Characteristics of Fuzzy Domination Number In Fuzzy Digraphs

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Abstract- The purpose of this paper is to introduce the concept of fuzzy in-dominating set ,fuzzy out-dominating set ,fuzzy characteristic function of fuzzy digraph .The theorems based on fuzzy kernel ,fuzzy absorbant ,fuzzy characteristic function are proved.

Index Terms- The fuzzy digraph- in-degree and out-degree in a fuzzy digraph-the fuzzy independence number the fuzzy out-dominating set- in-dominating set- the fuzzy characteristic function.

I. Introduction

RT osenfeld[12] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths,cycles and connectedness .A.Somasundram and S.Somasundram [11]discussed domination in fuzzy graph.They defined domination using effective edges in fuzzy graph.NagoorGani and Chandrasekaran [7]discussed domination in fuzzy graph using strong arc.

Recently, G.Nirmala ,M.Vijaya [9]&G.Nirmala,K.Dhanbal [10]in their studies discussed the various concepts on related with fuzzy graph along with their properties.Proceeding in the same chain, the domination number of the fuzzy digraph, the fuzzy kernel of the fuzzy digraph and the fuzzy absorbant of the fuzzy digraph are discussed in this paper. We also introduce some new concepts as fuzzy out-dominating set of the digraph, fuzzy in-dominating set of the fuzzy digraph, fuzzy characteristic function of the fuzzy digraph.

PRELIMINARIES1.1

Definition 1.1

A fuzzy digraph G_D= (σ_D,μ_D) is a pair of function σ_D :V→[0,1] and μ_D : V×V→[0,1] where $\mu_D(u,v) \leq \sigma_D(u) \wedge \sigma_D(v)$ for u,v ϵ V, σ_D a fuzzy set of V,(V$\times V$, μ_D) a fuzzy relation on V and μ_D is a set of fuzzy directed edges are called fuzzy arcs.

Let G_D= (σ_D,μ_D) be a fuzzy digraph .If $\sigma_D(u) > 0$, for u in V, then u is called a vertex of G_D.If$\sigma_D(u) = 0$ for u in V,then u is called an empty vertex of G_D.If$\mu_D(u,v)=0$, then (u,v) is called an empty arc of G_D.

Definition 1.2

Let $G_{D1}= (\sigma_{D1}, \mu_{D1})$ and $G_{D2}= (\sigma_{D2}, \mu_{D2})$ be two fuzzy digraphs of V . Then$G_{D2}= (\sigma_{D2}, \mu_{D2})$ called a fuzzy sub-digraph of $G_{D1}= (\sigma_{D1}, \mu_{D1})$ if

$\sigma_{D2}(u) \leq \sigma_{D1}(u)$for all u in V and

$\mu_{D2}(u,v) \leq \mu_{D1}(u,v)$ for all u,v in V, then write $G_{D2} \leq G_{D1}$.

Definition 1.3

An fuzzy arc μ(u,v) is said to be directed from σ(u) to σ(v) in which case σ(u) is said to be a predecessor of σ(v), σ(v) is a successor of σ(u). In this case also use the notational equivalence μ(u,v)≡σ(u)→σ(v).

Definition 1.4

For any real numberα,$0< \alpha \leq 1$.A fuzzy directed α walk w_α from a vertex $\sigma_D(x_j)$ to $\sigma_D(x_{j+1})$ is an alternating sequence of vertices and edges, beginning with$\sigma_D(x_j)$ and ending with$\sigma_D(x_{j+1})$, such that$\sigma_D(x_j) \geq \alpha$, $0 \leq j \leq n$, and $\mu_D(x_{i-1}, x_i) \geq \alpha$, $0 \leq i \leq n$ each arc is oriented from the vertex preceding it to the vertex following it. No arc in a fuzzy directed walk w_α appears more than once, but a vertex may appears more than once, as in the case of fuzzy undirected graphs .

Definition 1.5

In a fuzzy directed walk w_α, the starting and ending vertex are the same then the fuzzy directed walk $w_\alpha$is called fuzzy closed directed walk w_α.otherwise it is called a fuzzy open directed walk

Definition 1.6

A fuzzy semi-walk in a fuzzy digraph is a walk in the corresponding fuzzy undirected graph, but is not a fuzzy directed walk. fuzzy walk in a digraph can mean either a fuzzy directed walk or a fuzzy undirected walk.

Definition 1.7

For any real number α,$0< \alpha \leq 1$, a directed α -path ρ_α in a fuzzy digraph $G_D = (\sigma_D, \mu_D)$ is a sequence of distinct nodes $x_0, x_1, x_2 \dots \dots \dots x_n$ such that$\sigma_D(x_j) \geq \alpha$, $0 \leq j \leq n$, and

$\mu_D(x_{i-1}, x_i) \geq \alpha$, $0 \leq i \leq n$, $n \geq 0$, is called the length of ρ_α. In this case , write $\rho_\alpha=(x_0, x_1, x_2 \ldots \ldots \ldots . . x_n)$ and ρ_α is called a $(x_0, x_n)-\alpha$ path.

Definition 1.8

If the directed α-Path ρ_α has length $n > 0$, the strength of ρ_α is defined as $\{\bigwedge_{i=1}^{n} \mu_D(x_{i-1}, x_i)\} \wedge \{\bigwedge_{j=1}^{n} \sigma_D(x_j)\}$ if the directed α-path ρ_α has length 0, it is convenient to define its strength to be $\sigma_D(x_0)$.

Definition 1.9

A directed α-path $\rho_\alpha=(x_0, x_1, x_2 \ldots \ldots \ldots . . x_n)$ is called a directed α-cycle if $x_0 = x_n$ and $n \geq 3$.

Definition 1.10

A fuzzy digraph G_D is said to be strongly connected if there is at least one fuzzy directed path from every vertex to every other vertex.

Definition 1.11

A fuzzy digraph G_D is said to be weakly connected if its corresponding fuzzy undirected graph is connected but G_D is not fuzzy strongly connected.

Definition 1.12

In a fuzzy digraph G_D a fuzzy closed directed walk which traverses every arcs of G_D exactly once is called a fuzzy directed Euler line. A fuzzy digraph containing a fuzzy directed Euler line is called a fuzzy Euler digraph .

Definition 1.13

A fuzzy digraph G_D is said to be fuzzy unicursal if it contains an fuzzy open Euler walk.

Definition 1.14

Let $G(\sigma_D, \mu_D)$ be fuzzy digraph on G*(V,A).The vertex v_i , which edge e_k is incident out of, is called the initial vertex of e_k. The vertex v_j , which e_k is incident into, is called the terminal vertex of e_k.

Definition 1.15

Let $G(\sigma_D, \mu_D)$ be fuzzy digraph on G*(V,A).The outset of a vertex u is the set O(u)={v: (u,v)ϵA}.And the inset I(u)={w: (w,u)ϵA}.also $O[u]=O(u) \cup \{u\}$ and $I[u]=I(u) \cup \{u\}$

Definition 1.16

The in-degree of any vertex u in the fuzzy digraph G_D is the sum of membership of all those arcs which are incident into the vertex u. that is the in-degree of the vertex in(u)=ı|(u)|.

Definition 1.17

The out-degree of any vertex u in the fuzzy digraph G_D is the sum of membership of all those arcs which are incident out of the vertex u. that is the out degree of a vertex is od(u)=|O(u)|.

Definition 1.18

A fuzzy digraph is said to be fuzzy balanced digraph or fuzzy isograph if for every vertex $\sigma(v_i)$ the in-degree equals the out-degree

Definition 1.19

A fuzzy balanced digraph is said to be fuzzy regular digraph if every vertex has the same in-degree and out-degree as every other vertex.

Definition 1.20

Two vertices in a fuzzy digraphs G_D are said to be fuzzy independent if there is no strong arc between them.

Definition 1.21

A subset S of V is said to be fuzzy independent set of $G_D(\sigma_D, \mu_D)$ on G*(V,A) if every two vertices of S are fuzzy independent.

Definition 1.22

The fuzzy independence number $\beta_0(G_D)$ is the maximum cardinality of an independent set in G_D.

Definition 1.23

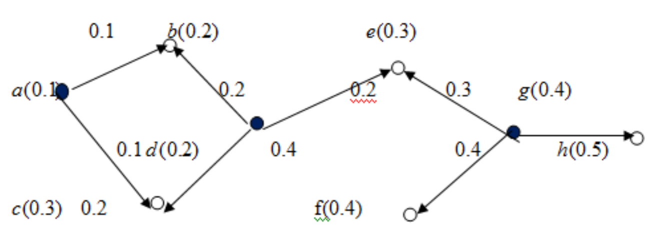
A set $S \subseteq V$ is called fuzzy absorbant if for every $v \in V$ - S, there exist a $u \in S$ which is a successor of v. that is $v \rightarrow u$ is an arc in μ_D

II. FUZZY DOMINATION IN FUZZY DIGRAPHS

Definition 2.1 A subset $S \subseteq V$ is a fuzzy out dominating set of G_D .If every vertex $v \in V - S$,there exists u in S such that $\mu_D(u,v)=\sigma_D(u) \wedge \sigma_D(v)$.

Definition 2.2

The fuzzy out - domination number of a fuzzy digraph G_D is the minimum cardinality taken over all fuzzy out dominating sets in G_D. *and it is denoted by* $\gamma_{od}(G_D)$



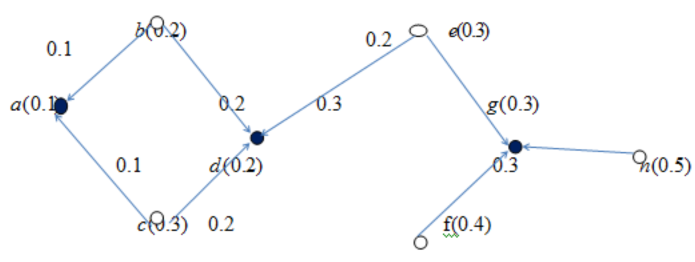
The fuzzy out dominating set S={a,d,g}

The fuzzy outdominating number $\gamma_{od}(G_D)$ =0.7

Definition 2.3 A subset S⊆V is a fuzzy in dominating set of G_D .If every vertex v ∈ V –S ,there exists u in S such that $\mu_D(v,u)=\sigma_D(v) \wedge \sigma_D(u)$.

Definition 2.4

The fuzzy in - domination number of a fuzzy digraph G_D is the minimum cardinality taken over all fuzzy in dominating sets in G_D.The fuzzy in-dominating number is denoted by $\gamma_{in}(G_D)$



The fuzzy in-dominating set S={a,d,g}

The fuzzy in-dominating number $\gamma_{in}(G_D)$ =0.6

Definition 2.5

Let $G_D(\sigma_D, \mu_D)$ be a fuzzy digraph on G(V,A) A set S ⊆ V is a fuzzy kernel in fuzzy digraph if it is both fuzzy independent and fuzzy absorbent in fuzzy digraph.

Definition 2.6

Let $G_D(\sigma_D, \mu_D)$ be a fuzzy digraph on G^*(V,A) and S be the fuzzy kernel of the fuzzy digraph then its characteristic function is defined by ϕ_{DS}(u)=1 if u∈S.

Theorem 2.1

A fuzzy digraph $G_D=(\sigma_D,\mu_D)$ is fuzzy regular and fuzzy Eulerian if and only if G_D is fuzzy strongly connected and for each of its vertices u, od(u)= in(u) that is
│O (u)│=│ I (u)│

Proof:

Necessity Let G_D be a fuzzy Euler,fuzzy regular digraph. Therefore, it contains a fuzzy Euler walk, say W_αIn traversing W_α, every time a vertex u is encountered we pass along an arc incident towards u and then an edge incident away from u. This is true for all the vertices of W_α, including the initial vertex ofW_α , say u, because we began W_α by traversing an arc incident away from initial vertex of W_α, and ended W_α by traversing an arc incident towards u.

Sufficiently Let for every vertex u in G_D, in(u) = od(u). For any arbitrary vertex u in G_D, we identify a fuzzy walk, starting at u and traversing the arcof G_D at most once each. This traversing is continued till it is impossible to traverse further. Since every vertex has the same number of edges incident towards it as away from it, we can leave any vertex that we enter along the walk and the traversal then stops at u. Let the walk traversed so far be denoted byW_α . IfW_α includes all arcs of μ_D, then the result follows. If not, we remove from G_D all the arcs of W_α and consider the remainder of μ_D. By assumption, each vertex in the remaining fuzzy digraph, say G_{D1}, is such that the number of arcs directed towards it equals the number of arcs directed away from it. Further, W_α andG_{D1} have a vertex, say v in common, since G_D is strongly connected. Starting at v, repeat the process of tracing a walk in G_{D1}. If this walk does not contain all the arcs of G_{D1}, the process is repeated until a fuzzy closed walk that traverses each of the arcs of G_D exactly once is obtained. Hence G_D is fuzzy Euler graph.

Theorem 2.2

A necessary and sufficient condition for a set S to be a fuzzy kernel of a fuzzy digraph $G_D=(\sigma_D, \mu_D)$ is that its characteristic function $\phi_{DS}(u)$ satisfy the condition: $\phi_{DS}(u)$=1-max{$\phi_{DS}(v)$:$v \in O$(u)}.

Proof:

Let S be fuzzy kernel of a digraph $G_D=(\sigma_D, \mu_D)$ and let $\phi_{DS}(u)$ be its characteristic function. Then by definition, if u∈S , then $\phi_{DS}(u)$=1.And since S is fuzzy independent , for all vertices $v \in O$(u),v∉ S, and $\phi_{DS}(v)$=0. Thus max{$\phi_{DS}(v)$:$v \in O$(u)}=0 and therefore, if u∈S, then $\phi_{DS}(u)$=1-max{$\phi_{DS}(v)$:$v \in O$(u)}.

Similarly, if u∉S then $\phi_{DS}(u)$=0.But since S is a fuzzy kernel, S is fuzzy absorbant, and therefore there must be a vertex v∈ S which is a successor of u,that is$v\in O$(u). This means that $\phi_{DS}(v)$=1 and max{$\phi_{DS}(v)$:$v \in O$(u)}=1.

Thus $\phi_{DS}(u)$=0=1-max{$\phi_{DS}(v)$:$v \in O$(u)}=1-1=0.

Thus $\phi_{DS}(u)$=1-max{$\phi_{DS}(v)$:$v \in O$(u)} holds for every vertex u$\in \sigma_D$.

Conversely, assume that Thus $\phi_{DS}(u)$=1-max{$\phi_{DS}(v)$:$v \in O$(u)} holds for some set S⊆V, its characteristic function ϕ_{DS}, and every vertex u$\in V$.To show that S is kernel of G_D, that is ,S is both fuzzy independent and fuzzy absorbant .Let u$\in S$.Then $\phi_{DS}(u)$=1 and $\phi_{DS}(v)$=0 for every $v \in O$(u).

But this implies that no vertex $v \in O$(u) is in S, that S is fuzzy independent.

Similarly, consider any vertex $v \in V$-S.We know that $\phi_{DS}(v)$=0 and that 0=1-max{$\phi_{DS}(u)$:$u \in O$(v)}.But this implies that max{ $\phi_{DS}(u)$: $u \in O(v)$}=1, which means that for at least one vertex $u \in O(v)$, $\phi_{DS}(u)$=1.Therefore, $u \in$ S and there is at least one vertex in S which is a successor of v.Thus,S is fuzzy absorbant. Since S is both fuzzy independent and fuzzy absorbant, it is a fuzzy kernel.

III. CONCLUSION

In this paper we define the concepts of fuzzy digraph, fuzzy out-dominating set ,fuzzy in-dominating set ,fuzzy characteristic function,further we proved the theorems based on fuzzy regular digraph,fuzzy characteristic function.

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