Two Phase Immiscible Fluids Flow Through a Porous Media: Viscosity and Drag Dependent on Pressure

Flomena Jeptanui[1], Jacob Bitok[2], Titus Rotich[3]

[1] Phd. student, Department of Mathematics and Computer Science, School of science, University of Eldoret, Eldoret, Kenya
[2] Department of Mathematics and Computer Science, School of Science University of Eldoret, Eldoret,

DOI: 10.29322/IJSRP.11.12.2021.p12068
http://dx.doi.org/10.29322/IJSRP.11.12.2021.p12068

Abstract- The groundwater remains a major source of water supply throughout the ages and groundwater quality is becoming water source problem. This problem is driven by contaminant transport equation involving advection and dispersion flux. Contaminant flow through a porous media meets tortuosity effects viscosity and drag coefficient must be considered. The study considers leaked petroleum from a transporting pipeline considered here as contaminant, enters the ground with pressure and infiltrate into groundwater reservoirs, hence contaminating aquifers. We let the viscosity and drag to depend on pressure. Petroleum and its product are extremely harmful to the natural environment and are considered carcinogenic. Numerical finite volume method is used to solve the modified momentum equation. Our aim is to determine the petroleum mobility and determination of horizontal distant for exploitation of uncontaminated groundwater after a given time. The temporal discretization employs the second order Crank-Nicolson scheme for diffusive phenomena and a second order explicit Adams-Bashforth scheme for advective terms. The obtained results indicated that the fluid released with pressure does not increase the volumetric flow rate in fact it decreases the flow rate by a certain percent.

Index Terms- Viscosity, drag, porous media and Finite volume method

I. Introduction

Groundwater plays crucial role as a source of uncontaminated drinking water and other human usage activities for millions of both urban and rural dwellers. Compared to surface water, groundwater is considered less susceptible to contamination and pollution.

Petroleum resource globally is considered the main source of energy that has been in use over centuries. It is an essential commodity that could be regarded as the backbone of today's global economy (2,3). The global ballooning population growth exerts high demand on energy and this has posed a threat to the natural environment when mishandled. Petroleum on transit through aged pipelines is prone to leakages due to corrosion. Once the petroleum has leaked on the ground surface, it penetrates deep into the ground and finally reaching the aquifers. Petroleum forms a layer on top of water due it insolubility. The water-insoluble components are the main source of contaminant that can infiltrate as far as underground water table. These in turn threatened fauna, flora and underground water reservoir.

According to study carried out by (1), benzene presents both in crude oil and gasoline has been identified as a causative agent of leukemia in humans. Its lowers white blood cells leading to immune suppression in human. Health studies conducted in oil-exposed communities in the Niger Delta region of Nigeria have reported an increased incidence of headaches, diarrhea, dizziness, anemia and respiratory problems (2). Chronic exposure either by drinking water contaminated with petroleum may also lead to degenerative changes in liver, kidney and spleen. The human activities have heavily affected the natural fields, the most prominent adverse effect being the soil and ground water contamination with petroleum products. A large number of physical, chemical and microbial processes and properties of the media affect contaminant transport in the soil and the groundwater. Most of the groundwater contaminants are reactive in nature and they infiltrate through the vadoze zone, reach the water table and continue to percolate in the direction of groundwater flow. Therefore, it is essential to understand the mobility process of contaminants through the subsurface porous media.

II. Petroleum Substances in Soil and Water Environment

The presences of petroleum remnant in soil and water results from two main sources namely point and linear source. The point source is attributed to the release of contaminant from a specific location: leaking underground fuel tanks. The linear source is where the contaminant emit from a source that may be along route where damaged vehicles or trucks transporting petroleum or cracked/corroded pipelines.

Contamination of water by oil products has negative impact on soil and water. It has toxic effect, mutagenic and carcinogenic and degradation of soil and aquifers. The constant deposition of petroleum product resulting from leaking pipelines has resulted to high accumulation of this substance in soil and water environment. To monitor the rate of petroleum product leakage to groundwater, mathematical approach is used to determine the mobility of petroleum contaminant when viscosity and drag friction is dependent on pressure at every stratum in the study domain.

2.1 Viscosity dependent on pressure

Darcy (1856) proposed a model to study the movement of fluids through porous media, which has been considered to this century, the most popular model. However, the model proposed by Darcy does not consider the fluids flow through porous media when high-pressure gradients. Several researchers have come up with different ideas. The argument of Forchheimer (7) is that the inertial effects cannot be neglected if the Reynolds number exceeds a value about 10. Modifications have been made to the Darcy's model in order to capture the inertial effects of the fluid and the deformation of the solid matrix**.** Brinkman (6) in his studies captured the importance of viscous stress, which has been neglected in Darcy model. As a result, his model may be extended to Darcy-Brinkman-Forchheimer model by including suggested corrections.

The application of fluid flow through porous media with high pressures and pressure gradients in the flow domain is recently of interest. A few examples of such applications are the problems of enhanced oil recovery and carbon dioxide sequestration, wherein the fluid is subject to high pressures and pressure gradients. The Darcy-brinkman model inadequately accounts for such problems and the effects of viscosity of the fluid and the drag coefficient depended on the pressure should be accounted for, though they both can vary by orders of magnitude in the flow field. Experimental studies carried out by difference researchers have shown that the viscosity of a fluid depends on the pressure and this dependence can be very significant, for example exponential, (4,5). This leads to conclude that the drag coefficient would also depend on the pressure in the fluid. (8) studied the flow of a fluid within a porous solid under pressure gradient and they found out that the fluid flow through the porous solid achieved an asymptotic value as the driving pressure gradient increased unlike the predictions of the classical Darcy model that the flux has to increase linearly. Although Darcy's model accounts for the effect of drag, but ignoring the effect of viscosity within the fluid, that is, it ignores the viscous dissipation within the fluid. The study by (9) generalize the brinkman model by accounting for the viscous effects within the fluid flow and letting the fluid viscosity and drag coefficients to depend on the fluid flow pressure. Although they restricted their study to steady flows, and in agreement with the results of (8), they found an increase in volumetric flux. We are interested in the problems of flow of petroleum, and since the flow in these problems are inherently unsteady and the fluid leaks from a pipeline at high pressures, in this paper we allow for the possibility that the flow is unsteady and that the viscosity and drag are dependent on the pressure, but we assume that the solid is homogeneous and isotropic. Our aim is to determine whether the leaked petroleum under high pressure would enhance the volumetric flux, a phenomenon that is observed in fluid flows in pipes. To our knowledge, this is the first study wherein the unsteady flow of a fluid with viscosity dependent on pressure through a porous solid is carried out.

2.2 Momentum Balance Equation

We now consider the constant leakage of petroleum from a pipeline in a porous medium of low permeability initially saturated with water, which is a case of importance in petroleum engineering. We expect the fluid-solid interface to play a much important role in flow rate in this case, especially when fluid phases flow in distinct pore networks and momentum exchange between the two fluid phases is negligible.

Let us consider the momentum conservation equation

$$\rho\left(\frac{\partial u}{\partial t}+u.\nabla u\right)=-\nabla \mathrm{p}+\mu\nabla^2 u+F \tag{1}$$

The first term on the left hand side of equation (1) is the force per unit fluid volume due to unsteady acceleration. The second term on the left hand side describes the force per unit fluid volume due to convective acceleration of fluid particles. Convection refers to the internal current changes within fluids. The right hand side of equation (1) is the sum of body forces on a fluid volume. The first term represents forces due to pressure gradients in the fluid, the second term is the viscous force due to shear stresses and the third term is the effect of acceleration of gravity as mentioned. Since the petroleum jets out from the pipe with high pressure, the flow will be slowed by friction of the fluid with the solid phase. Based on the experimental investigations done by Barus (5) it showed that viscosity of the fluid depends on the pressure and suggested an exponential dependence of the viscosity on the pressure as

$$\mu(P)=\mu_0 e^{\beta P} \qquad \mu_0>0, \quad \beta\geq 0 \tag{2}$$

Where μ_0 and β are constants, with μ_0 having the dimension of viscosity and β the dimension that is the inverse of that of the pressure. In some of the research done, it has been shown that in most liquids the variation in the viscosity can be of the order of 10^8 (4). Rajagopal (8) realized that the fluid densities variation with variation in pressure was insignificant, and hence we can assume that the liquid is incompressible while the viscosity varies exponentially with the pressure.

Without ignoring the frictional effects in fluids and the assumption that the viscosity of the fluid depends on the pressure, the Cauchy stress **T** in the fluids is given by

$$\mathbf{T} = -p1 + 2\mu(p)R \quad \textbf{(3)}$$

Where p1 is the indeterminate part of the stress due to the constraint of incompressibility μ (p) is the viscosity dependent on pressure and R denotes the symmetric part of the velocity gradient that is

$$R = \frac{1}{2}\left[\left(\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^{T}\right)\right] \quad (4)$$

The balance of linear momentum for the fluid is reduce to

$$\rho\left(\frac{\partial u}{\partial t} + u.\nabla u\right) = -\frac{\partial P}{\partial x} + \mathrm{div}[\mathbf{T}] + F \quad (5)$$

2.3 A Simple Model of F for Immiscible flow in a Porous Media

The variable F in equation (1) above represent the solid-fluid interaction commonly described as the frictional resistance at the solid pores on the flowing fluid. In this study we assumed a rigid porous media, hence ignoring its balance of linear momentum. Here, we are only concerned with the balance of linear momentum for the fluid. Generally, there could be different interaction mechanisms between the flowing fluid and the porous media: drag (resulting from relative velocity between the solid and the fluid), virtual mass effect, magnus effect, basset forces, faxen forces, density gradient effects, temperature gradient effects, etc. We shall assume that the only interaction mechanism is that due to drag and that F is given by

$$F = -\sigma(P)u \qquad (6)$$

The “drag coefficient" is related to the viscosity μ through $\sigma = \frac{\mu}{K}$ where k is the coefficient of permeability and hence the expression (2) leads to a drag coefficient" $\sigma(P)$ of the form

$$\sigma(P)u = \sigma_0 e^{\beta P}, \qquad \sigma \geq 0 \qquad (7)$$

It follows from (2)-(8) the appropriate governing equation becomes

$$\rho_i \left(\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + \mathrm{u}.\nabla \mathrm{u}\right)_i = \nabla P_i + (\mathrm{div}[2\mu(\mathrm{P})\mathrm{R}])_i - (\sigma(\mathrm{P})\mathrm{u})_i \qquad (8)$$

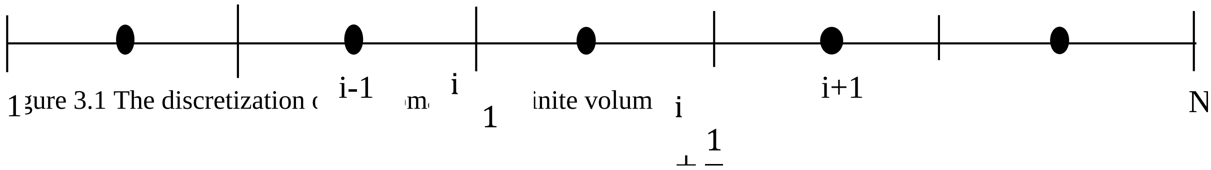
III. Numerical method

3.1 Finite Volume Discretization(FVM).

The solution of the partial differential equation (1) will be obtained numerically by employing the finite volume method. Adams-Bashforth Differencing was chosen to obtain the solution equation (3). The solution domain consists of a space and time domain. The time discretisation required for fluid flow problems is broken into a set of equal time steps written as Δt.

Discretization of space requires the subdivision of the domain into a number of discrete volumes as shown in figure 3.1. Each volume will have a representative value located at the centre. For one dimension, the cells are numbered from 1 to N. The cell point will be located at i and having neighbors i-1 and i+1 and the interfaces with those neighboring volume being $i - \frac{1}{2}$ and $i + \frac{1}{2}$. The boundaries (or faces) of control volumes are positioned mid-way between adjacent nodes. Thus each node is surrounded by a control volume or cell. It is common practice to set up control volumes near the edge of the domain in such a way that the physical boundaries coincide with the control volume boundaries.

We will illustrate the position of the control volume using 1D as shown in figures 3.1 and 3.2



gure 3.1 The discretization (m. inite volum

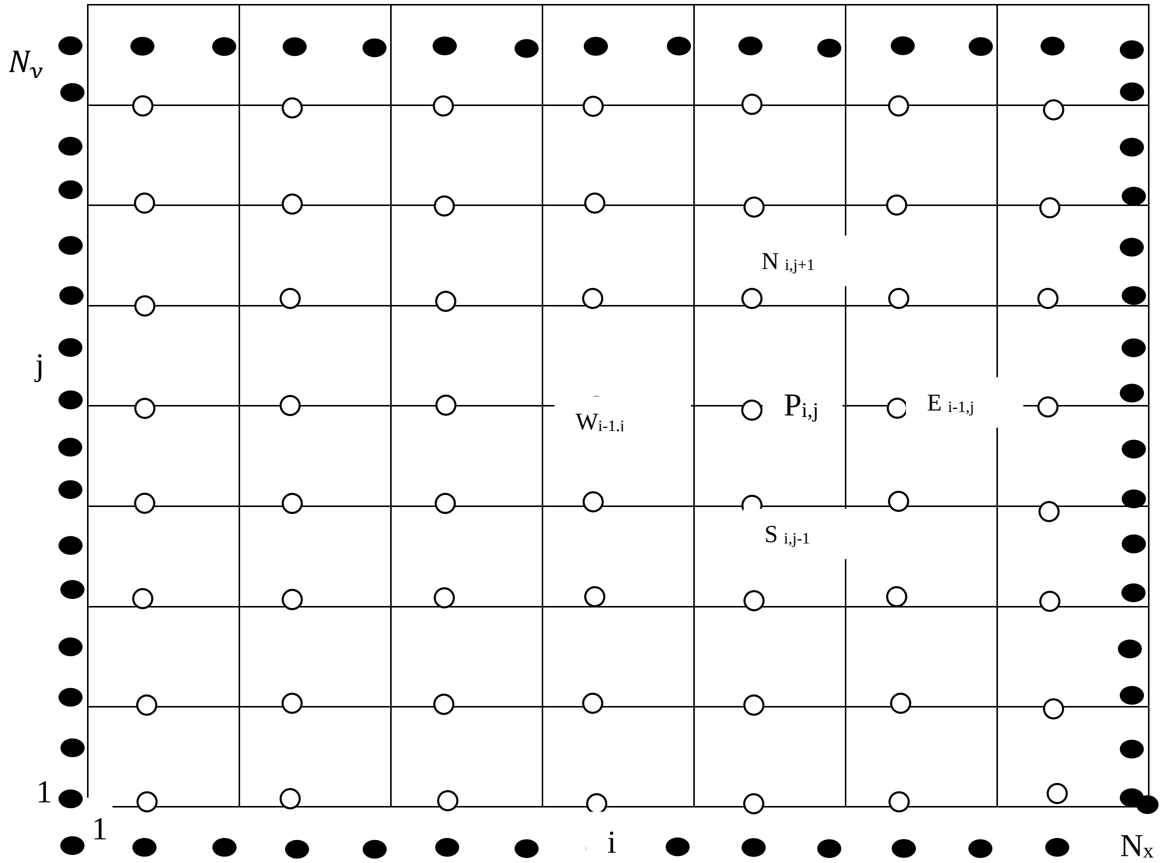


Figure 3.2 The discretization of 2D domain into Cartesian finite volume

3.2 Momentum equation

The momentum equation (8) given by

$$\rho\left(\frac{\partial u}{\partial t}+u.\nabla u\right)=-\frac{\partial P}{\partial x}+\operatorname{div}[2\mu(P)R]-\sigma(P)u$$

Taking integral on both sides to obtained

$$\int_V \frac{\partial \rho u}{\partial t}dV+\int_{\partial V}(\rho uu).nds=-\int_V \nabla p\,dV+\int_V \nabla(2\mu(P)R)dV-\int_V (\sigma(P)u)\,dV \quad (9)$$

The discretisation of the advective term in the momentum equation after applying Adams-Bashforth differencing scheme becomes

$$\rho u_x\left(\frac{3}{4}(u_P^n+u_E^n)-(u_P^n+u_W^n)-\frac{1}{4}((u_P^{n-1}+u_E^{n-1})-(u_P^{n-1}+u_W^{n-1})\right)$$
$$+\rho u_z\left(\frac{3}{4}(u_P^n+u_N^n)-(u_P^n+u_S^n)-\frac{1}{4}((u_P^{n-1}+u_N^{n-1})+(u_P^{n-1}+u_S^{n-1})\right) \quad (10)$$

While the pressure equation using implicit CNS

$$\int_V \nabla p\,dV\approx\frac{1}{4}(p_E^{n+1}-p_W^{n+1}+p_N^{n+1}-p_S^{n+1}+p_E^n-p_W^n+p_N^n-p_S^n) \quad (11)$$

$$\int_V \nabla(2\mu(P)R)dV\approx\mu\;\; R^{n+1}(p_E^{n+1}+p_W^{n+1}-4p_P^{n+1}+p_N^{n+1}+p_S^{n+1})+\;\;\mu\;\; R^n(p_E^n+p_W^n+p_N^n+p_S^n-4p_P^n) \quad (12)$$

$$\int_V (\sigma(P)u)\approx\sigma u^{n+1}(p_E^{n+1}+p_W^{n+1}-4p_P^{n+1}+p_N^{n+1}+p_S^{n+1})+\sigma u^n(p_E^n+p_W^n-4p_P^n+p_N^n+p_S^n) \quad (13)$$

The discretized equation (10),(11),(12) and (13) are used in equation (9) to obtain the discretized momentum equation

$$\Rightarrow \rho u_x\left(\frac{3}{4}(u_P^n + u_E^n) - (u_P^n + u_W^n) - \frac{1}{4}\left((u_P^{n-1} + u_E^{n-1}) - (u_P^{n-1} + u_W^{n-1})\right)\right) + \rho u_z\left(\frac{3}{4}(u_P^n + u_N^n) - (u_P^n + u_S^n) - \frac{1}{4}\left((u_P^{n-1} + u_N^{n-1}) - (u_P^{n-1} + u_S^{n-1})\right)\right)$$
$$= -\frac{1}{4}(p_E^{n+1} - p_W^{n+1} + p_N^{n+1} - p_S^{n+1} + p_E^n - p_W^n + p_N^n - p_S^n) + \mu^{n+1}R^{n+1}(p_E^{n+1} + p_W^{n+1} - 4p_P^{n+1} + p_N^{n+1} + p_S^{n+1}) + \mu^n R^n(p_E^n + p_W^n - 4p_P^n + p_N^n + p_S^n) - \left(\sigma u^{n+1}(p_E^{n+1} + p_W^{n+1} - 4p_P^{n+1} + p_N^{n+1} + p_S^{n+1}) - \sigma u^n(p_E^n + p_W^n - 4p_P^n + p_N^n + p_S^n)\right) \tag{14}$$

IV. Model setup

Once the models equations have been transformed to discrete equations, the next step to be followed is the setup of the model. At this point the physical parameters are determined, more so the porous media properties and configurations. The design of the model is a simple rectangle with confined condition at the bottom side. The following are simple explanation of the boundary and initial conditions:

i. Dirichlet boundary condition: This is where the primary variables values are constant along the model boundary (12), hence the contaminant concentration (C) have a constant value.
ii. Neumann boundary condition: It represents the variables conditions on the perpendicular model boundary (12). These are related to contaminant and groundwater flux across the model boundaries. The top and bottom is impermeable layer, simulating the model into a confined condition. The following conditions holds:

$$\frac{\partial C}{\partial z} = 0$$

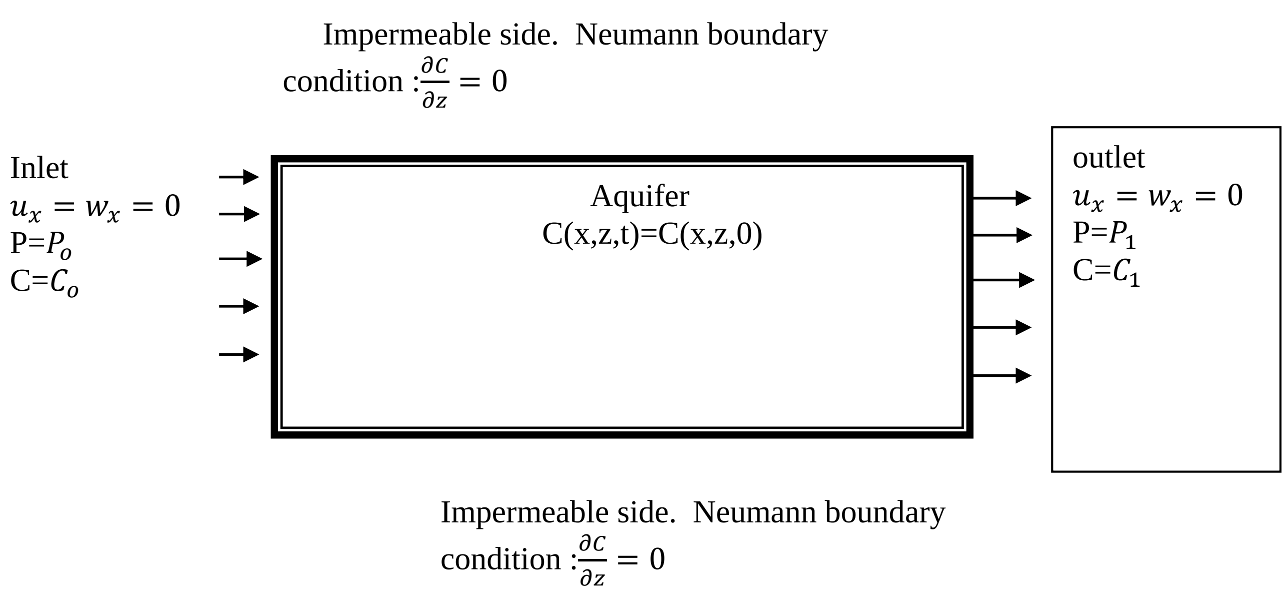
The initial conditions for the study

$$u(x,0) = w(z,0) = 0 = u(x,z)$$

And boundary conditions

$$P(0,z) = P_o, P(x,z) = P_1$$
$$C(0,z) = C_o, C(x,z) = C_1$$

These conditions can be diagrammatically represented as shown in figure 3.2



The groundwater contaminant simulation will be carried out through a continuous sources scenario where these sources were taken as boundary conditions. The physical parameters and media configuration need to be determined before simulating the results. We shall assume the simulation media is sandy.

The hypothetical physical parameters and configuration as adopted from (Ngakanputu.P.,Herr.S. and Dwinanti.R.

Figure 3.2: boundary condition

Table 3.2: Physical parameters

Parameter	Unit	Values
Model width (x-direction)	M	100
Model height (y-direction)	M	30
Longitudinal dispersitivity D_x	M	10
Transversal dispersitivity D_y	M	1.5
Porosity		0.35
Gravitational acceleration, g	m/s^2	9.81
Groundwater density	Kg/m^3	1000
Initial contaminant Concentration		50

V. SIMULATION OF RESULTS

Once the model equations and grid generation have been defined, the results are formulated using matlab software. The porous domain is taken to be 100m long and width of 30m. In this simulation, the spatial domain is divided into an equidistant grid with $\Delta x = 500cm\ and\ \Delta z = 300cm$. The source term is set to zero meaning $\tilde{q}_i = 0 \quad i = o, w$. The duration of the simulation is 1 year.

4.1 Effects of pressure viscosity coefficient on petroleum flux

After generation and analyses of contaminant dispersion, velocity and time result, we summarize the study by analyzing the effects of pressure viscosity on petroleum flux. According to the study carried out by Shriram (9), the value of the pressure coefficient varies from $\beta = 0, \beta = 2 \times 10^{-3}\ to\ \beta = 4 \times 10^{-4}$. Results were simulated for the difference values of β, the pressure viscosity coefficient and graphed as shown in figure 4. Shriram (9) found out that the use of Brinkman model($\beta = 0$) predicted that pulsating the forcing pressure about the non zero mean value would not change the flow rate, but our method showed a reduction in flow rate. This leads to conclusion that the petroleum leaking with high pressure from a transporting pipeline cannot increase flow rate through a porous media. From figure 4 it can be observed that as we increase the value of β, the volumetric flux decreases at that instant. The contaminants flux oscillated as shown in Figure 4 shows the volumetric flux as time increases from 1-12 months when the value of pressure viscosity is $\beta = 0$ from the graph we can deduce that there is decrease of contaminant concentration as distance increases. This shows that clean groundwater would be obtained in not less than 50m horizontally from the point of leakage.

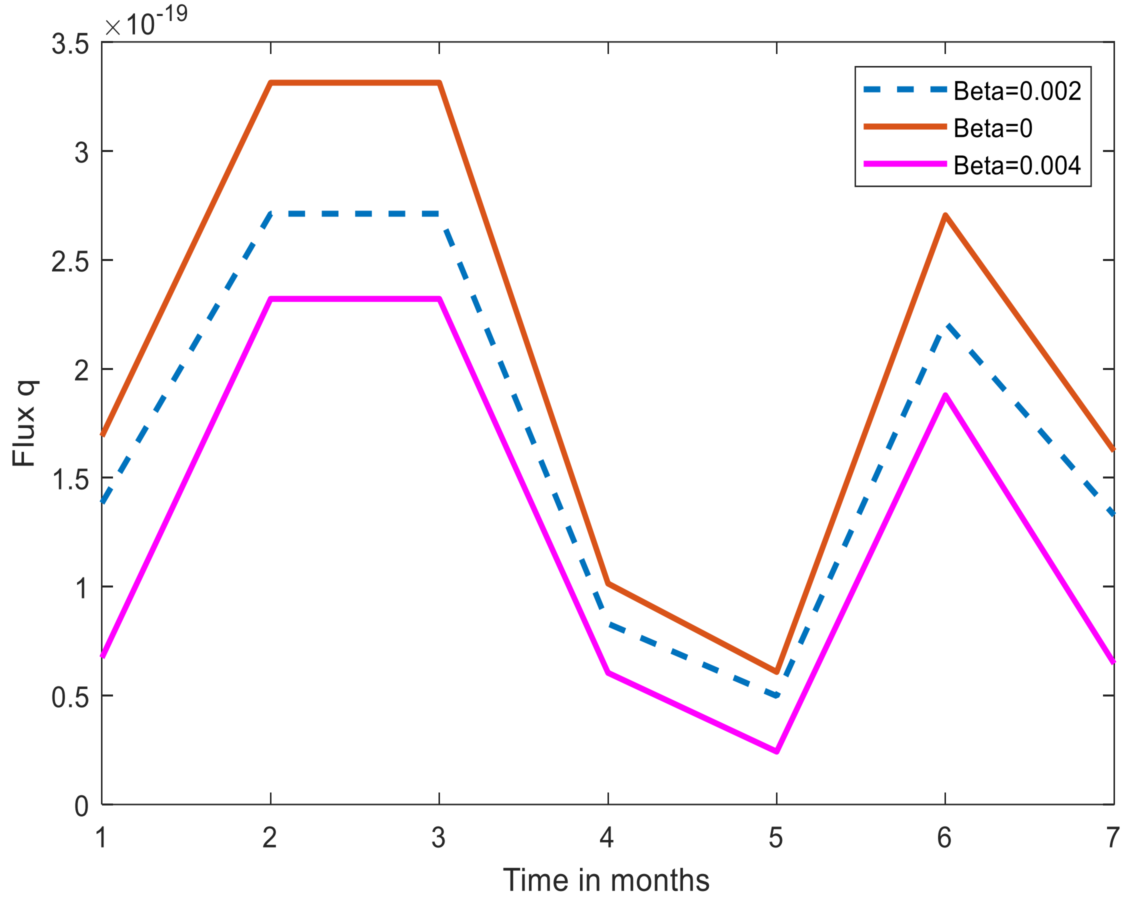


Figure 4: volumetric flux

VI. Conclusion

This study considered a problem that addresses unsteady contaminant flow through a porous media while considering the viscosity dependence on pressure. The obtained results indicated that the fluid released with pressure does not increase the volumetric flow rate in fact it decreases the flow rate by a certain percent.

This study tackled a two dimensional problem which can be extended to three dimension to account fully for flow of contaminant in a porous media.

References

[1] Kponee, K.Z, Chiger, A., Kakulu, I.I.,Vorhees. D & Helger.B.W. (2015). Petroleum contaminated water and health symptoms: a cross-sectional pilot study in a rural Nigerian community. Environ Health 14, (86). https://doi.org/10.1186/s12940-015-0073-0

[2] Kponee, K.Z, Nwanyi-Enwevem, J.C., Fu, X, Kakulu, I.I, Weisskof, M.G., & Jia, C. (2018). Elevated indoor volatile organic compound exposure in the Niger Delta Region of Nigeria. International Journal Environment resident Public Health 15(9). https://doi.org/10.3390/ijerph-15091949

[3] Aguilera, F., J. Méndez, E. Pásaro, and B. Laffon,(2010). "Review on the effects of exposure to spilled oils on human health," Journal of Applied Toxicology, 30 (4). 291-301.

[4] Bair,S., & Kottke,P.(2003): Pressure-Viscosity Relationships for Elasto hydrodynamics, Tribology Transactions, 46(3) 289-295

[5] Barus, C. (1893). Isotherms, isopiestics and isometrics relative to viscosity. American journal of science, 45(87) 96-102.

[6] Brinkman,H.C.(1901). A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. Applied scientific research Journal. 1,27-34.

[7] Forchheimer. C. (1901). Wasserbewegung Durch boden. Zeitschrift des vereines deutscher ingeneieure, 45, 1782-1788.

[8] Rajagopal, K. R. (2008). A semi-inverse problem of flows of fluids with pressure-dependent viscosities. Inverse problems in science and engineering, 16(3),269-280.

[9] Shriram .S, Bonito .A., & Rajagopal. K.R.(2013). Flow of a fluid through a porous solid due to high pressure gradients. Journal of porous media, 16(3),193-203.

[10] Asghar, H. N., H. M. Rafique, Z. A. Zahir, M. Y. Khan, M. J. Akhtar, M. Naveed, and M. Saleem, "Petroleum HydrocarbonsContaminated Soils: Remediation Approaches," Soil Science: Agricultural and Environmental Prospectives, K. R. Hakeem, J. Akhtar and M. Sabir, eds., pp. 105-129, Cham: Springer International Publishing, 2016

[11] Henry Darcy. (1856) Les Fontaines Publiques de la Ville de Dijon. Victor Dalmont, Paris.

[12] Guo, W., & Langevin, C.(2002). User's Guide to Seawat: A Computer Program for Simulation of Three-Dimensional Variable Density Ground-Water Flow. Florida: U.S Geological Survey.

AUTHORS

First Author – Flomena Jeptanui, Phd. student, Department of Mathematics and Computer Science, School of science, University of Eldoret, Eldoret, Kenya
Second Author – Jacob Bitok, Department of Mathematics and Computer Science, School of Science University of Eldoret, Eldoret,
Third Author – Titus Rotich, Department of Mathematics and Computer Science, School of Science University of Eldoret, Eldoret,

Correspondence Author – E-mail:flomenamaiyo@gmail.com