Nano Regular Generalized Irresolute Maps in Nano Topological Spaces

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Abstract: In this paper we introduce and study new class of maps, namely Nano regular generalized irresolute maps, strongly Nano regular generalized continuous maps and perfectly Nano regular generalized continuous maps in Nano topological spaces. We define and analyze some of the properties of these mappings in terms of Nano rg-closed map, Nano rg-open map.

Keywords: *Nano rg-closed map, Nano rg-open map, Nano rg-closure, Nano-continuity, Nano rg-continuity, Nano-irresolute, Nano rg-irresolute.*

1. Introduction

The concept of regular continuous and completely continuous functions was first introduced by Arya .S.P and Gupta.R[1]. Later Y.Gnanambal [4] studied the concept of generalized pre regular continuous fuctions. R.S Wali et al [3] introduced and studied the properties of rgw-irresolute maps and strongly rgw-continuous maps, perfectly rgw-continuous maps. N.Nagaveni[9] introduced and studied the concept of regular generalized irresolute maps. Bhuvaneswari et al [2] introduced and studied some properties of Nano generalized irresolute maps, Nano semi generalized irresolute maps and Nano pre generalized irresolute maps in Nano topological spaces. In this aim of this paper concepts of Nano irresolute maps, Nano regular generalized irresolute maps in Nano topological spaces and some of their properties.

2. Preliminaries

Definition 2.1: [7] A subset A of a space $(U, \tau_R(X))$ is called a regular generalized closed set if $rcl(A) \subseteq V$ whenever $A \subseteq V$ and V is regular open.

Definition 2.2: [7] The regular generalized closure of a subset A of a space U is the intersection of all rg-closed sets containing A and is denoted by rgcl(A).

Definition 2.3: [7] The regular generalized interior of a subset A of a space U is the union of all rg-open sets contained in A and is denoted by rgInt (A).

Definition2.4: [7] A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is regular generalized continuous (rg-continuous) if $f^{-1}(B)$ is rg-closed set in U for every closed set B of V or equivalently a function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is rg-continuous if and only if the inverse image of each open set is rg-open set.

Definition2.5: [4] A function f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be irresolute if the inverse image of every regular open set in V is regular open in U.

Definition2.6: [4] A function f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be rg-irresolute if the inverse image of every rg-closed set in V is rg-closed in U.

Definition2.7: [6] A function f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be strongly continuous if $f^{-1}(B)$ is both open and closed in (U, $\tau_R(X)$) for each subset B or (V, τ_R'(Y)).

Definition2.8: [7] A function f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be strongly rg-continuous if the inverse image of every rg-open set in V is open in U.

Definition2.9: [7] A function f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be perfectly rg-continuous if the inverse image of every rg-open set in V is both open and closed in U.

Definition 2.10: [8] Let U be a non-empty finite set of objects called the Universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U\ \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x $\in$U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as an X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi\}$.

3. The boundary region of X with respect to R is the set of all objects , which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) – L_R(X)$.

Definition 2.11: [8] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $\beta = \{U_R(X), L_R(X), B_R(X)\}$ is the basis for τ_R (X).

Definition 2.12: [8] If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X$\subseteq$ U and if A $\subseteq$U, then

(i) The Nano interior of the set A is defined as the union of all open subsets contained in A and it is denoted by NInt (A). That is, NInt (A) is the largest Nano open subsets of A.

(ii) The Nano closure of A is defined as the intersection of all Nano closed sets containing A and is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

Definition 2.13: [8] If (U, τ_R (X)) is a Nano topological space with respect to X where X$\subseteq$ U and if A$\subseteq$U, then

(i) The Nano regular interior of A is defined as the union of all Nano regular open subsets of A and it is denoted by NrInt(A). That is, NrInt(A) is the largest Nano regular open subsets of A.

(ii) The Nano regular closure of A is defined as the intersection of all Nano regular closed sets containing A and is denoted by NrCl(A). That is, NrCl(A) is the smallest Nano regular closed set containing A.

Definition 2.14: [8] A subset A of (U, $\tau_R(X)$) is called Nano regular-generalized closed set (briefly Nrg-closed) if Nrcl(A) $\subseteq$ V and A$\subseteq$V and V is Nano regular-open in (U, $\tau_R(X)$). The subset A is called Nano rg-open in (U, $\tau_R(X)$) if A^C is Nano rg-closed.

Definition 2.15: [9] If (U, $\tau_R(X)$) is a Nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then

(i) The Nano regular generalized interior of A is defined as the union of all Nano regular generalized open subsets of A and it is denoted by NrgInt (A). That is, NrgInt (A) is the largest Nano regular generalized open subsets of A.

(ii) The Nano regular generalized closure of A is defined as the intersection of all Nano regular generalized closed sets containing A and is denoted by NrgCl (A). That is, NrgCl (A) is the smallest Nano regular generalized closed set containing A.

Definition2.16: [10] Let (U, $\tau_R(X)$) and (V, τ_R'(Y)) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is Nano continuous on U if the inverse image of every Nano open set in V is Nano open in U.

Definition2.17: [10] Let (U, $\tau_R(X)$) and (V, τ_R'(Y)) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is Nano rg-continuous on U if the inverse image of every Nano open set in V is Nano rg-open in U.

3. Nrg - IRRESOLUTE MAPS

In this section, we introduce the concept of Nano irresolute maps, Nrg-irresolute maps in Nano topological spaces and investigate some of their properties.

Definition 3.1: A map f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is Nano irresolute map if the inverse image of every Nano regular open set A in V is Nano regular open in U.

Definition 3.2: A map f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is said to be Nano regular generalized irresolute map (briefly Nrg-irresolute) if the inverse image of every Nrg-closed set A in V is Nrg- closed in U.

Example 3.3: Let U = {a, b, c, d} with U|R = {{a}, {c}, {b, d}} and X = {a, b}.Then $\tau_R(X)$ = {U, φ, {a}, {b, d}, {a, b, d}}. Let V = {x, y, z, w} with V|R' = {{y}, {w}, {x, z}}. Then τ_R'(Y) = {V, φ, {y}, {x, y, z}, {x, z}}. Define a mapping f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) as, f (a) = x, f (b) = y, f (c) = w, f (d) = z. Then f^{-1}(V) = U, f^{-1}(ϕ) = ϕ, f^{-1}({x, y, w}) = {a, b, c}, f^{-1}({x, z, w}) = {a, c, d}, f^{-1}({y, z, w}) = {b, c, d}, f^{-1}({x}) = {a}, f^{-1}({y}) = {b}, f^{-1}({w}) = {c}, f^{-1}({z}) = {d}, f^{-1}({x, z}) = {a, d}. Thus the inverse image of every Nrg- closed set in V is Nrg-closed set in U. Hence f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) is Nrg-irresolute.

Theorem 3.4: Let (U, $\tau_R(X)$) and (V, τ_R'(Y)) be any two Nano topological spaces, where "every Nrg - closed subset is Nano closed" and f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) be a map. If f is Nrg-irresolute map then it is Nrg-continuous but not conversely.

Proof: Let f: (U, $\tau_R(X)$) $\rightarrow$ (V, τ_R'(Y)) be an Nrg-irresolute map. Then the inverse image f^{-1}(A) of every Nrg-closed set A in V is Nrg-closed set in U. Since every Nano closed set is Nano regular generalized closed, the inverse image of every Nano closed set in V is Nrg-closed in U. whenever the inverse image of Nrg-closed is Nrg-closed. Hence Nrg-irresolute function is Nrg-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Consider U={a, b, c, d} with U|R={{a},{c},{b, d}}; X={a, b} $\subseteq$ U $\tau_R(X)$={U,ϕ,{a},{a, b, d},{b, d}} and V={a, b, c, d} with V|R' ={[a},{b, d}} ; Y = {a, b} $\subseteq$ V. τ_R'(Y) ={U, ϕ,{a},{b, d},{a, b, d}}. Define f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) as the identity map. Then f is Nrg-continuous but not Nrg-irresolute map.

Theorem 3.6: Let (U, $\tau_R(X)$), (V, τ_R'(Y)) and (W, τ_R' (Z)) be Nano topological spaces. If f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. If f is Nrg-irresolute and g is Nrg-continuous then gof is Nrg-continuous.

Proof: Let A be a Nano closed set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is Nrg-continuous the inverse image g^{-1} (A) is Nrg-closed set in V. since the map f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is Nrg-irresolute, the inverse $f^{-1}(g^{-1}$ (A)) of Nrg-closed set g^{-1}(A) in V is Nrg-closed in U. Thus the inverse image $(g \text{ o } f)^{-1}$(A) is Nrg-closed in U for every Nrg closed set A in W. Hence the composition (g o f): (U, $\tau_R(X)$) → (W, τ_R' (Z)) is Nrg-continuous.

Theorem 3.7: Let f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. If f is Nrg-continuous and g is Nrg-irresolute then g o f is Nrg-continuous.

Proof: It is obvious.

Theorem 3.8: Let f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. If f and g are both Nrg-irresolute then (g of) is Nrg-continuous.

Proof: As the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is Nrg-irresolute, the inverse image g^{-1}(A) of every Nrg-open set A in W is Nrg-open in V. Hence g^{-1}(A) is Nrg-open set in V and f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) being Nrg-irresolute implies that $f^{-1}[g^{-1}(A)]$ is Nrg-open in U. Thus $(g \text{ o } f)^{-1}(A) = f^{-1}$[g-1(A)] is Nrg-open set in U for every Nrg-open set g^{-1}(A) in V. Hence (g o f) is Nrg - continuous.

Theorem 3.9: Let (U, $\tau_R(X)$), (V, τ_R'(Y)) and (W, τ_R' (Z)) be Nano topological spaces. If f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. If f and g are both Nrg-irresolute then gof is Nrg - irresolute.

Proof: As the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is Nrg-irresolute, the inverse image g^{-1}(A) of every Nrg-open set A in W is Nrg-open in V. Hence g^{-1}(A) is Nrg-open set in V and f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) being Nrg-irresolute implies that $f^{-1}[g^{-1}(A)]$ is Nrg-open in U. Thus $(g \text{ o } f)^{-1}(A) = f^{-1}$[g-1(A)] is Nrg-open set in U for every Nrg-open set g^{-1}(A) in V. Hence (g o f) is Nrg-irresolute.

Theorem 3.10: If f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is bijective, Nano pre regular open and Nrg-continuous then f is Nrg-irresolute.

Proof: Assume that A is a Nrg-closed set in V. Let X be any Nano regular open set in U such that $f^{-1}(A) \subset X$. Then $A \subset f(X)$ where f(X) is Nano regular open. Since A is Nrg- closed. NCl (A) $\subset$ f(X). Which implies f-1[NCl (A) $\subset$ X]. By assumption f is Nrg-continuous. Therefore f^{-1}(NCl(A)) is Nrg-closed. Hence NCl(f^{-1}(NCl(A))) $\subset$ f(X). That is NCl (f^{-1}(A)) $\subset$ NCl(f^{-1}(NCl(A))) $\subseteq$ X. Thus f^{-1}(A) is Nrg-closed set in U.

Theorem 3.11: A map f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is Nrg – irresolute if and only if the inverse image of an Nrg – open set in V is Nrg- open in U.

Proof: Let f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) be a Nrg – irresolute and A be a Nrg – open set in V. Then A^c is Nrg – closed in V. Since f is Nrg – irresolute, $f^{-1}(A^c)$ is Nrg – closed in U. Since $f^{-1}(A^c) = (f^{-1}(A))^c$ and so f^{-1} (A) is Nrg – open in U.

Conversely, assume that f^{-1} (A) is Nrg – open in U for each Nrg – open set A in V. Let F be Nrg – closed set in V and by assumption $f^{-1}(F^c)$ is Nrg – open in U. Since, $f^{-1}(F^c) = (f^{-1}(F))^c$, we have f^{-1} (F) is Nrg – closed in U and so f is Nrg – irresolute.

Definition 3.12: A map f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is said to be Nrg-closed map if the image of every Nano closed set in U is Nrg – closed set in V.

Remark 3.13: Every Nano closed map is Nrg-closed but not conversely.

Proof: Let f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) be Nano closed map. Let A be any Nano closed set in U. Then f(U) is Nano closed set but every Nano closed set is Nrg – closed set.Hence f is Nrg – closed map.

The converse of the above remark need not be true as seen from the following example,

Example 3.14: Consider U={a, b, c, d} with U|R={{a},{c},{b, d}} X={a ,b} ⊆ U $\tau_R(X)$={U, ϕ,{a},{a, b, d},{b ,d}} and V={a, b ,c, d} with V|R' ={[a},{b, d}} Y = {a, b} ⊆ V τ_R'(Y)={{V, ϕ,{c},{a, c},{b, c, d}}. If f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is the identity map. Then f is Nrg-closed but not Nano closed.

Theorem 3.15: If f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is Nano continuous, Nrg-closed and A is a Nano generalized closed set of U then f (A) is Nrg-closed.

Proof: Suppose that S is Nano regular open set such that f (A) ⊂ S then A ⊂ $f^{-1}(S)$ where $f^{-1}(S)$ is Nano open. Since A is Nano generalized closed set. NCl (A) ⊂ $f^{-1}(S)$. Therefore f (NCl(A)) is Nrg-closed by assumption. Hence NCl(f (A)) ⊂ NCl (f (NCl (A))). Thus f (A) is Nrg-closed.

Theorem 3.16: If f: (U,$\tau_R(X)$) → (V,τ_R'(Y)) is Nrg - irresolute, Nrg-closed and A is an Nrg-closed set of U then f (A) is Nrg-closed.

Proof: Let f(A) ⊆ G, where G is a Nrg – open in V. Since f is Nrg – irresolute, $f^{-1}(G)$ is Nrg – open in U, by definition 2.6 and A ⊆ $f^{-1}(G)$. Since A is a Nrg – closed set in U, NCl(Nint(A)) ⊆ $f^{-1}(G)$. Since f is Nrg – closed , f (NCl(Nint(A))) is Nrg – closed set contained in Nrg – open set G implies that NCl(Nint(f(NCl(Nint(A))))) ⊆ f (NCl(Nint(A))) ⊆ G and so Ncl(NInt (f(A))) ⊆ G. Hence, f(A) is Nrg – closed set in V.

The following example proves that the composition of two Nrg-closed maps need not be Nrg-closed.

Example 3.17: Consider U={a, b, c, d} with U|R={{a},{c}, {b, d}} X= {a, b} ⊆ U $\tau_R(X)$={U, ϕ,{a},{a ,b, d},{b, d}} and V={a, b, c, d} with V|R' ={[a},{b, d}} Y = {a, b} ⊆ V. Define f : (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) and g: (V, $\tau_R'(Y)$) → (W, $\tau_R'(Z)$) as follows: f(a)=b, f(b)=c ,f(c)=a, f(d)=d, g(a)=b, g(b)=a, g(c)=d, g(d)=c. Then f and g are Nrg-closed maps. But their composition (g o f) is not Nrg-closed. Since for the Nano closed set{c}, (g o f(c)) =g [f(c)] =g (a) =b is not Nrg-closed.

Definition 3.18: A map f: (U, $\tau_R(X)$) → (V, τ_R'(Y)) is said to be Nrg – open map if the image of every Nano open set in U is Nrg – open set in V.

Theorem 3.19: A map f : (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is Nrg-closed if and only if for each subset S of V and for each Nano open X containing $f^{-1}(S)$ there is a Nrg-open set Y of V such that S ⊂ X and $f^{-1}(Y)$ ⊂ X.

Proof: Necessity: Let S be a subset of Y and X be a Nano open set of U such that $f^{-1}(S)$ ⊂ X, then V\f (U\X) = Y is an Nrg-open set containing S such that $f^{-1}(Y)$ ⊂ X.

Sufficiency: Let B be a Nano closed set of U. Then f^{-1}(V\f (B)) ⊂U\B and U\B is Nano open. By hypothesis there is an Nrg-open set Y of V such that V\f (B) ⊂Y and $f^{-1}(Y)$ ⊂ U\B. Hence it follows that B⊂ U\$f^{-1}(Y)$ and Y\V⊂ f (B) ⊂ V\Y. This proves that f (B) =V\Y. Since V\Y is Nrg-closed then f (B) is Nrg-closed.

Theorem 3.20: If f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is Nrg – open then f^{-1}(Nrg – cl (A)) ⊆ Ncl ($f^{-1}(A)$) for each subset of A of V.

Proof: Let f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) be Nrg – open and A be any subset of V. Then $f^{-1}(A)$ ⊆ Ncl($f^{-1}(A)$) and Ncl($f^{-1}(A)$) is Nano closed set in U. Then by the above theorem 3.18, there exists a Nrg – closed set B of V such that A ⊆ B and $f^{-1}(B)$ ⊆ Ncl($f^{-1}(A)$). Now, Nrg – cl(A) ⊆ Nrg - cl(B) = B. Therefore f^{-1}(Nrg – cl (A)) ⊆ $f^{-1}(B)$ and so f^{-1}(Nrg – cl (A)) ⊆ $f^{-1}(B)$ ⊆ Ncl($f^{-1}(A)$). Thus, f^{-1}(Nrg – cl (A)) ⊆ Ncl($f^{-1}(A)$) for each subset of A of V.

Definition 3.21: A space (U, $\tau_R(X)$) is said to be Nrg-normal if for any two disjoint Nano regular closed sets A and B there exist disjoint Nrg-open sets X and Y such that $A \subset X$; $B \subset Y$.

Theorem 3.22: If f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is Nano continuous, Nrg-closed surjection from a normal space U onto a space V, then V is Nrg- normal.

Proof: Let A and B be two disjoint Nano regular closed sets of V. Then A and B are Nano closed sets in V. Which implies that $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint Nano closed sets of U. Since U is normal, there are disjoint Nano open sets X, Y in U such that $f^{-1}(A) \subset X$ and $f^{-1}(B) \subset Y$. Then there are Nrg-open sets F, S in V such that $A \subset F$, $B \subset S$ and $f^{-1}(F) \subset X$; $f^{-1}(S) \subset Y$. Since X and Y are disjoint, that is $f^{-1}(F) \cap f^{-1}(S) \subset X \cap Y = \phi$. Hence the space V is Nrg-normal.

Definition3.23: Let (U, $\tau_R(X)$) and (V, $\tau_R'(Y)$) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nano continuous if $f^{-1}(A)$ is both Nano open and Nano closed in (U, $\tau_R(X)$) for each subset A of (V, $\tau_R'(Y)$).

Definition3.24: Let (U, $\tau_R(X)$) and (V, $\tau_R'(Y)$) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nano rg-continuous (briefly strongly Nrg-continuous) if $f^{-1}(A)$ is Nano open in (U, $\tau_R(X)$) for every Nrg-open set A in (V, $\tau_R'(Y)$).

Theorem 3.25: If the map f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nano rg-continuous, then it is Nano continuous but not conversely.

Proof: Let A be a Nano open set in (V, $\tau_R'(Y)$). As every Nano open set is Nrg-open, A is Nrg-open in V. By the given hypothesis, f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nrg-continuous and hence $f^{-1}(A)$ is Nano open in U for every Nrg-open set A in V. Hence f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is Nano continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.26: Let U = {a, b, c, d} with U|R = {{a}, {c}, {b, d}} and X = {a, b}.Then $\tau_R(X)$ = {U, φ, {a}, {b, d}, {a, b, d}}. Let V = {x, y, z, w} with V|R' = {{y}, {w}, {x, z}}. Then $\tau_R'(Y)$ = {V, φ, {y}, {x, y, z}, {x, z}}. Define a mapping f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) as, f (a) = x, f (b) = y, f (c) = w, f (d) = z. Then f is Nano continuous. Since the inverse image of every Nano open set in V is Nano open in U. But f is not strongly Nrg-continuous. Since f^{-1}({x, z, w}) = {a, c, d} is not Nano open in U even though {x, z, w} is Nrg-open in V. Hence f is Nano continuous is not strongly nrg-continuous.

Theorem 3.27: If the map f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nano continuous, then it is strongly Nrg- continuous but not conversely.

Proof: Let A be a Nrg-open set in (V, $\tau_R'(Y)$). The map f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nano continuous and hence the inverse image of every subset of V is both Nano open and Nano closed in (U, $\tau_R(X)$). Hence f -1(A) is Nano open in (U, $\tau_R(X)$) for every Nrg-open subset A of (V, $\tau_R'(Y)$). Hence f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) is strongly Nrg-continuous.

Corollary 3.28: If the map f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) and g: (V, $\tau_R'(Y)$) → (W, $\tau_R'(Z)$) are strongly Nrg-continuous, then their composition (g o f): (U, $\tau_R(X)$) → (W, $\tau_R'(Z)$) is also strongly Nrg- continuous.

Theorem 3.29: Let f: (U, $\tau_R(X)$) → (V, $\tau_R'(Y)$) and g: (V, $\tau_R'(Y)$) → (W, $\tau_R'(Z)$) are any two maps. Then their composition (g o f): (U, $\tau_R(X)$) → (W, $\tau_R'(Z)$) is

(i) Strongly Nrg- continuous if g is strongly Nrg-continuous and f is Nano continuous.

(ii) Strongly Nrg-continuous if g is strongly Nano continuous and f is Nano irresolute.

(iii) Nano continuous if g is Nrg-continuous and f is strongly Nrg-continuous.

Proof: (i) Let A be a Nrg-open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is strongly Nrg-continuous, g^{-1} (A) is Nano open set in V. since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is Nano continuous, the inverse $f^{-1}(g^{-1}$ (A)) is Nano open in U. Thus the inverse image (g o f)$^{-1}$(A) = $f^{-1}(g^{-1}$ (A)) is Nano open in U for every Nrg-open set A in W. Hence the composition (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is strongly Nrg-continuous.

(ii)Let A be a Nrg-open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is strongly Nano-continuous, g^{-1} (A) is both Nano open and Nano closed set in V. As every Nano open set is nano regular-open, since g^{-1} (A) is Nano regular-open in V. since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is Nano irresolute, the inverse $f^{-1}(g^{-1}$ (A)) is Nano-regular open in U. Thus the inverse image (g o f)$^{-1}$(A) = $f^{-1}(g^{-1}$ (A)) is Nano regular- open in U and g is strongly Nano continuous. Hence the map (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is strongly Nrg-continuous.

(iii) Let A be a Nano-open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is Nrg-continuous, g^{-1} (A) is Nrg-open in V. since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is strongly Nrg-continuous, the inverse $f^{-1}(g^{-1}$ (A)) is Nano-open in U. Thus the inverse image (g o f)$^{-1}$(A) = $f^{-1}(g^{-1}$ (A)) is Nano - open in U. Hence the map (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is Nano-continuous for every Nano open set A in (W, τ_R' (Z))

Theorem3.30: Let f: (U, τ_R(X)) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. If f is Nrg-continuous and g is strongly Nrg-continuous then gof is Nrg-irresolute.

Proof: Let A be a Nano open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is Nrg-continuous the inverse image g^{-1} (A) is Nrg-closed set in V. since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is strongly Nrg-continuous, the inverse $f^{-1}(g^{-1}$ (A)) of Nrg- open set in U. Thus the inverse image (g o f)$^{-1}$(A) = $f^{-1}(g^{-1}$ (A)) is Nrg-open in U for every Nrg open set A in W. Hence the map (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is Nrg-irresolute.

Definition 3.31: A function f: (U, τ_R(X)) → (V, τ_R'(Y)) is said to be perfectly Nrg-continuous if the inverse image of every Nrg-open set in V is both Nano-open and Nano-closed in U.

Theorem 3.32: Let f: (U, τ_R(X)) → (V, τ_R'(Y)) and g: (V, τ_R'(Y)) → (W, τ_R' (Z)) be two maps. Then their compositions (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is

(i) If f is Nrg-continuous and g is perfectly Nrg-continuous then gof is Nrg-irresolute

(ii) If f is perfectly Nrg-continuous and g is strongly Nano-continuous then gof is perfectly Nrg-continuous.

Proof: (i) Let A be a Nrg-open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is perfectly Nrg-continuous, the inverse image g^{-1} (A) is both Nano open and Nano closed in V. since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is Nrg-continuous, the inverse $f^{-1}(g^{-1}$ (A)) of Nrg- open set in U. Thus the inverse image (g o f)$^{-1}$(A) = $f^{-1}(g^{-1}$ (A)) is Nrg-open in U for every Nrg open set A in W. Hence the map (g o f): (U, τ_R(X)) → (W, τ_R' (Z)) is Nrg-irresolute.
is Nrg-irresolute.

(ii)Let A be a Nrg-open set in W. since the map g: (V, τ_R'(Y)) → (W, τ_R' (Z)) is strongly Nrg-continuous, the inverse image g^{-1} (A) is both Nano open and Nano closed in V. As every Nano open set is Nrg-open, g^{-1} (A) is both Nrg- open and Nrg closed. Since the map f: (U, τ_R(X)) → (V, τ_R'(Y)) is perfectly Nrg-continuous, the inverse $f^{-1}(g^{-1}$ (A)) of Nano open and Nano closed set in U. Thus the inverse image $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is both Nano-open and Nano-closed in U for every Nrg open set A in W. Hence the map $(g \circ f): (U, \tau_R(X)) \rightarrow (W, \tau_R'(Z))$ is perfectly Nrg - continuous.

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