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An Optimization Model to Generate a Finite Sequence Subject to the Given Conditions

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Abstract- In this paper we develop an optimization model to generate a finite sequence, where i th term of the sequence is the number of occurrence of i in that sequence. This model is formulated as a 0-1 Integer Programming Problem and is solved using Branch-and- Bound algorithm.

Index Terms- Integer Programming problem.

I. Introduction

Our objective is to generate a finite sequence by placing the integers which are less than or equal to a given integer, say n ($n \neq 1,2$ and 3), as a finite sequence, in which i th term, *i* = 0,1…n-1, of the sequence is the num of occurrence of i in that sequence. For an in-depth review of sequences refer [2],[3]and [4]. The optimization model presented here is used by Truls Flatberg [1] in his OPL tutorial.

The rest of the paper is organized as follows. The next section describes research elaborations and the subsequent sections present the results and appendix. Finally, we conclude the paper with our conclusion.

II. RESEARCH ELABORATIONS

Below we illustrate the formulation of the optimization model which is used to generate the sequence described above for an integer n:

Let a_{ij} be a 0-1 variable defined as below:

$$a_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ term of the sequence has the value } j \\ 0, & \text{otherwise} \end{cases}$$

The constraints of the 0-1 integer linear programming model can be given as below:

$$\sum_{j=0}^{n-1} a_{ij} = 1 \quad , i = 0,1,\ldots,n-1 \ldots\ldots\ldots\ldots 1$$

$$\sum_{t=0}^{n-1} a_{ti} - j \leq n(1 - a_{ij}), \qquad i, j = 0,1,\ldots,n-1 \ldots\ldots\ldots\ldots\ldots 2$$

$$\sum_{t=0}^{n-1} a_{ti} - j \geq n(1 - a_{ij}), \qquad i, j = 0,1,\ldots,n-1 \ldots\ldots\ldots\ldots. 3$$

$$a_{ij} \in \{0,1\} \ldots\ldots\ldots\ldots. 4$$

The second and the third constraints given above ensure that $a_{ij} = 1$ implies that $\sum_{t=0}^{n-1} a_{ti} = j$.

Since we are interested in finding a feasible solution to the mathematical model, the objective function can be any arbitrary function. Therefore, in our study we take the objective function as:

Minimization 0.

A computer programme using C++ programming language is written to generate the constraints of the 0-1 Integer Linear Programming problem. This computer programme is capable of generating the constraints for any integer value of *n*. The C++ code can be found in the Appendix.

III. RESULT

The following tables exhibit some generated sequences obtained for certain values of n by solving the mathematical model developed above:

Identify the constructs of a Journal – Essentially a journal consists of five major sections. The number of pages may vary depending upon the topic of research work but generally comprises up to 5 to 7 pages. These are:

Table I: *n* = 4

Term	0	1	2	3
Value	2	0	2	0

Table II: *n* = 5

Term	0	1	2	3	4
Value	2	1	2	0	0

Table III: *n* =10

Term	0	1	2	3	4	5	6	7	8	9
Value	6	2	1	0	0	0	1	0	0	0

Table IV: *n* =20

Term	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Value	16	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

IV. Conclusion

In this paper we presented a mathematical method to generate finite sequences subject to the conditions mentioned above. We conclude that the model failed to generate sequences when *n* takes the values 1,2 and 3. Also, we conclude that as *n* gets larger the computational time increases to generate the sequences.

APPENDIX

This appendix presents the C++ code that is used to generate the constraints of the 0-1 Integer Linear Programming problem.

```
#include<iostream.h>
#include <conio.h>
#include<fstream.h>
const int n=20;
void main()
{
clrscr();
ofstream testFile1("C:\\Param\\sequence.txt");
cout<<"model:"<<"\n";
testFile1<<"model:"<<"\n";

for (int i=0;i<n;i++)
{
for (int j=0;j<n;j++)
{
cout <<"x"<<i<<"_"<<j;
testFile1<<"x"<<i<<"_"<<j;
if (j!=n-1)
{
cout<< "+";
testFile1<<"+";
}
}
cout<<"=1;"<<"\n";
testFile1<<"=1;"<<"\n";
}
cout<<"\n";
testFile1<<"\n";


for (int p=0;p<n;p++)
{
for (int q=0;q<n;q++)
         {
for (int k=0;k<n;k++)
                  {
                  cout <<"x"<<k<<"_"<<p;
                  testFile1<<"x"<<k<<"_"<<p;
                           if (k!=n-1)
{
cout<< "+";
testFile1<<"+";
}


                  }
cout<<"-"<<q<<"<="<<n<<"*(1-x"<<p<<"_"<<q<<");";
         testFile1<<"-"<<q<<"<="<<n<<"*(1-
x"<<p<<"_"<<q<<");";
         cout<<"\n";
         testFile1<<"\n";
         }
cout<<"\n";
testFile1<<"\n";
}

for (int s=0;s<n;s++)
{
for (int t=0;t<n;t++)
         {
for (int w=0;w<n;w++)
```

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```
                    {
                    cout <<"x"<<w<<"_"<<s;
                    testFile1<<"x"<<w<<"_"<<s;
                              if (w!=n-1)
{
cout<< "+";
testFile1<<"+";
}

                    }
          cout<<"-"<<t<<">="<<n<<"*(x"<<s<<"_"<<t<<"-1);";
          testFile1<<"-"<<t<<">="<<n<<"*(x"<<s<<"_"<<t<<"-
1);";
          cout<<"\n";
          testFile1<<"\n";
          }
cout<<"\n";
testFile1<<"\n";
  }


for (int e=0;e<n;e++)
{
for (int f=0;f<n;f++)
{
cout<<"@bin(x"<<e<<"_"<<f<<");" ;
testFile1<<"@bin(x"<<e<<"_"<<f<<");";
cout<<"\n";
testFile1<<"\n";
}
cout<<"\n";
testFile1<<"\n";
}
cout<<"end";
testFile1<<"end";
getch();
}
```

REFERENCES

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