Derivation Of Results Centered On Range Of Appearance Of Two Disparate Sequences

[1]V. Pandichelvi and [2]S. Saranya

[1]Assistant Professor, PG & Research Department of Mathematics, Urumu Dhanalakshmi College, Trichy.
(Affiliated to Bharathidasan University).
[2] Research Scholar, PG & Research Department of Mathematics, Urumu Dhanalakshmi College, Trichy.
(Affiliated to Bharathidasan University).

DOI: 10.29322/IJSRP.12.12.2022.p13243
http://dx.doi.org/10.29322/IJSRP.12.12.2022.p13243

Paper Received Date: 5th November 2022
Paper Acceptance Date: 5th December 2022
Paper Publication Date: 20th December 2022

Abstract- In this paper, the range of appearance of totality of two familiar sequences involving Icosagonal number and square pyramidal number so-called Icospyramidal result number and addition of the range of appearance of the above cited sequences separately are assessed. Furthermore, some results centered on range of appearance of those numbers are perceived and confirmed by separate python programs.

Index Terms- Polygonal number, divisibility, $p-$ adic range, Range of appearance.

I. Introduction

"Polygonal number is a number denoted as dots or pebbles organized in the form of a systematic polygon. It is a two dimensional figurate number. An Icosagonal number is of the form $9i^2-8i$ and it is a twenty sided polygon. Pyramid number or square pyramidal number $\frac{j(j+1)(2j+1)}{6}$ shows three dimensional figurate number" [1, 4, 5]. In [2] deals the modulus condition used in Fibonacci number. In [3] author shows the order of Fibonacci and Lucas number.[6-10] display the order of appearance in different forms like a product, upper bound and Diophantine equation involving of Fibonacci number. In this communication, the range of appearance of sum of two peculiar sequences concerning Icosagonal number and square pyramidal number termed as Icospyramidal number and sum of the range of appearance of the already quoted sequences distinctly are evaluated. Also, few results based on range of appearance of those numbers are presented and verified by python programs.

II. RANGE OF APPEARANCE IN ICOSPYRAMIIDAL NUMBER

The r^{th} term of sequence of Icosagonal number is taken as

$$\mathfrak{s}_r = 9r^2 - 8r.$$

The r^{th} term of sequence of Square pyramidal number is consider as

$$\mathfrak{m}_r = \frac{r(r+1)(2r+1)}{6}$$

The r^{th} term of the novel sequence named as Icospyramidal sequence received by adding like terms in both of the above mentioned sequences is denoted by

$$\mathfrak{f}_r = \mathfrak{s}_r + \mathfrak{m}_r = \frac{2r^3+57r^2-47r}{6}$$

Then, the sequence of Icospyramidal number is given by $2,25,71,142,240,367,525,716,942,1205,1507, \ldots ..$

"The range of appearance of a positive integer $\mathfrak{f}_r$ in the sequence of Icospyramidal number is denoted by $\mathfrak{u}(\mathfrak{f}_r)$ and is defined by the least positive integer c such that $\mathfrak{f}_r|c$ "[10].

Few range of appearance of Icospyramidal number are enlisted in table 2.1

ISSN 2250-3153

Table 2.1

f_r	1	2	3	4	5	6	7	8	9	10	11	12
$I(f_r))$	1	1	5	5	2	5	7	5	16	5	11	5
f_r	13	14	15	16	17	18	19	20	21	22	23	24
$I(f_r))$	13	21	5	5	15	16	19	5	7	33	23	5
f_r	25	26	27	28	29	30	31	32	33	34	35	36
$I(f_r))$	2	13	16	21	22	5	31	37	77	16	7	16
f_r	37	38	39	40	41	42	43	44	45	46	47	48
$I(f_r))$	12	57	52	5	41	77	13	77	27	69	42	5
f_r	49	50	51	52	53	54	55	56	57	58	59	60
$I(f_r))$	49	12	16	13	53	16	22	21	95	29	27	5
f_r	61	62	63	64	65	66	67	68	69	70	71	72
$I(f_r))$	61	93	70	101	52	77	67	16	23	77	3	16
f_r	73	74	75	76	77	78	79	80	81	82	83	84
$I(f_r))$	25	12	7	133	77	52	79	5	97	41	83	77
f_r	85	86	87	88	89	90	91	92	93	94	95	96
$I(f_r))$	15	13	167	165	89	77	91	69	124	89	57	293
f_r	97	98	99	100								
$I(f_r))$	57	49	77	32								

The range of appearance in Icospyramidal number can be estimated with the assistance of the following Python program 1

Program 1

```
i = int(input("Enter r value: "))
t = []
for k in range(1, i + 1) :
  a = (9 * (k ** 2) - 8 * k)
  b = (k * (k + 1) * ((2 * k) + 1))/6
  a = a + b
  t.append(int(a))
print(t)
r = int(input("Enter r value: "))
arr = list(range(1, r + 1))
print("fr: ", arr)
z = []
for j in range(r) :
  v = []
  for k in range(i):
    if (t[k]%arr[j]) == 0:
       v.append(k + 1)
  if not v:
```

```
    z.append(" * ")
  else:
    z.append(min(v))
print("i(fr): ", z)
```

Result 2.1

Let r be a positive integer of the form $\mathfrak{f}_r = P^t$ where P is a prime number

i. If $P = 2$ then $I(2^t) \leq \frac{5}{3}(2^t)$ for $t \geq 1$.

ii. If $P = 3$ then $I(3^t) \leq \frac{5}{2}(3^t)$ for $t \geq 1$.

iii. If $P = 5$ then $I(5^t) \leq (5^t)$ for $t \geq 1$.

iv. If $P > 5$ then $I(P^t) \leq \left(3P - \frac{2}{P}\right) P^{t-1}$, for $t \geq 1$.

This result can be verified by the Python program 2.

Program 2

```
i = int(input("Enter r value: "))
m = [ ]
for k in range(1, i + 1) :
  a = (9 * (k ** 2) − 8 * k)
  b = (k * (k + 1) * ((2 * k) + 1))/6
  a = a + b
  m.append(int(a))
print(m)
t = int(input("Enter t value: "))
t_arr = []
for j in range(1, t + 1):
  t_arr.append(2 ** j)
print("fr: ", t_arr)
z = [ ]
for j in range(t):
   v = []
   for l in range(i):
      if (m[l]%t_arr[j]) == 0:
         v.append(l + 1)
   if not v:
      z.append(" * ")
   else:
      z.append(min(v))
print("i(fr): ", z)
```

Result 2.2

Let x be any integer and $d_P(x)$,$\wedge(x)$ are p-adic and prime conjunction function where $\mathfrak{S}_x = \begin{cases} 0 \; if \; x \nmid 5 \\ 1 \; if \; x|5 \end{cases}$

i. If $d_P(x) = 1$,then $I(\mathfrak{f}_r) \leq \begin{cases} \frac{5}{2}\, x \,, if \;\wedge(x) = 2 \; and \; x \nmid 5 \\ 2x \;, if \;\wedge(x) = 2 \; and \; x|5 \\ 4(\frac{2}{3})^{\wedge(x)-\mathfrak{S}_x-2} x \,, if \;\wedge(x) > 2 \end{cases}$

ii. If $d_P(x) = 2$,then $I(\mathfrak{f}_r) \leq \begin{cases} \frac{7}{2}\, x \,, if \;\wedge(x) = 2 \; and \; x \nmid 5 \\ 2x \;, if \;\wedge(x) = 2 \; and \; x|5 \\ 3(\frac{5}{2})^{\wedge(x)-\mathfrak{S}_x-2} x \,, if \;\wedge(x) > 2 \end{cases}$

iii. If $d_P(x) = 3$,then $I(\mathfrak{f}_r) \leq \begin{cases} \frac{5}{2}\ x\ , if\ \ \wedge(x) = 2\ and\ x \nmid 5 \\ 3x\ \ , if\ \ \wedge(x) = 2\ and\ x|5 \\ 5(\frac{1}{2})^{\wedge(x)-\mathfrak{S}_x-2}x\ , if\ \ \wedge(x) > 2 \end{cases}$

This result can be substantiated by the Python program 3.

Program 3

```
i = int(input("Enter r value: "))
m = [ ]
for k in range(1, i + 1) :
  a = (9 * (k ** 2) − 8 * k)
  b = (k * (k + 1) * ((2 * k) + 1))/6
  a = a + b
  m.append(int(a))
print(m)
p = int(input("Enter p value: "))
n = 2 * p
print("fr: ", n)
z = [ ]
v = []
for j in range(i):
  if (m[j]%n) == 0:
    v.append(j + 1)
if not v:
  z.append(" * ")
else:
  z.append(min(v))
print("i(fr): ", z)
```

III. SUM OF RANGE OF APPEARANCE OF ICOSAGONAL AND SQUARE PYRAMIDAL NUMBERS

Let $\mathfrak{u}(\mathfrak{s}_r)$ and $\mathfrak{u}(\mathfrak{m}_r)$ be the range of appearance of Icosagonal and square pyramidal number respectively then $\mathfrak{u}(\mathfrak{t}_r) = \mathfrak{u}(\mathfrak{s}_r) + \mathfrak{u}(\mathfrak{m}_r)$ is the sum of range of appearance in Icosgonal and square pyramidal number.

Some numerical values of range of appearance of Icosagonal numbers are listed in table 3.1.

Table 3.1

$\mathfrak{s}_r$	1	2	3	4	5	6	7	8	9	10	11	12
$I(\mathfrak{s}_r)$	1	2	3	2	2	6	4	4	9	2	7	6
$\mathfrak{s}_r$	13	14	15	16	17	18	19	20	21	22	23	24
$I(\mathfrak{s}_r)$	11	4	12	4	16	18	3	2	18	18	6	12
$\mathfrak{s}_r$	25	26	27	21	29	30	31	32	33	34	35	36
$I(\mathfrak{s}_r)$	12	24	27	4	17	12	25	8	18	16	7	18
$\mathfrak{s}_r$	37	38	39	40	41	42	43	44	45	46	47	48
$I(\mathfrak{s}_r)$	5	22	24	12	10	18	27	6	27	12	39	12
$\mathfrak{s}_r$	49	50	51	52	53	54	55	56	57	58	59	60
$I(\mathfrak{s}_r)$	39	12	33	24	48	54	7	4	3	46	14	12
$\mathfrak{s}_r$	61	62	63	64	65	66	67	68	69	70	71	72
$I(\mathfrak{s}_r)$	28	56	18	8	37	18	53	16	6	32	64	36

ISSN 2250-3153

$\mathfrak{s}_r$	73	74	75	76	77	78	79	80	81	82	83	84
$I(\mathfrak{s}_r)$	9	42	12	22	7	24	36	12	81	10	47	18
$\mathfrak{s}_r$	85	86	87	88	89	90	91	92	93	94	95	96
$I(\mathfrak{s}_r)$	17	20	75	40	80	72	11	6	87	74	22	24
$\mathfrak{s}_r$	97	98	99	100								
$I(\mathfrak{s}_r)$	44	88	18	12								

Few range of appearance of square pyramidal numbers are scheduled in table 3.2.

Table 3.2

$\mathfrak{m}_r$	1	2	3	4	5	6	7	8	9	10	11	12
$I(\mathfrak{m}_r)$	1	3	4	7	2	4	3	15	12	4	5	8
$\mathfrak{m}_r$	13	14	15	16	17	18	19	20	21	22	23	24
$I(\mathfrak{m}_r)$	6	3	4	31	8	27	9	7	13	11	11	31
$\mathfrak{m}_r$	25	26	27	28	29	30	31	32	33	34	35	36
$I(\mathfrak{m}_r)$	12	12	40	7	14	4	15	63	22	8	7	40
$\mathfrak{m}_r$	37	38	39	40	41	42	43	44	45	46	47	48
$I(\mathfrak{m}_r)$	18	19	13	15	20	27	21	16	27	11	23	31
$\mathfrak{m}_r$	49	50	51	52	53	54	55	56	57	58	59	60
$I(\mathfrak{m}_r)$	24	12	8	32	26	40	5	31	9	28	29	40
$\mathfrak{m}_r$	61	62	63	64	65	66	67	68	69	70	71	72
$I(\mathfrak{m}_r)$	30	15	13	127	12	27	33	8	22	7	35	80
$\mathfrak{m}_r$	73	74	75	76	77	78	79	80	81	82	83	84
$I(\mathfrak{m}_r)$	36	36	49	47	10	71	39	32	121	20	41	31
$\mathfrak{m}_r$	85	86	87	88	89	90	91	92	93	94	95	96
$I(\mathfrak{m}_r)$	17	43	58	16	44	27	6	23	31	23	9	63
$\mathfrak{m}_r$	97	98	99	100								
$I(\mathfrak{m}_r)$	48	24	27	24								

The succeeding Python program 4 ensured the values of $\mathfrak{u}(\mathfrak{s}_r)$ and $\mathfrak{u}(\mathfrak{m}_r)$.

Program 4

```
i = int(input("Enter r value: "))
t = []
p = []
for k in range(1, i + 1):
  a = (9 * (k ** 2) - 8 * k)
  t.append(int(a))
print(t)
j = int(input("Enter r value: "))
for k in range(1, j + 1):
  b = (k * (k + 1) * ((2 * k) + 1))/6
  p.append(int(b))
```

```
print(p)
r = int(input("Enter sr value: "))
arr = list(range(1, r + 1))
print("sr: ", arr)
z = [ ]
for y in range(r):
  v = []
  for k in range(i):
    if (t[k]%arr[y]) == 0:
       v.append(k + 1)
  if not v:
    z.append(" * ")
  else:
    z.append(min(v))
print("i(sr): ", z)
m = int(input("Enter mr value: "))
m_arr = list(range(1, m + 1))
print("mr: ", m_arr)
z = [ ]
for y in range(m):
  v = []
  for k in range(i):
    if (p[k]%m_arr[y]) == 0:
       v.append(k + 1)
  if not v:
    z.append(" * ")
  else:
    z.append(min(v))
print("i(mr): ", z)
```

Result 3.1

Let x be a positive integer of the form $x = P^t$, where P is a prime number

i. If $P = 2$ then $I(2^t) \leq \frac{5}{2}(2^t)$ for $t \geq 1$.

ii. If $P = 3$ then $I(3^t) \leq \frac{7}{3}(3^t)$ for $t \geq 1$.

iii. If $P = 5$ then $I(5^t) \leq \frac{5}{3}(5^t)$ for $t \geq 1$.

iv. If $P > 5$ then $I(P^t) \leq 5\left(P - \frac{6}{P}\right)P^{t-1}$, for $t \geq 1$.

This result can be verified by Python program 5

Program 5

```
i = int(input("Enter r value: "))
m = [ ]
p = [ ]
for k in range(1, i + 1):
  a = (9 * (k ** 2) - 8 * k)
  m.append(int(a))
print(m)
j = int(input("Enter r value: "))
for k in range(1, j + 1):
  b = (k * (k + 1) * ((2 * k) + 1))/6
  p.append(int(b))
```

```
print(p)
t = int(input("Enter t value: "))
t_arr = []
for u in range(1, t + 1):
  t_arr.append(2 ** u)
print("sr: ", t_arr)
z = [ ]
for e in range(t):
   v = []
   for l in range(i):
      if (m[l]%t_arr[e]) == 0:
         v.append(l + 1)
   if not v:
      z.append(" * ")
   else:
      z.append(min(v))
print("i(sr): ", z)
print("mr: ", t_arr)
x = [ ]
for e in range(t):
   v = []
   for l in range(j):
      if (p[l]%t_arr[e]) == 0:
         v.append(l + 1)
   if not v:
      x.append(" * ")
   else:
      x.append(min(v))
print("i(mr): ", x)
```

Result 3.2

Let x be any integer and $d_P(x)$,$\wedge(x)$ are p-adic and omega function where $\mathfrak{S}_x = \begin{cases} 0 \ if \ x \nmid 5 \\ 1 \ if \ x|5 \end{cases}$

i. If $d_P(x) = 1$,then $I(\mathrm{t}_r) \leq \begin{cases} \frac{7}{2}x \ , if \ \wedge(x) = 2 \ and \ x \nmid 5 \\ 3x \ , if \ \wedge(x) = 2 \ and \ x|5 \\ 4(\frac{1}{2})^{\wedge(x)-\mathfrak{S}_x-2}x \ , if \ \wedge(x) > 2 \end{cases}$

ii. If $d_P(x) = 2$,then $I(\mathrm{t}_r) \leq \begin{cases} \frac{9}{2}x \ , if \ \wedge(x) = 2 \ and \ x \nmid 5 \\ 3x \ , if \ \wedge(x) = 2 \ and \ x|5 \\ 4(\frac{3}{2})^{\not{g}(x)-\mathfrak{S}_x-2}x \ , if \ \wedge(x) > 2 \end{cases}$

iii. If $d_P(x) = 3$,then $I(\mathrm{t}_r) \leq \begin{cases} \frac{9}{2}x \ , if \ \wedge(x) = 2 \ and \ x \nmid 5 \\ 3x \ , if \ \wedge(x) = 2 \ and \ x|5 \\ 4(\frac{5}{2})^{\wedge(x)-\mathfrak{S}_x-2}x \ , if \ \wedge(x) > 2 \end{cases}$

This result can be confirmed by the Python program 6.

Program 6

```
i = int(input("Enter r value: "))
m = [ ]
```

```
x = []
for k in range(1, i + 1):
  a = (9 * (k ** 2) - 8 * k)
  m.append(int(a))
print(m)
j = int(input("Enter r value: "))
for k in range(1, j + 1):
  b = (k * (k + 1) * ((2 * k) + 1))/6
  x.append(int(b))
print(x)
p = int(input("Enter p value: "))
n = 2 * p
print("sr: ", n)
z = []
v = []
for e in range(i):
  if (m[e]%n) == 0:
    v.append(e + 1)
if not v:
  z.append(" * ")
else:
  z.append(min(v))
print("i(sr): ", z)
print("mr: ", n)
q = []
y = []
for e in range(j):
  if (x[e]%n) == 0:
    y.append(e + 1)
if not y:
  q.append(" * ")
else:
  q.append(min(y))
print("i(mr): ", q)
```

IV. CONCLUSION

In this article, few results using the concept of range of appearance of number patterns are evaluated. All the derived results are verified by various python programs. In a similar way, one can scrutinize numerous results by modifying the definition of range of appearance of different kinds of numbers.

REFERENCES

[1] Mordell, L.J., “Diophantine Equations”, Academic press, London (1969).
[2] E. Jacobson. "Distribution of the Fibonacci Numbers Mod 2^k" , the Fibonacci Quarterly 30(3),1992,211-15.
[3] T.Lengyel,“The order of the Fibonacci and Lucas numbers” Fibonacci Q.33,234–239, 1995.
[4] Ivan Niven, Hibert S, Zuckerman and Huge L, Montgomery, “An Introduction to the Theory of Numbers”, Fifth Edn., John Wiley & Sons Inc., 2004.
[5] Coppel, William A. Number Theory: “An introduction to mathematics” Springer Science & Business Media, 2009.
[6] Marques D, “Fixed points of the order of appearance in the Fibonacci sequence”. Fibonacci Q. 50, 2012, 346–352.
[7] Marques D, “The order of appearance of product of consecutive Fibonacci numbers”. Fibonacci Q. 50, 2012, 132–139.
[8] Marques D, “Sharper upper bounds for the order of appearance in the Fibonacci sequence”. Fibonacci Q. 51”, 2013, 233–238.
[9] Khaochim, Narissara, and Prapanpong Pongsriiam. "On the order of appearance of products of Fibonacci numbers." Contributions to Discrete Mathematics 13.2 (2018): 45-62.
[10] Eva Trojovs, “On the Diophantine Equation $z(n) = \left(2 - \frac{1}{k}\right) n$ involving the order of appearance in the Fibonacci Sequence”, Mathematics 2020,1-8.

AUTHORS

First Author – V. Pandichelvi, Assistant Professor, PG & Research Department of Mathematics, Urumu Dhanalakshmi College, Trichy., (Affiliated to Bharathidasan University)., E-mail: mvpmahesh2017@gmail.com
Second Author – S. Saranya, Research Scholar, PG & Research Department of Mathematics, Urumu Dhanalakshmi College, Trichy.
(Affiliated to Bharathidasan University)., E-mail:srsaranya1995@gmail.com