Analysis of Students' Thinking Process in Solving Comparison Story Questions

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Abstract- Solving mathematical problems is the focus of mathematics learning which develops students' reasoning skill. This qualitative research aims at identifying the thinking process of students in solving comparison story questions. The subject of this research includes 6 students of Mathematics Education from Universitas Wisnuwardhana, Malang, East Java. The techniques of data collection are essay test and semi-structured interview. Resource triangulation technique is used as the analysis technique. According to the students' work, the result shows that in solving comparison story questions for scale on map, there are 2 students used conceptual thinking process, 1 student used semi-conceptual thinking process, 1 student used intuitive thinking process, and 2 students used semi-conceptual and conceptual thinking process.

Index Terms- Thinking process, computational thinking, procedural thinking, semi-conceptual thinking, comparison.

I. INTRODUCTION

Thinking is a process which produces new mental representation through transformation of information involving complex interaction between mental attributes. These mental attributes are abstraction, logic, imagination, and problem solving [1]. Problem solving has a prominent place within scientific reasoning because of its impact on changes and increasing emotional, cognitive, and psychomotor development. The relation between thinking and problem solving considers problem solving consisting of basic processes, strategies, and knowledge resources [2]. Problem solving is the heart of mathematics and problem solving in mathematics lies in that it is the goal and final outcome of the learning and teaching process, as problem solving is perceived as the right way toward practicing thinking in general. In other words, there is no math without thinking and there is no thinking without problem. Problem solving skills are important in order to develop the abilities of solving problem in mathematics and finding the solution of problems in daily life. Students can apply their knowledge and problem solving skills to be useful in daily life and a variety of unfamiliar situations. On the other hand, problem solving is also a skill that helps individuals in developing logical thinking and improving their decision making skills by the use of logical processes such as induction and deduction, as well as using algorithms when needed to work out daily situations [3,6].

Problem solving is a process which begins with the initial contact with the problem and ends when the answer is received in the light of the given information. Problem solving is a complex process to learn and multiple set of step process where the problem solver must find the relationship between past experience and the problem at hand then act upon a solution [4]. Component in problem solving consists of cognitive, behavioral, and attitudinal. Problem solving ability according to Polya (1973) is identified as the ability to 1) understanding the problem; 2) devising plan; 3) carrying out the plan; and 4) looking back [4,5,7]. Analog, Herald suggest that one may solve daily problems using a method I.D.E.A.L, where each letter of the acronym represents a step in the problem solving process, presented i.e. 1) identify the problem; 2) describe the possibilities; 3) evaluate the ideas; 4) act out a plan; 5) learn for the future [6].

Every stage of problem solving, according to Polya, lies the thinking process which takes place inside it. Thinking process is a sequence of mental events happening naturally or planned and systematic in a context of time, space, and media used which produce an object change influencing them. Process of thinking includes manipulating and transforming information into memory which are often done in order to form reasoned concept, critical thinking, decision making, creative thinking, and problem solving [8,9]. Thinking process is categorized into 3 kinds namely 1) conceptual thinking process, 2) semi-conceptual thinking process, and 3) computational thinking process. Conceptual thinking process is a way of thinking which always solve problems using owned concept based on the result of assessment. Semi-conceptual thinking process is a way of thinking which, in solving problem, tends to use concept but with less understanding of that concept so that the completion is mixed with intuitive way of completion. Computational thinking

process is a way of thinking that generally, in solving problem, tends to use intuition. The indicator to identify the thinking process includes the ability to express what is known from the question with student's own sentences, the ability to answer which has the tendency to use the concept that has been learned before, and the ability to mention the elements of concept [10]. Therefore, the goal of this research is to analyze students' process of thinking in solving comparison story questions.

II.   METHOD

Participant

This research is descriptive qualitative research with 6 students of Mathematics Education from Universitas Wisnuwardhana, Malang, East Java, as the subjects of the research.

Instruments and Data Collection Technique.

The technique of data collection in this research is giving tests and semi-structured interview. The instruments of this research are essay test and interview guidelines. The essay test consists of 5 scale and map comparison questions. In this material, there are three variables such as 1) actual distance, 2) scale, and 3) distance on map. The relationship can be formulated in the following.

$$\text{scale} = \frac{\text{size on image}}{\text{actual size}}$$

Scale $1:n$ means that 1 cm size on the image represent n cm to the actual size. There are 2 factors as in the following.

1. Diminution factor: Scale $= 1:n$ where the actual size is minimized.
2. Magnification factor: Scale $= n:1$ where the actual size is enlarged.

The 5 questions include 1) actual size determination question if the scale and size on the image are known; 2) scale determination question if the size on the image and actual size are known; 3) size on the image determination question if the scale and actual size are known; 4) actual size determination question if the model of image is minimized and 5) size determination question if the image is enlarged. Below is the example of the questions.

Question 1

Two cities has a distance of 15 km and will be drawn on the map with the scale of 1:250.000. Determine the distance of both cities on the map!

Question 2

The distance of two ports is 240 km. If the distance of both ports on the map is 8 cm, determine the scale!

Question 3

A map is made with the scale of 1:350.000. If the distance between two cities is 4.2 cm, determine the actual distance!

Question 4

A map has a scale of 1:1.500.000. The map is 80% copied. If the distance between two cities is the result of map's copy which is 4,8 cm then determine the actual distance of both cities!

Question 5

A map has a scale of 1:350.000. The map is 120% photocopied. If the actual distance is 17.5 km, then determine the distance between both cities on the map that is the result of photocopy!

Data Analysis Technique

The data analysis technique of this research is the resource triangulation technique.

III.   RESULT

The students' thinking process in solving comparison story questions is in the following.

Question 1

In answering question 1, there are several subjects who answered systematically correspond with Polya's stages. Some subjects directly answered. Below is the subject's answer for Question 1.

Figure 1. Answer of Subject 1 to Question 1

Figure 2. Answer of Subject 4 to Question 1

First stage, Subject 1 represented the understanding of the question given in the form of image. On the other hand, Subject 4 represented the answer with sentences. Second stage, Subject 1 made a stair of unit length then continue to analyze the relationship between the scale on the map and the actual distance while Subject 2 formulated a formula that is:

Scale = DM (distance on the map) : AD (actual distance)

Third stage, Subject 1 and Subject 4 did a calculation resulting of the distance on the map as 6 cm. For the fourth stage, Subject 1 and Subject 4 did a recheck. Subject 1 and Subject 4 used the conceptual thinking process because they have understood the scale concept on the map and applied it in order to solve the given question.

The answer of Subject 2 can be seen in the next Figure 3.

Figure 3. Answer of Subject 2 to Question 1

First stage was done by Subject 2 in understanding the problem given by writing the acquired information which are a distance of 15 km = 1500; scale of 1:250.000 and distance of both cities= 250.000. Based on the result of Subject 2's work, it can be seen that this subject did not understand the conversion of unit length from kilometer to meter. Besides, Subject 2 did not understand the meaning of 1:250.000 scale. Therefore, Subject 2 interpreted 1:250.000 as the distance of both cities such as 250.000 km. Second stage, Subject 2 thought about the relationship of scale, actual distance, and distance on the map. This subject could understand the relationship of the three so that Subject 2 wrote 6 as the result of stage 3. However, unit length used in the calculation was still incorrect. In the final stage, Subject 2 did not recheck the answer given. According to the answer of Subject 2, the thinking process used is the semi-conceptual thinking process. It is shown that Subject 2 actually understood the concept of scale on the map but the subject was not careful in calculating the answer so that the subject only interpreted some parts of the information there.

The answer of subject 3 can be seen in the following Figure 4.

Figure 4. Answer of Subject 3 to Question 1.

The first step done by Subject 3 was writing the known information from the question then Subject 3 did not do a reflection on the relationship of scale, actual distance, and the distance on the map. Subject 3 directly did the algebra operation resulting 16,6 cm as the answer. Finally, Subject 3 did not reevaluate. The thinking process done by Subject 3 is intuitive thinking process because Subject 3 did not understand the concept and used the intuition to understand the relationship between scale on the map and the actual distance in order to determine the distance on the map.

The answer of Subject 5 can be seen in Figure 5.

Figure 5. Answer of Subject 5 to Question 1

Subject 5 has understood the question and the concept on Question 1. The first step done in order to solve this question was

by converting the unit length from 15 km to $15 \cdot 10^5$ cm. Then, Subject 5 enter the formula in order to determine the distance on the map resulting 6 cm as the answer. The thinking process used is conceptual thinking process.

The answer of Subject 6 can be seen in the next Figure 6.

Figure 6. Answer of Subject 6 to Question 1.

Subject 6 did not do Polya's stages in sequence. The subject remembered what the teacher taught that the relationship of the actual distance (AD) is the multiplication of distance on the map (DM) with the scale on the map (SM) pictured in the image below.
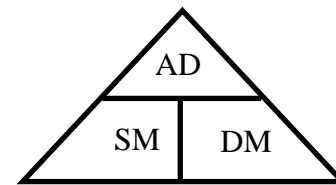

Figure 7. Relationship of actual distance, distance on the map, and scale on the map.

According the interview, Subject 6 directly answered with formula taught and the subject understood about map scale question that needed to be equalized first. The result was 6 cm. The thinking process used is semi-conceptual thinking process.

Question 2

In answering Question 2, there are several subjects answered systematically with Polya's stages. There are some subjects directly answered and even did not answer the question. The subjects' answers to Question 2 are as follow.

Figure 8. Answer of Subject 1 to Question 2

Figure 9. Answer of Subject 4 to Question 2

Analog of the completion of Question 1 in the first stage is that Subject 1 represented the understanding of the question given in the form of image while Subject 4 represented the answer with sentences. Second stage, Subject 1 conversed the unit length made in Question 1 the the subject made a comparison of

distance on the map with the actual distance in order to acquire the map scale such as:

$$8: 24.000.000 \Leftrightarrow 1: 3.000.000$$

On the other hand, Subject 4 made a stair of unit length first before making the comparison of distance on the map with the actual distance. The thinking process used is Conceptual thinking process.

The answer of Subject 2 can be seen in the Figure 10.

Figure 10. Answer of Subject 2 to Question 2.

Subject 2 directly answered and determined the scale of the map by comparing the distance on the map and the actual distance. According to the interview, Subject 2 explained the formula to determine the scale of the map. Subject 2 was incorrect in giving the period sign so that the scale acquired was 1:300.0000 not 1:3.000.000. Therefore, the thinking process of this subject is semi-conceptual thinking process.

The answer of Subject 3 can be seen in the Figure 11.

Figure 11. Answer of Subject 3 to Question 2.

Subject 3 did not answer the Question 2 because the subject was confused and could not formulate the solution to the problem.

The answer of Subject 5 can be seen in the next Figure 12.

Figure 12. Answer of Subject 5 to Question 2.

Analog to Question 1, Subject 5 has understood the concept of Question 2 where the first step taken by the subject in order to solve the problem was converting the unit length of 240 km to $240 \cdot 10^5$ cm. Then, the subject entered it to the formula to determine the scale resulting $1: 30(10^5) = 1: 3000000$. The thinking process used is conceptual thinking process.

The answer of Subject 6 can be seen in the following Figure 13.

Figure 13. Answer of Subject 6 to Question 2.

Based on the answer of Subject 6 on Figure 13, it can be seen that Subject 6 in determining the scale of the map used the division of distance on the map with the actual distance by converting the unit length first. The process of thinking used is semi-conceptual.

Question 3

Based on the results of all 6 subjects, all of them did not write what was known but they understood the meaning of question number 3. All of the subjects directly used the formulated formula. The error happened was that the students were incorrect in calculating or including the unit length. The answers of the students for question 3 are as follow.

The answer of Subject 4, 5, 6, and 2 can be seen in the following Figure 14, 15, 16, and 17.

Figure 14. Answer of Subject 4 to Question 3

Figure 15. Answer of Subject 5 to Question 3.

Figure 16. Answer of Subject 6 to Question 3.

Figure 17. Answer of Subject 2 to Question 3.

Based on the results of the answers and the interview on Subject 4,5,6 above, basically they already understood the meaning of question number 3 and they understood in determining the actual distance acquired from multiplying the distance on the map with the scale. However, the difference was that the step on each of the calculation. Meanwhile, Subject 2 wrote the information known from the question first then determined the actual distance by multiplying the scale and the distance on the map divided with the actual distance. In formulating the formula or determining the equal comparison, Subject 2 made some error. The process of thinking used by the three subjects is conceptual thinking process while Subject 2 used the semi-conceptual thinking process.

The answer of Subject 1 can be seen in the next Figure 18.

Figure 18. Answer of Subject 1 to Question 3.

Analog on questions 1 and 2, in order to understand the question given, Subject 1 used image representation. To determine the actual distance, Subject 1 used the equal comparison but the result obtained was incorrect because the subject made error in multiplying 4.2 with 350.000. The process of thinking used is conceptual thinking process.

The answer of Subject 3 can be seen in Figure 19.

Figure 19. Answer of Subject 3 to Question 3.

Based on the result of the interview, Subject 3 was still confused to determine the actual distance and the result was also incorrect. The process of thinking used is intuition.

Question 4 and Question 5 were the question about the scale of the map that have been minimized and enlarged. Based on the students' answers, some students could solve the questions and some students could not. For questions number 4 and 5, the students who could solve them were only 3 students which were Subjects 1, 5, and 6. The results of question number 4 can be seen on Figures 1, 5, and 6. Also, the results of question number 4 can be seen on Figure 20, 21, and 22. For question number 5, it can be seen on Figure 23, 24, and 25.
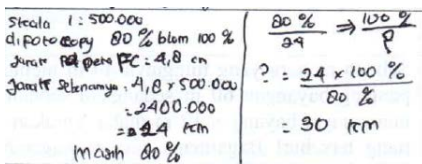

Figure 20. Answer of Subject 1 to Question 4.

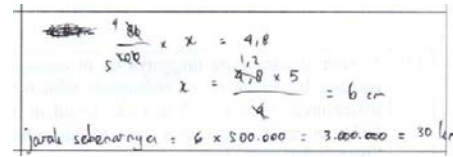**Figure 21.** Answer of Subject 5 to Question 4.

Figure 22. Answer of Subject 6 to Question 4.

Based on Figures 20 and 21 above, Subject 1 wrote the information known, then the subject determine the actual distance by multiplying the distance on the map that had been 80% minimized with 100% scale of the map then the result was the distance on the map that has been minimized. Meanwhile, Subject 5 did not wrote the information known in the question but directly determined the distance on the actual map and resulting the distance on the map which was 6 cm. Next, the subject determined the actual distance with the distance on the map formula divided with the scale (in fraction). For the answer of Subject 6 (Figure 22), in the beginning the subject determined variable x that symbolized the distance on the actual map. The subject determined x then determined the actual distance. The result of the actual distance was 30 km. The process of thinking used by the three subjects is conceptual thinking process.
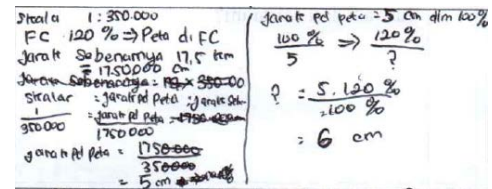

Figure 23. Answer of Subject 1 to Question 5

Figure 24. Answer of Subject 5 to Question 5.

Figure 25. Answer of Subject 6 to Question 5.

Based on Figure 23, Subject 1 wrote the known information, then determined the distance on the map which has not been enlarged. Next, the result on the map was made to a comparison that had the same value as the distance when the map was enlarged 120%. Analog on the answer of Subject 1, Subject 5 didn't wrote information of the problem and he only solve the answer. In the beginning, the subject determined the distance on the actual map then multiplied it with 120% to get the distance on the map that had been enlarged 120%. For the answer of Subject 6 (Figure 25), the subject first determined the x as the actual distance for the map photocopied 120% and got the result of 21 km. Then the value of x was converted into meter and divided by the scale. The result of the distance on the map

obtained was 6 cm. Even though the final result was the same, the thinking logic of Subject 6 was still based on the formula on Figure 7 without seeing the content which was that the actual distance would still be the same even though the map was enlarged. If the map was enlarged or minimized, the ones undergone changes was the distance on the map and the scale remained the same. The process of thinking used by both subjects was the conceptual thinking process.

The answer of Subject 2 to Question 4 can be seen in Figure 26.

Figure 26. Answer of Subject 2 to Question 4.

In the beginning, Subject 2 wrote the scale of 1:500.000 and formed it as a fraction. Next, the subject multiplied the scale with 4,8 divided by x. The value of x in this context was the actual distance. It should not be multiplied, but the subject should form the comparison equal to the following.

$$\frac{1}{500.000} = \frac{4,8}{x} \Leftrightarrow x = 4,8 \times 500.000$$

$$\Leftrightarrow x = 2.400.000 \text{ cm} = 24 \text{ km}$$

The value showed the actual distance on the map that had been minimized 80% and to determine the actual distance on the actual map there should be a formulation of a comparison as follows.

$$\frac{80}{100} = \frac{24}{x} \Leftrightarrow x = \frac{2400}{80} = 30 \text{ km}$$

It shows that Subject 2 understood the concept but was not entirely correct so that the thinking process used is semi-conceptual.

The answer of Subject 4 on Question 4 can be seen in Figure 27.

Figure 27. Answer of Subject 2 to Question 4.

Based on the answer of subject 4 above, it shown that Subject 4 formed a comparison equal to determining the actual distance which was 24 km. Though the subject did not understand that it was for 80% minimized map while the actual distance to the actual map had not been determined yet by the subject. The thinking process used is semi-conceptual thinking process.

The answer of Subject 3 on Question 4 can be seen in the following Figure 28.

Figure 28. Answer of Subject 3 to Question 4.

Subject 3 actually could determine the comparison equal to question number 4 though not entirely correct. The subject solved the problem to have the actual distance. According to the interview result, Subject 3 was in doubt with the comparison equal to the one the subject made so that the subject did not continue the calculation. The thinking process used is intuitive thinking process because the subject could not relate to the concept.

The answer of Subject 2 on Question 5 can be seen in Figure 29 below.

Figure 29. Answer of Subject 2 to Question 5.

Based on Subject2's answer above 2 (Figure 29), it should not be a multiplication operation but rather a comparison equals to the following.

$$\frac{1}{350.000} = \frac{x}{17,5 \text{ km}} \Leftrightarrow \frac{1}{350.000} = \frac{x}{1750000}$$

$$\Leftrightarrow x = \frac{1750000}{350000} = 5 \text{ cm}$$

After this the distance on the map that had been enlarged 120% should be determined. After being interviewed, Subject 2 actually understood about the question but the subject was confused in making the equal comparison. The process of thinking used was semi-conceptual thinking process.

The answer of Subject 3 on Question 5 can be seen in Figure 30.

Figure 30. The answer of Subject 3 to Question 5.

Analog on the answer of Question 4, the subject actually could determine the equal comparison from the question number 5 though not entirely correct. The subject could not solve the problem to get the distance on the map. Based on the interview result, Subject 3 was in doubt with the equal comparison made so that the subject did not continue it. The thinking process used is intuitive thinking process because the subject could not relate it with the concept.

The answer of Subject 4 on Question 5 can be seen in the following Figure 31.

Figure 31. Answer of Subject 4 to Question 5.

Analog to question number 4, according to Subject 4's answer, the subject formulated the equal comparison in order to determine the distance on the map resulting 5 cm. However, the subject did not understand that it was for 120% enlarged map. The thinking process used is semi-conceptual thinking process.

IV. DISCUSSION

Based on the students' work results, their process of thinking in solving the comparison question for the scale on the map can be seen on the following Table 1.

Table 1. The Student' Process of Thinking

	Question 1	Question 2	Question 3	Question 4	Question 5
Subject 1	C	C	C	C	C
Subject 2	SC	SC	SC	SC	SC
Subject 3	I	-	I	I	I
Subject 4	C	C	C	SC	SC
Subject 5	C	C	C	C	C
Subject 6	SC	SC	C	C	C

Information: C = Conceptual; SC = Semi Conceptual and I = Intuition

Based on Table 1, it can be seen that in solving the map scale comparison story question, there are 2 subjects who used the conceptual thinking process, 1 subject who used semi-conceptual thinking process and 1 subject who used intuitive thinking process when solving a variety of questions; there was 1 subject that used the conceptual process of thinking when solving basic questions (principal) and used the semi-conceptual when solving developing questions; and also there was 1 subject that used semi-conceptual thinking process when solving basic questions and used the conceptual process of thinking when solving developing questions. Basicly, student's thinking process until conceptual level but not all student until this level. Lecture must design the learning that can improve thinking process level.

V. CONCLUSION

The result of the research shows that in solving the map scale comparison story questions, there were 2 students that used the conceptual thinking process, 1 student that used the semi-conceptual thinking process, 1 student that used the intuitive thinking process, and also 2 students that used semi-conceptual and conceptual thinking process.

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