Mathematical Model For The Compressive Strength Of A Sedimentary Rock Aggregate Concrete

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Abstract- In this work 12mm maximum size sedimentary rock aggregate from Nkwere-Ezunaka in Anambra State, Nigeria, was studied in order to determine its suitability for structural concrete production as it has very visible silt content. In so doing sieve analysis test was conducted on the sample of aggregate. The strength of the concrete for a predetermined number of mix proportions was obtained and a mathematical model formed, and optimized. It was observed that the aggregate contained 21.9% silt/clay, and had an optimum strength of concrete 11.403 N/mm2 for a mix proportion of 1: 1.35: 2.65, being ratios of cement, sand and gravel respectively, at a water/cement ratio 0.54. It was concluded that the contamination was high and it was the cause of the general low strength of the concrete made from the aggregate as optimum strength of 11.403N/mm2 was half of an expected value of 20N/mm2 obtainable from an alternative clean granite aggregate. It was recommended that this size of aggregate from Nkwere-Ezunaka quarry be disused as structural concrete aggregate where high stress is expected. The aggregate should be restricted to concretes for lintels and columns of two storey buildings. Further processing of the aggregate by sieving was recommended before use to improve the strength of the concrete.

Index Terms- Aggregate, Contaminations, Concrete strength, Sedimentary rocks, Simplex method, Optimization.

I. INTRODUCTION

There are two main types of rock aggregates used in Nigeria for making concrete. They are sedimentary rock and granite rock aggregates. Granite aggregates are more uniform in chemical composition and strength than the sedimentary rock which vary in strength, density, hardness, toughness and silt contamination from one rock outcrop to the other. One advantage of sedimentary rock aggregate is that they are often found in abundance and are much cheaper than granite in most cases. In fact fifty percent of the surface area of Nigeria land mass is covered by sedimentary rock [1]. The problem with sedimentary rock aggregates is that it is difficult to predict the strength of concrete made with it by any known mix design process as a result of high variation in the properties of the aggregate. This work in concerned with 12–millimeter (maximum size) aggregates from Nkwere-Ezunaka which contains a lot of clay and silt contaminations and which can cause the strength of concrete made with it to be unexpectedly very low. The aim of this report is to produce a mathematical model for the strength of concrete made with the aggregate, using Scheffe's simplest method, within a pre-determined range of water-to-cement, coarse aggregate and fine aggregate ratios that are commonly used in making concrete. The model can then be optimized to obtain optimum mix proportion and associated strength, with the objective of advising engineers and other users of the aggregate on how to safely use the aggregate to prevent structural failures.

1.1 Scheffe's Simplex Method of Optimization

A fair knowledge of simplex is needed for understanding Scheffe's simplex method of optimization. Akhanazarove and Kafarove[2] defined a simplex as a convex polyhedron formed by K intersecting hyper planes and having K + 1 vertices. In other words a simplex is a geometrical figure having K + 1 vertices, where K is the number of intersecting planes forming it. A simplex is a straight line if it is formed by one plane, and is called a one-dimensional simplex. It is a triangle if it is formed by two planes, and is called a two-dimensional simplex. A simplex is a tetrahedron shape, if it is formed by three planes and is also called three-dimensional simplex. If all the sides of a simplex are equal, it is regarded as a regular simplex. A simplex can be used to relate data information graphically.

Scheffe[3] used a regular simplex of (q - 1) dimension, where q is the number of components in the mixture, to describe a domain for selected mixtures, such that a polynomial model can be formed to relate any property of the mixture studied with the concentration of the components of the mixture. The properties of the mixture were assumed to be entirely dependent on the concentrations of the components.

For concrete, the number of mixture components is usually four, and a simplex formed by three intersecting hyper planes with four vertices (a tetrahedron shape) is required to describe the domain of the mixtures involved in the study, and for which a general polynomial model can be formed, following Scheffe's principles, and this is used to describe the property of any particular mixture in the domain with respect to the concentrations of its components. Traditionally a high degree polynomial of the general form in Eq 1 is used for this purpose. The number of coefficients of the resulting model which depends on the number of components q and the degree of the polynomial is given by the combination, .

Knowing also that the sum of the mixture component ratios (or concentrations) must be equal to unity (Eq 2)

,where represents concentrations of the various components in the mixture, Scheffe[3] was able to derive a new polynomial with fewer number of coefficients given by , thereby reducing the bulk of experimental work required to evaluate the coefficients. From Scheffe's derivations, the second degree polynomial (n = 2) for 4 component mixtures (q = 4) needed for concrete is given by Eq 3

.

(3a)

where

(3b)

and are responses from the various experimental trials. Eq 3 has 10 coefficient instead of 15 that is obtainable from direct use of Eq 1. The number of experimental trials required to evaluate the coefficients in Eq 3a is ten, given by Eq 4.

(4)

These ten mixtures are carefully chosen so that they are uniformly distributed on the edges of the simplex. In this way the domain or boundary space of the mixtures and the relative proportions of the mixtures' components are carefully described, see Fig1.

Fig. 1: Factor notations on the simplex

The information in Fig1 is further presented in a tabular format with extra column for responses from which the coefficient of the model, $_i$ and $_{ij}$ of Eq3b can then be calculated after the experiments (see Table 1)

Table 1: Matrix table of mixture components and responses for Scheffe's second degree polynomial in 4 components

Component Concentrations					**Responses**
S/N	**X_1**	**X_2**	**X_3**	**X_4**	
1	1	0	0	0	Y_1
2	0	1	0	0	Y_2
3	0	0	1	0	Y_3
4	0	0	0	1	Y_4
5	½	½	0	0	Y_{12}
6	½	0	½	0	Y_{13}
7	½	0	0	½	Y_{14}
8	0	½	½	0	Y_{23}
9	0	½	0	½	Y_{24}
10	0	0	½	½	Y_{34}

Obviously, Fig 1 and Table1 do not display mixtures that are similar to concrete mix proportions, and has to be changed to proportion that resembles concrete mixtures. This is done by carefully choosing 4 concrete mix proportions that will show the range of water-to-cement ratio (w/c), cement, fine aggregate and coarse aggregate components that the proposed model is likely to cover. These are placed on the vertices of the simplex and the rest of the required test points to replace X_{12}, X_{13}, X_{14}, X_{23}, X_{24}, X_{34} are obtained by interpolation. Choosing X_1, X_2, X_3 and X_4 as 0.6: 1.0: 1.5: 4, 0.5:1.0:1.0:1½, 0.55:1.0:1½:3 and 0.555:1.0:1½:4, respectively, for this purpose we have Fig2 to replace Fig1 as shown.

Fig. 2 Factor Notations on the simplex for concrete mix proportions.

A transpose of a matrix formed from the chosen concrete proportions becomes a conversion factor from Fig1 to Fig 2. Point X_{12} in Fig2 is for example obtained as follows:

0.6 1.0 1.5 4.0
T 0.5
0.5 1.0 1.0 1.5
0.5
0.55 1.0 1.5 3.0
0
0.55 1.0 1.5 4.0
0

0.6 0.5 0.55 0.555
½ 0.55
1.0 1.0 1.0 1.0
½ = 1.0
1.5 1.0 1.5 1.5
0 1.25
4.0 1.5 3.0 4.0
0 2.75

The rest of other points are obtained in a similar manner and are shown in Fig2.

Table 1 is then modified as shown in Table 2

Table 2: Matrix table for scheffe's second degree polynomial for concrete

S/N	Pseudo-Components				Responses	Real Components			
X	X_1	X_2	X_3	X_4		W/C Z_1	Cement Z_2	Fine Aggregate Z_3	Coarse Aggregate Z_4
•	1	0	0	0	Y_1	0.6	1.0	1.5	4.0
•	0	1	0	0	Y_2	0.5	1.0	1.0	1.5
•	0	0	1	0	Y_3	0.55	1.0	1.5	3.0
•	0	0	0	1	Y_4	0.555	1.0	1.5	4.0
•	½	½	0	0	Y_{12}	0.55	1.0	1.25	2.75
•	½	0	½	0	Y_{13}	0.575	1.0	1.5	3.5
•	½	0	0	½	Y_{14}	0.578	1.0	2.0	4.0
•	0	½	½	0	Y_{23}	0.525	1.0	1.25	2.25
•	0	½	0	½	Y_{24}	0.528	1.0	1.75	2.75
•	0	0	½	½	Y_{34}	0.553	1.0	2.0	3.5

The mix proportions shown as Real-components on the left hand side of Table 2 is instead used for the experiments and the various responses,, etc. resulting from the experiments recorded appropriately for calculation of the coefficient of Scheffe's polynomial model for concrete.

II. METHODOLOGY

2.1 Material and Equipment

Sample of 12-mm coarse sedimentary rock aggregate was obtained from Nkwere-Ezunaka. A washed river sand was also obtained from Onitsha (River Niger) both of them were stored in-doors to prevent excessive moisture variation. Portland cement from dealers was also obtained and store in a dry place under room temperature. Laboratory equipment employed in the research include; universal crushing machine, 150x150x150-mm cube mold, weighing balance, curing tank, trowel, B S Sieves and sieve shaker.

2.2 Methods

(i) Concrete Strength Test

For each mix proportion shown in Table 2, water, cement, fine aggregate and coarse aggregate were weighed out in such a quantity that would serve for three cubes. The materials were thoroughly mixed together inside a non-absorbent container, before water was added and final mixing was done. The fresh concrete was filled into the moulds in three layers, each layer tamped 25 times and leveled off. It was left for 24 hours to set and harden, after which the moulds were removed and the cubes immersed in water in the curing tank for 28 days. Sixty cubes were made in the total – 3 cubes for each mix proportion or test point. At the expiration of 28 days, the cubes were crush in the universal crushing machine. The compressive cube strength was calculated for each cube and the average cube strength for each mix proportion in Table 2 was calculated and all recorded in a table . Ten extra test points were included for validation of the model.

The model was formed, using the responses (average strength response) from the first ten experimental prints, as in Table 2, by substituting them into Eq 3b to obtain the model coefficients. The coefficients were then substituted into Eq 3 to obtain the model.

(ii) Validation of the Model

The model was validated for adequacy using Fisher's variance ratio test (F-ratio) given by

(5)

Where

(6)

and (7)

In the above formula F is Fishers ratio, is sum of squares of goodness of fit, is sum of squares related to error mean square. N is number of experimental points, m is the number of replicate trials in each experimental point and is the number of coefficient in the model. F-ratio value obtained by calculation using Eq 5 must not be greater than the value read from any table of F-distribution using the degree of freedoms and at significant level of 0.05.

(iii) Optimization of the Model

The model obtained was used to optimize the strength of the concrete through a Quick Basic Computer Program with the flow-chart is shown in Fig (3)

(iv) Sieve Analysis Test

Sieve analysis test for the coarse aggregate was carried out in accordance with BS 812: Part 1 requirements. The sieves were selected in the prescribed manner and arranged in descending order of sizes with the largest on top of the stalk and the smallest at the bottom and finally the tray. 1000g of air-dried sample was introduced into the stack and vibrated until passage of aggregates stopped. The weight of aggregate retained on each sieve was weighed and tabulated. The percentage passing each sieve was calculated and presented graphically in a semi logarithmic plot.

III. RESULTS

The result of the concrete strength test is presented in Table 3. The mix proportions from which the concretes were cast are tabulated in the last four columns and headed 'Real Components'. The cube strength values for the three replicate tests and their averages are shown at the middle columns of the table and labeled "Replicate Response" and "Average Response" respectively. The prediction from the model is also given and is headed ‘Predictions’. The first ten results were used to build the model and the remaining ten were incorporated for validation and as control so that predictions from the model can easily be compared with experimental results. The grading curve for the coarse aggregate is shown in Fig4 in which percentage passing sieve size of 0.075mm or 75 is regarded as clay or silt contamination.

Table 3: Responses from Experiments and Predictions from Model

Pseudo-component					Response symbol	Replicate Response (N/mm^2)			Average Response	predictions	Real-components		
N	X_1	X_2	X_3	X_4		1	2	3	(N/mm^2)	(N/mm^2)	W/C	Cement	Fine a
•	1	0	0	0	Y_1	4.43	4.43	4.87	4.58	4.58	0.6	1	1½
•	0	1	0	0	Y_2	9.72	7.97	8.41	8.7	8.7	0.5	1	1
•	0	0	1	0	Y_3	7.09	6.64	8.87	6.2	6.2	0.55	1	1½
•	0	0	0	1	Y_4	4.87	4.87	4.87	4.87	4.87	0.555	1	2½
•	½	½	0	0	Y_{12}	11.07	11.51	9.72	10.77	10.77	0.55	1	2 ¼
•	½	0	½	0	Y_{13}	9.74	8.15	8.00	8.96	8.96	0.575	1	1½
•	½	0	0	½	Y_{14}	7.97	8.41	7.97	8.12	8.12	0.578	1	2
•	0	½	½	0	Y_{23}	10.04	9.00	11.51	10.18	10.18	0.525	1	1 ¼
•	0	½	0	½	Y_{24}	9.74	9.74	10.19	9.89	9.89	0.528	1	1 ¾
•	0	0	½	½	Y_{34}	7.53	7.53	8.41	7.82	7.82	0.533	1	2
CONTROLS													
•	½	0	¼	¼	C_1	7.33	8.67	9.11	8.37	9.11	0.576	1	
•	¼	0	½	¼	C_2	9.56	9.11	6.44	8.37	9.24	0.564	1	
•	¼	¼	¼	¼	C_3	8.33	11.33	8.78	9.48	10.89	0.557	1	
•	$^{2}/_{3}$	0	0	$^{1}/_{3}$	C_4	7.22	10.44	7.00	8.22	7.69	0.585	1	
•	¼	¼	½	0	C_5	9.78	11.33	10.44	10.52	10.60	0.55	1	
•	¼	½	0	¼	C_6	10.00	8.67	11.78	10.15	11.18	0.539	1	
•	¼	0	¼	½	C_7	9.11	8.44	9.33	8.96	8.86	0.535	1	
•	½	¼	0	¼	C_8	9.89	10.78	9.22	9.96	10.22	0.564	1	
•	¼	½	$^{1}/_{8}$	$^{1}/_{8}$	C_9	10.69	11.56	10.67	10.97	11.14	0.538	1	
•	$^{1}/_{3}$	$^{1}/_{3}$	0	$^{1}/_{3}$	C_{10}	10.67	10.87	9.78	10.44	10.77	0.552	1	

Fig. 4 Grading curve for Nkwere-Ezunaka sample

The model developed using the data in Table 3 and Eqs3a and 3b is given here as Eq 8

(8)

The model was validated using Eqs 5 – 7. The F-ratio value obtained at a significant level of 0.05 was 1.9 which was less than 2.1 obtained from the table of F-distribution with V_1=10 and V_2 = 40;and this indicates that there were no statistically significant differences between the experimental and predicted strength values. The predictions of the model can be compared with the controls in Table 3.

The optimization of the strength of the concrete was done using the model, Eqn 8, in a Quick Basic Computer Program whose flow chart is given in Fig 3. The maximum strength and corresponding mix proportion within the range shown in the simplex was obtained as 11.403N/mm^2 (for strength) and 0.541: 1.0: 1.35: 2.65 (for mix ratio of water, cement, sand and gravels). The contamination in the coarse aggregate or gravels is 21.9%.

IV. DISCUSSION OF RESULTS

In Table 3, the concrete strengths (responses and predictions) are all less than the prescribed value for structural design[4] which is 20N/mm^2. A good granite aggregate (the gold standard) for which this is stipulated must be clean, but the contamination in the aggregate under study is 21.9%. This contamination is the reason for the low strength [5]. For this aggregate to be good enough for structural concrete it must be cleaned of the dirt to a level of less than 5%[6].

V. CONCLUSION AND RECOMMENDATIONS

Based on the results and discussion, it is clear that the strength of concrete from Nkwere-Ezunaka gravel (12-mm maximum size) is very low because of high contamination (dirt in the aggregate). The following recommendations are therefore made:

- The aggregate should not be used for bridges, culverts, columns of building above 3 storeys and any other structural element where stress is known to be above the obtained optimum value, unless it is washed thoroughly.

- It is recommended for short lintels of all building and columns of two storey building without cleaning.
- Washing or sieving the aggregate is recommended for improved concrete strength before it can be used for other purposes.

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