
33	10	21	16	28	3	**111**
111	**106**	**117**	**105**	**116**	**111**	**111**

(c) Central block $\begin{bmatrix} 18 & 13 \\ 24 & 19 \end{bmatrix}$

(i)

34	9	16	21	27	4	**111**
2	29	17	20	11	32	**111**
31	30	18	13	12	1	**105**
6	7	24	19	25	36	**117**
5	26	14	23	8	35	**111**
33	10	22	15	28	3	**111**
111	**111**	**111**	**111**	**111**	**111**	**111**

(II)

34	9	16	21	27	4	**111**
2	29	17	20	11	32	**111**
31	30	18	13	12	7	**111**
6	1	24	19	25	36	**111**
5	26	14	23	8	35	**111**
33	10	22	15	28	3	**111**
111	**117**	**111**	**111**	**111**	**105**	**111**

(d) Making rows and columns sums equal effects the diagonal sums (unequal)

34	9	16	21	27	4	**111**
2	29	17	20	11	32	**111**
31	30	24	13	12	1	**111**
6	7	18	19	25	36	**111**
5	26	14	23	8	35	**111**
33	10	22	15	28	3	**111**
111	**111**	**111**	**111**	**111**	**111**	**117**

Here, the diagonal sums are affected
It gives the diagonal sums as 105 and 117 (not equal)

VII. CONCLUSION

The technique can be used for finding magic squares from basic Latin Squares of any order ($n \geq 1$, for n is doubly even). The construction is done by fixing the column, associated with the adjacent elements to the pivot element and, assigning it as diagonal element and arranging other elements in an orderly manner with minor adjustments on the elements corresponding to the magic parametric constant and sub parametric constants.

However, for singly even cases, the technique can generate weak magic squares only and therefore separate techniques are to be applied further for making magic squares. For crypto-logical studies, magic squares and weak magic squares in different structures can be applied.

The application of magic squares, derived from Latin squares (when n is singly even) is expected to provide more security against using the actual magic squares, derived for any n (odd or doubly even).

VIII. REFERENCES

[1]. Abe, G.: Unsolved Problems on Magic Squares; *Disc. Math.* **127**, 3-13, 1994

[2] Barnard, F. A. P: Theory of Magic Squares and Cubes; *Memoirs Natl. Acad. Sci.* **4**, 209-270, 1888.

[3] Carl, B Boyer (Revised by Uta, C. Merzbach): A History of Mathematics, Revised Edition, 1998

[4] Gardner, M: Magic Squares and Cubes; Ch. 17 in *Time Travel and Other Mathematical Bewilderments.* New York: W. H. Freeman, pp 213-225, 1988

[5] Flannery, S. and Flannery, D.: In code: A Mathematical Journey, London's Profile Books, p16-24, 2000

[6] Heinz, H and Hendricks J. R.: Magic Squares Lexicor, Illustrated Self Published, 2001

[7] Hirayama, A. and Abe, G: *Researches in Magic Squares;* Osaka, Japan: Osaka Kyoikutosho, 1983.

[8]. McCranie, Judson: Magic Squares of All Orders, Mathematics Teacher, 674-678, 1988

[9] Pickover, C. A.: *The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures Across Dimensions.* Princeton, NJ: Princeton University Press, 2002

[10] Tomba I. A Technique for constructing Odd-order Magic Squares using Basic Latin Squares, International Journal of scientific and research Publications, Volume-2, Issue-5, May 2012 [ISSN 2250-3153]