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Vibrational Analysis of Passive Suspension System for Solar Vehicle

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Abstract- Suspension system in an automobile determines the riding comfort of passengers and the amount of damage to the vehicle. In this paper, the passive suspension system, quarter car model is analyzed to improve handling and ride performance of a vehicle. Two degree and three degree of freedom with and without tire damping is analyzed by using Matlab Simulink. Comparisons between passive suspension system with tire damping and without tire damping are performed by using different damping ratios range from 0.2 to 0.4. As a results, the sprung mass displacement of two degree of freedom suspension system with and without tire damping for front suspension are 0.1503 m and 0.1509 m respectively and rear suspension are 0.1548 m and 0.1564 m. For three degree of freedom, the sprung mass displacements of front suspension for with and without tire damping are 0.1301 m and 0.1351 m respectively. Sprung mass displacement for rear suspension for with and without tire damping are 0.1331 m and 0.1393 m. Seat mass displacement of front suspension for with and without tire damping are 0.1581 m and 0.1836 m. Seat mass displacement of rear suspension for with and without tire damping are 0.1590 m and 0.1854 m. The results show that suspension system of both types with tire damping is more passenger comfortable than suspension system without tire damping. Moreover, it is observed that the greater the damping effect, the more the ride handling and ride comfort of passenger.

Index Terms- Automobile, Damping effect, Riding comfort, Solar vehicle, Suspension system

I. Introduction

A solar vehicle is an electric vehicle powered completely or significantly by direct solar energy. Usually, photovoltaic (PV) cells contained in solar panels convert the sun's energy directly into electric energy. The term "solar vehicle" usually implies that solar energy is used to power all or part of a vehicle's propulsion [9]. The brief study of solar car is efficient in our daily life because now day's pollution and fuel rate is very big problem and many people having fuel cars. Solar energy is being used for car, besides the control of vehicular pollution in the city, less consumption of fuel, solar cars are effective reducing global warming and environment problem in big frame.

There are many components in a solar car such as solar panel, chassis frame, steering system, transmission system, suspension system, brake system, axle, wheel, motor, etc. A solar car works as the following principle. Firstly, solar panel converts light energy from the sun into the electrical power. Solar controller converts the energy collected from the solar array to the proper system voltage, so that the batteries and motor can use it. Then, motor controller adjusts the amount of energy that flows to the motor. Finally, the motor uses that energy to drive the transmission system.

A car suspension system is the mechanism that physically separates the car body from the wheels of the car. The performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Suspension consists of the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other meaning, suspension system is a mechanism that physically separates the car body from the car wheel. The main function of the vehicle suspension system is to minimize the vertical acceleration transmitted to the passenger which directly provides road comfort [5].

At present, three types of vehicle suspensions are used: passive, active and semi-active suspension system. The passive system is the most used type in automobile suspensions. The main reasons are the simplicity, low cost and reliability of this solution. A spring and a damper compose this suspension system, both fixed between the wheel supporting structure (unsprung mass) and the vehicle body (sprung mass). The damper is a cylinder filled with hydraulic oil or a compressed gas. Inside the cylinder there is a piston driven from the outside by a rod. Allowing the piston displacement there is a hole, which permits the fluid pass between the parts of the cylinder. This fluid flow generates a reaction force that is proportional to the relative speed between sprung and unsprung masses. The damping is achieved converting the energy of the oscillations in heat[5].

The active suspension system is also comprised of an actuator, sensors, and a control programming unit (CPU). Actually, the shock absorber is replaced by an active force actuator. The operational conditions of the vehicle are continuously controlled by sensors that measure the velocity of the sprung and un-sprung masses and lead it to the CPU that ensures correct impulses for the actuator, which creates the desired active damping forces when required. Semi-active suspension systems were first proposed in the early 1970's. The semi-active suspension system is based on passive and active systems. The presented one contains instead a passive shock absorber,

and a variable shock absorber as an active damping force that is automatically controlled by an integrated regulator. In this type of system, the conventional spring element is retained, but the damper is replaced with a controllable damper [6].

In this work, two degrees and three degrees of freedom passive suspension system are designed. Sprung mass displacement for passive suspension system with and without tire damping are analysed by changing different damping effects. Matlab Simulink is used for analysing of the suspension system.

II. Quarter Car Model

A quarter car model consists of the wheel and its attachments, the tire, the suspension elements and quarter chassis and its rigidly connected parts. Quarter car model is very often used for suspension analysis because it is simple and can capture important characteristics of full model. The quarter car model is used extensively in studying the vehicle dynamics. Many researchers consider this model as a two degree of freedom by considering only the tire wheel assembly and the sprung mass. In this work, not only two degree of freedom but also three degree of freedom quarter model is analyzed by considering the passengers and seat. Parameters of quarter car model for front and rear suspension are shown in Table I.

TABLE I
Parameter of Quarter Car Model Suspension System

Parameters	Front suspension	Rear suspension	Units
Passenger seat stiffness	3923.966	6014.868	N/m
Suspension spring stiffness	17822.932	36049.070	N/m
Tire stiffness	190000	190000	N/m
Tire damping coefficient	1150	1150	Ns/m
Road input	0.1	0.1	m

In design calculation of passive suspension system, firstly, the total sprung weight, unsprung weight and passenger seat weight are calculated. Then, sprung mass displacement of suspension system are analysed by using Matlab program. Table II shows the specification and technical data of a solar vehicle.

TABLE II
Specifications of Solar Vehicle

Technical Category	Dimensions	units
Vehicle gross weight	490	kg
Vehicle net weight	310	kg
Weight of solar panel	46	kg
Weight of motor	24	kg
Weight of batteries	168	kg
Seating capacity	2	-
Weight of person	180	kg
Weight of seat	5	kg

The design procedure of total sprung weight involves the following steps:

A. *Calculation of the unsprung weight*

In today's standard size automobile, the weight of unsprung components is normally in the range of 13 to15 percent of the vehicle net weight.

$$\text{Unsprung weight} = 0.15 \times \text{vehicle net weight} \tag{1}$$

B. *Calculation of total unsprung weight*

Total unsprung weight can be calculated as

$$\text{Total unsprung weight} = \text{unsprung weight} + \text{motor weight} \tag{2}$$

C. Calculation of the sprung weight

Gross vehicle weight is the sum of unsprung weight and sprung weight. Sprung weight can be calculated as

$$\text{Sprung weight} = \text{gross weight} - \text{unsprung weight} \tag{3}$$

D. Calculation of total sprung weight

The the total sprung weight of solar vehicle can be calculated as

$$\text{Total sprung weight} = \text{sprung weight} + \text{solar weight} \tag{4}$$

E. Calculation of passenger and seat weight

The passenger and seat mass of solar vehicle can be calculated as

$$\text{Passenger and seat weight} = \text{person weight} + \text{seat weight} \tag{5}$$

F. Calculation of weight on quarter car model

By using quarter car model approach, load on each suspension is one fourth of the weight on suspension.

$$\text{Load on each suspension} = \frac{1}{4} \times \text{weight on suspension} \tag{6}$$

G. Calculation of critical damping

Critical damping can be calculated as

$$c_c = 2\sqrt{k \times m} \tag{7}$$

H. Calculation of damping coefficient

Suspension damping and seat damping can be calculated as follow:

Damping ratio is 0.3 [2]

$$c_s = \xi \times c_c \tag{8}$$

The result table of weights acting on quarter car model are shown in Table III

TABLE III
RESULTS TABLE OF WEIGHT ACTION ON QUARTER CAR MODEL

Parameters	Results	units
Total sprung weight	116.375	kg
Total unsprung weight	17.625	kg
Seat and passenger weight	46.25	kg
Front suspension damping coefficient	864.113	N/m
Rear suspension damping coefficient	1228.933	N/m
Front seat damping coefficient	255.605	Ns/m
Rear seat damping coefficient	316.461	Ns/m

III. TWO DEGREE OF FREEDOM PASSIVE SUSPENSION

Sprung mass displacement is considered depend on two degree of freedom quarter car model depend on without and without tire damping in this part.

A. Two Degree of Freedom without Tire Damping

Modeling of two degree of freedom quarter car suspension without tire damping is shown in Figure 1.

Let $x_1 \rangle x_2 \rangle y$

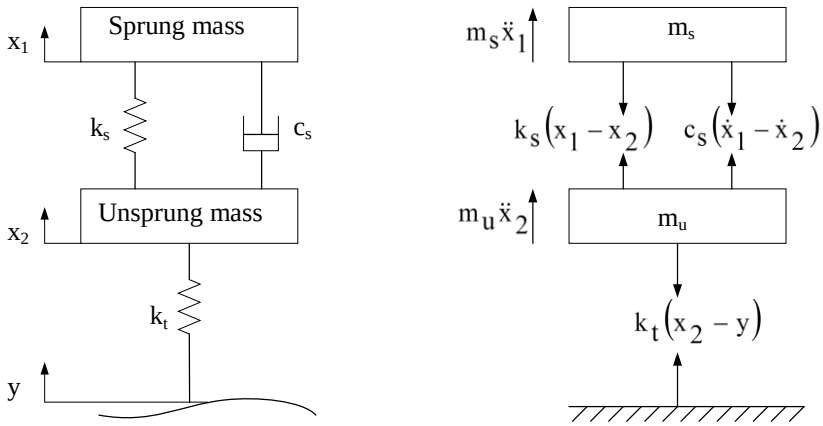

Figure 1. Modeling of Two Degree of Freedom without Tire Damping

where,

m_s	=	sprung mass
m_u	=	unsprung mass
k_s	=	suspension spring stiffness
k_t	=	tire stiffness
c_s	=	suspension damping coefficient
y	=	road input (height of speed bump)
x_1	=	sprung mass vertical movement
x_2	=	unsprung mass vertical movement

The equations of motion for sprung and unsprung mass are written as follows:

For sprung mass,

$$m_s\ddot{x}_1 = -k_s(x_1 - x_2) - c_s(\dot{x}_1 - \dot{x}_2)$$

For unsprung mass,

$$m_u\ddot{x}_2 = -k_t(x_2 - y_r) + k_s(x_1 - x_2) + c_s(\dot{x}_1 - \dot{x}_2)$$

Let $Z_1 = x_1, \quad Z_2 = \dot{x}_1,$

$Z_3 = x_2, \quad Z_4 = \dot{x}_2$

Sprung and unsprung mass equation are become

$$\dot{Z}_2 = -\frac{k_s}{m_s}Z_1 + \frac{k_s}{m_s}Z_3 - \frac{c_s}{m_s}Z_2 + \frac{c_s}{m_s}Z_4$$

$$\dot{Z}_4 = \frac{k_s}{m_u}Z_1 - \frac{k_s}{m_u}Z_3 + \frac{c_s}{m_u}Z_2 - \frac{c_s}{m_u}Z_4 - \frac{k_t}{m_u}Z_3 + \frac{k_t}{m_u}y$$

State space model

$$\dot{Z} = AZ + Bu$$

$$Y = CZ + Du$$

where,

A	=	state space matrix
B	=	input matrix
C	=	output matrix
D	=	direct transmission matrix
u	=	input of system

State variable are Z_1, Z_2, Z_3, Z_4

State space vector matrix can be written as

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{(k_s+k_t)}{m_u} & -\frac{c_s}{m_u} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [y]$$

Output matrix can be written as

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [y]$$

Matlab program can be used to analyze sprung mass displacement and settling time of a two degree of freedom quarter car model.

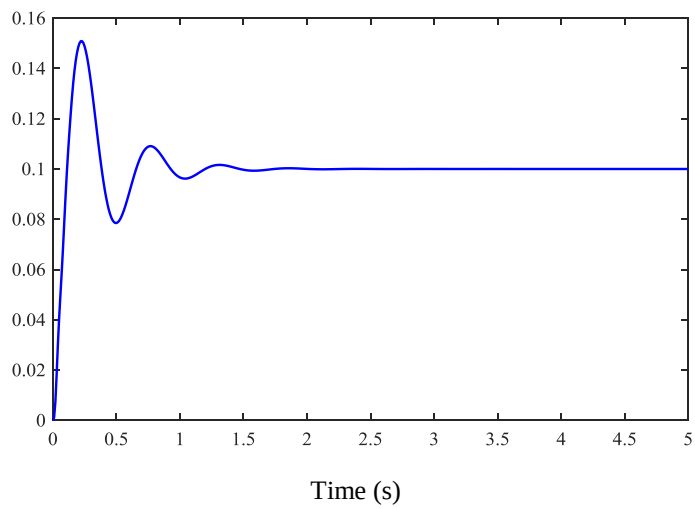

Figure 2. Sprung Mass Displacement without Tire Damping for Front Suspension

The sprung mass displacement of front suspension without tire damping is shown in Figure 2. According from these figures, maximum overshoot is 0.1559 m and settling time occurs at 2 sec.

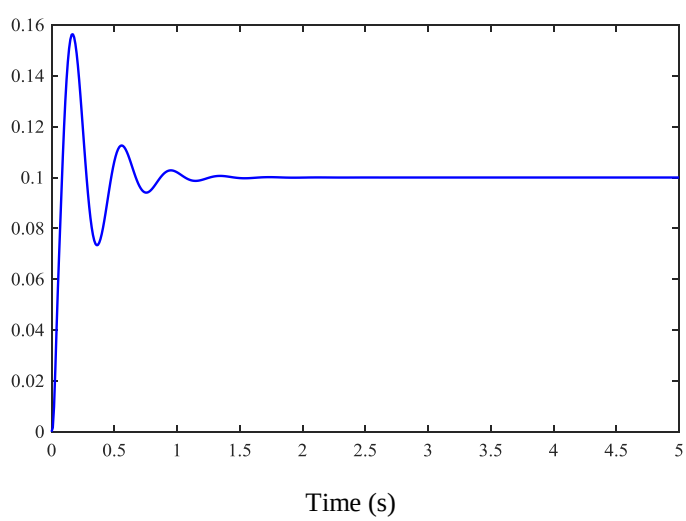

Figure 3. Sprung Mass Displacement without Tire Damping for Rear Suspension

The sprung mass displacement of rear suspension without tire damping is shown in Figure 3. According from these figures, maximum overshoot is 0.1564 m and settling time occur at 2 sec.

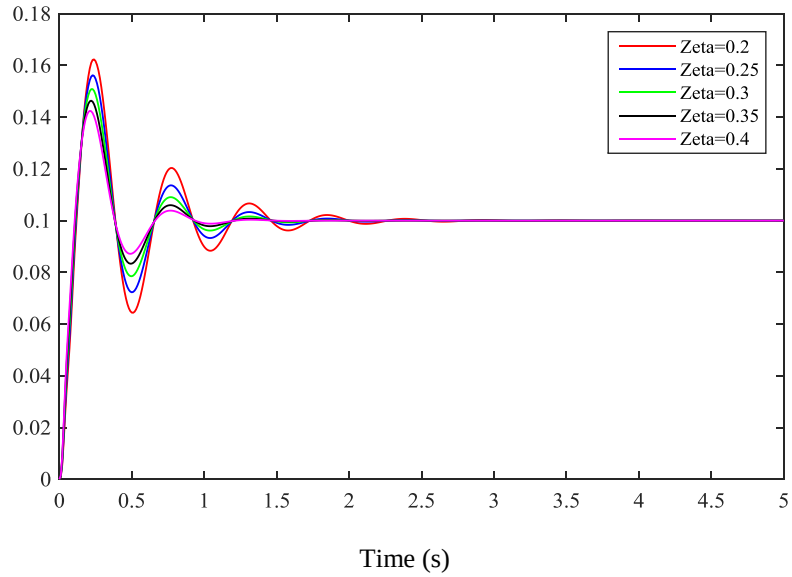

Figure 4. Sprung Mass Displacement using Various Damping Ratio for Front Suspension without Tire Damping

Sprung mass displacement is also analyzed by using different damping ratios. Figure 4 show sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1624m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

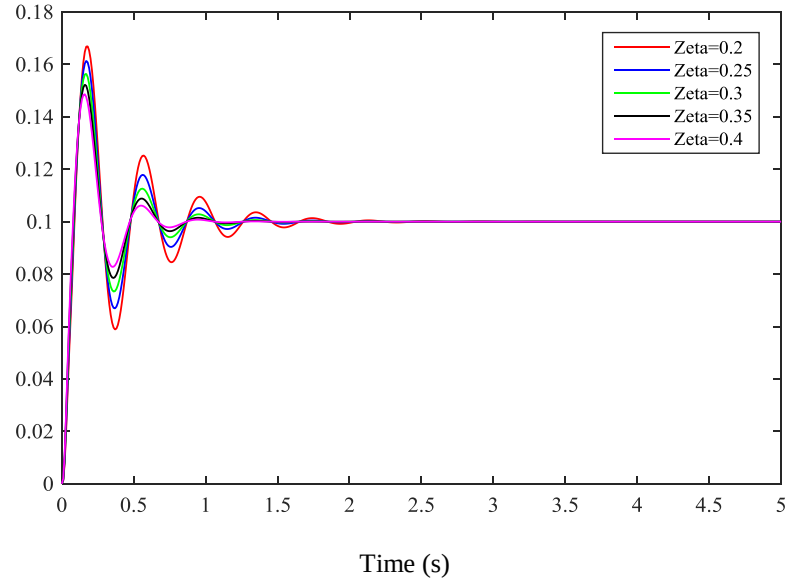

Figure 5. Sprung Mass Displacement using Various Damping Ratio for Rear Suspension without Tire Damping

Sprung mass displacement using various damping ratio for rear suspension without tire damping is shown in Figure 5. As a result, maximum displacement is 0.1669m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

B. Two Degree of Freedom Suspension System with Tire Damping

Modeling of two degree of freedom quarter car suspension with tire damping is shown in Figure 6.

Let $x_1 \rangle x_2 \rangle y$

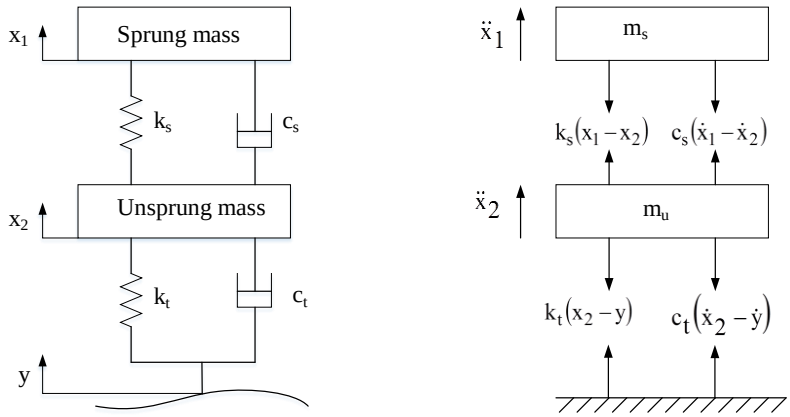

Figure 6. Modeling of Two Degree of Freedom with Tire Damping

The equations of motion for sprung and unsprung mass are written as follows:

For sprung mass,

$$m_s\ddot{x}_1 = -k_s(x_1 - x_2) - c_s(\dot{x}_1 - \dot{x}_2)$$

For unsprung mass,

$$m_u\ddot{x}_2 = k_s(x_1 - x_2) - k_t(x_2 - y_r) - c_t(\dot{x}_2 - \dot{y}_r) + c_s(\dot{x}_1 - \dot{x}_2)$$

Let, $\dot{x}_1 = V_1, \quad \dot{x}_2 = V_2, \quad \Delta = (x_1 - x_2), \quad \dot{\Delta} = (V_1 - V_2)$

From sprung mass equation

$$m_s\dot{V}_1 + k_s\Delta + c_s(V_1 - V_2) = 0$$

$$\dot{V}_1 = -\frac{k_s}{m_s}\Delta - \frac{c_s}{m_s}(V_1 - V_2) \tag{9}$$

From unsprung mass equation

$$m_u\dot{V}_2 = -k_t x_2 - c_t V_2 + k_s\Delta + c_s(V_1 - V_2) + k_t y + c_t\dot{y}$$

$$\dot{V}_2 - \frac{c_t}{m_u}\dot{y}_r = -\frac{k_t}{m_u}x_2 + \frac{k_s}{m_u}\Delta + \frac{c_s}{m_u}V_1 - \left(\frac{c_s + c_t}{m_u}\right)V_2 + \frac{k_t}{m_u}y \tag{10}$$

Let $\dot{T} = V_2 - \frac{c_t}{m_u}\dot{y}_r$

$$T = V_2 - \frac{c_t}{m_u}y$$

$$V_2 = T + \frac{c_t}{m_u}y \tag{11}$$

Substitute Equation (11) in Equation (10)

$$\dot{T} = -\frac{k_t}{m_u}x_2 + \frac{k_s}{m_u}\Delta + \frac{c_s}{m_u}V_1 - \left(\frac{c_s + c_t}{m_u}\right)T + \left[-\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2} + \frac{k_t}{m_u}\right]y$$

$$\dot{\Delta} = V_1 - \left[T + \frac{c_t}{m_u}y\right]$$

From Equation (8)

$$\dot{V}_1 = -\frac{k_s}{m_s}\Delta - \frac{c_s}{m_s}V_1 + \frac{c_s}{m_s}\left[T + \frac{c_t}{m_u}y\right]$$

$$\dot{V}_1 = -\frac{k_s}{m_s}\Delta - \frac{c_s}{m_s}V_1 + \frac{c_s}{m_s}T + \left(\frac{c_s c_t}{m_s m_u}\right)y$$

State space model

$$\dot{Z} = AZ + Bu$$

$$Y = CZ + Du$$

State variable are x_2, Δ, V_1, T

State space vector matrix can be written as

$$\begin{bmatrix} \dot{x}_2 \\ \dot{\Delta} \\ \dot{V}_1 \\ \dot{T} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ -\frac{k_t}{m_u} & \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{(c_s + c_t)}{m_u} \end{bmatrix} \begin{bmatrix} x_2 \\ \Delta \\ V_1 \\ T \end{bmatrix} + \begin{bmatrix} \frac{c_t}{m_u} \\ -\frac{c_t}{m_u} \\ \frac{c_s c_t}{m_s m_u} \\ -\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2} + \frac{k_t}{m_u} \end{bmatrix} [y]$$

Output matrix

$$\begin{bmatrix} x_1 \\ V_1 \\ x_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ \Delta \\ V_1 \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix} [y]$$

Matlab program can be used to analyze sprung mass displacement and settling time of a two degree of freedom quarter car model.

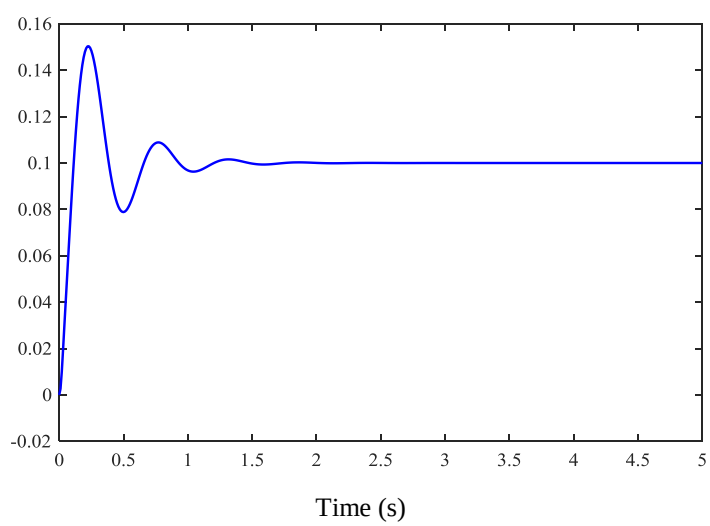

Figure 7. Sprung Mass Displacement with Tire Damping for Front Suspension

The sprung mass displacement of front suspension with tire damping is shown in Figure 7. In this figure, maximum overshoot is 0.1503 m and settling time occurs at 2 sec.

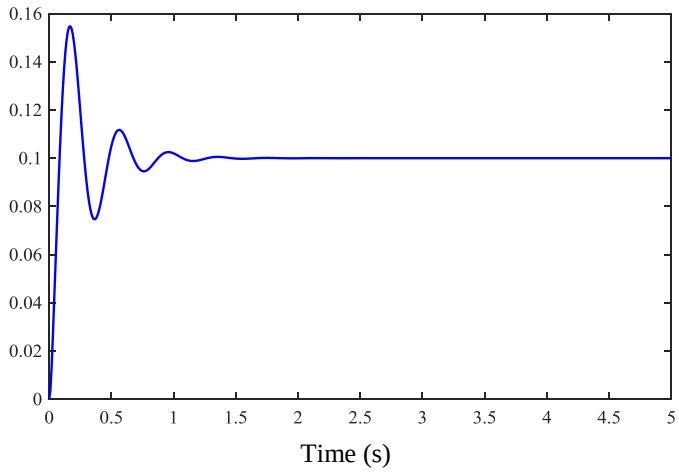

Figure 8. Sprung Mass Displacement with Tire Damping for Rear Suspension

The sprung mass displacement of rear suspension with tire damping is shown in Figure 8. In this figure, maximum overshoot is 0.1548 m and settling time occurs at 2 sec.

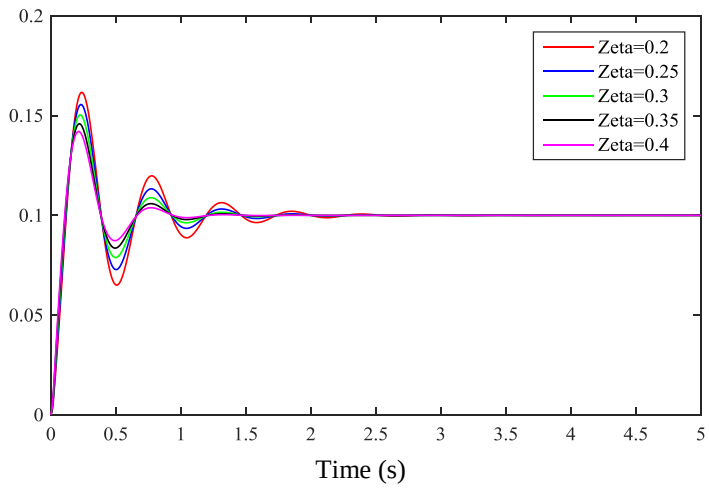

Figure 9. Sprung Mass Displacement using Various Damping Ratio for Front Suspension with Tire Damping

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 9 show sprung mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1617m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

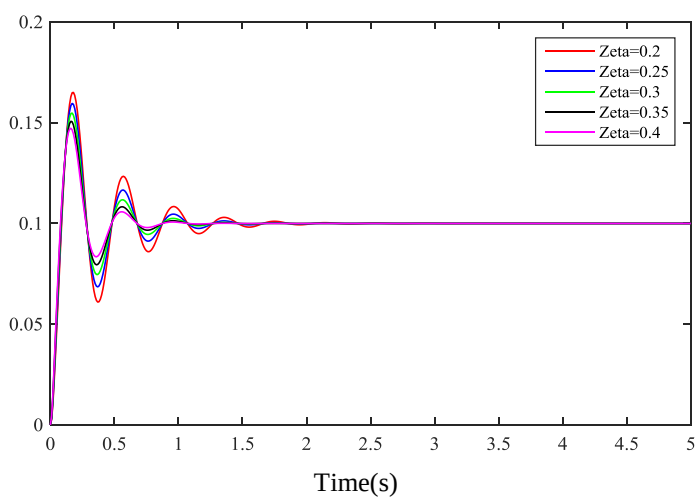

Figure 10. Sprung Mass Displacement using Various Damping Ratio for Rear Suspension with Tire Damping

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 10 shows sprung mass displacement using various damping ratio for rear suspension with tire damping. As a result, maximum displacement is 0.1651 m and maximum settling time is 2.5s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

C. Comparison of Two Degree of Freedom Suspension System without and with Tire Damping

Sprung mass displacement of two DOF suspension system without tire damping is compared with suspension system with tire damping for front and rear suspension. The comparison for front suspension is shown in Table IV.

TABLE IV
COMPARISON OF SPRUNG MASS DISPLACEMENT FOR FRONT SUSPENSION

Damping ratio ξ	Sprung mass displacement with tire damping , X_1 , m	Sprung mass displacement without tire damping, X_1 , m
$\xi = 0.2$	0.1617	0.1624
$\xi = 0.25$	0.1556	0.1562
$\xi = 0.3$	0.1503	0.1509
$\xi = 0.35$	0.1460	0.1464
$\xi = 0.4$	0.1421	0.1425

In Table IV, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

TABLE V
COMPARISON OF SPRUNG MASS DISPLACEMENT FOR REAR SUSPENSION

Damping ratio ξ	Sprung mass displacement with tire damping , X_1 , m	Sprung mass displacement without tire damping, X_1 , m
$\xi = 0.2$	0.1651	0.1669
$\xi = 0.25$	0.1596	0.1614
$\xi = 0.3$	0.1548	0.1564
$\xi = 0.35$	0.1507	0.1523
$\xi = 0.4$	0.1449	0.1486

The comparison of sprung mass displacement with and without tire damping for rear suspension is shown in Table V. In Table V, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

IV. THREE DEGREE OF FREEDOM PASSIVE SUSPENSION

Seat displacement and sprung mass displacement are also considered depend on three degree of freedom quarter car model.

A. Three Degree of Freedom Suspension System without Tire Damping

Modeling of three degree of freedom, quarter car model without tire damping is shown in Figure 11.

Let $x_1 \rangle x_2 \rangle x_3 \rangle y$

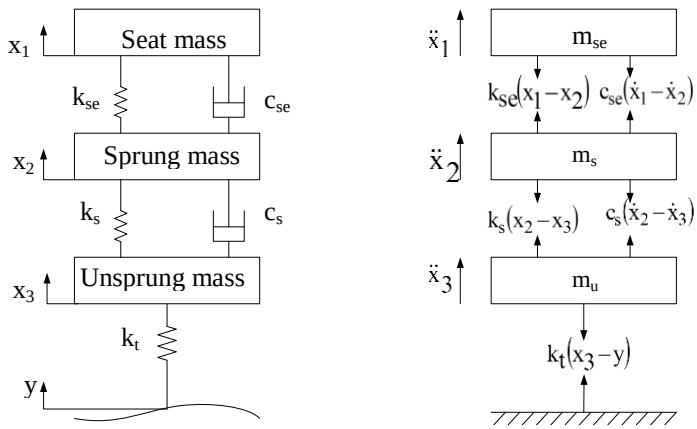

Figure 11. Modeling of Three Degree of Freedom without Tire Damping

where,

m_{se}	=	passenger and seat mass
k_{se}	=	passenger seat stiffness
k_s	=	suspension spring stiffness
k_t	=	tire stiffness
c_s	=	suspension damping coefficient
x_1	=	passenger seat mass vertical movement
x_2	=	sprung mass vertical movement
x_3	=	unsprung mass vertical movement

Equation of motion can be written from the free body diagram;

For seat and person mass:

$$m_{se}\ddot{x}_1 = -k_{se}(x_1 - x_2) - c_{se}(\dot{x}_1 - \dot{x}_2)$$

For sprung mass:

$$m_s\ddot{x}_2 = k_{se}(x_1 - x_2) + c_{se}(\dot{x}_1 - \dot{x}_2) - k_s(x_2 - x_3) - c_s(\dot{x}_2 - \dot{x}_3)$$

For unsprung mass:

$$m_u\ddot{x}_3 = k_s(x_2 - x_3) + c_s(\dot{x}_2 - \dot{x}_3) - k_t(x_3 - y)$$

Let $Z_1 = x_1$, $Z_2 = \dot{x}_1$, $Z_3 = x_2$, $Z_4 = \dot{x}_2$, $Z_5 = x_3$, $Z_6 = \dot{x}_3$

From seat and person mass equation

$$m_{se}\dot{Z}_2 = -k_{se}(Z_1 - Z_3) - c_{se}(Z_2 - Z_4)$$

$$\dot{Z}_2 = -\frac{k_{se}}{m_{se}}Z_1 - \frac{c_{se}}{m_{se}}Z_2 + \frac{k_{se}}{m_{se}}Z_3 + \frac{c_{se}}{m_{se}}Z_4$$

From sprung mass equation

$$m_s\dot{Z}_4 = k_{se}(Z_1 - Z_3) + c_{se}(Z_2 - Z_4) - k_s(Z_3 - Z_5) - c_s(Z_4 - Z_6)$$

$$\dot{Z}_4 = \frac{k_{se}}{m_s}Z_1 + \frac{c_{se}}{m_s}Z_2 - \frac{k_{se}}{m_s}Z_3 - \frac{c_{se}}{m_s}Z_4 - \frac{k_s}{m_s}Z_3 + \frac{k_s}{m_s}Z_5 - \frac{c_s}{m_s}Z_4 + \frac{c_s}{m_s}Z_6$$

$$= \frac{k_{se}}{m_s}Z_1 + \frac{c_{se}}{m_s}Z_2 - \left(\frac{k_{se}}{m_s} + \frac{k_s}{m_s}\right)Z_3 - \left(\frac{c_{se}}{m_s} + \frac{c_s}{m_s}\right)Z_4 + \frac{k_s}{m_s}Z_5 + \frac{c_s}{m_s}Z_6$$

From unsprung mass equation

$$m_u \dot{Z}_6 = k_s(Z_3 - Z_5) + c_s(Z_4 - Z_6) - k_t(Z_5 - y)$$

$$\dot{Z}_6 = \frac{k_s}{m_u} Z_3 - \frac{k_s}{m_s} Z_5 + \frac{c_s}{m_u} Z_4 - \frac{c_s}{m_u} Z_6 - \frac{k_t}{m_u} Z_5 + \frac{k_t}{m_u} y$$

State space model

$$\dot{Z} = AZ + Bu$$

$$Y = CZ + Du$$

State variable are Z_1, Z_2, Z_3, Z_4, Z_5, Z_6

State space vector matrix can be written as

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \\ \dot{Z}_5 \\ \dot{Z}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_{se}}{m_{se}} & -\frac{c_{se}}{m_{se}} & \frac{k_{se}}{m_{se}} & \frac{c_{se}}{m_{se}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_{se}}{m_s} & \frac{c_{se}}{m_s} & -\left(\frac{k_{se}+k_s}{m_s}\right) & -\left(\frac{c_{se}+c_s}{m_s}\right) & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\left(\frac{k_s+k_t}{m_u}\right) & -\frac{c_s}{m_u} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix} [y]$$

Output matrix can be written as

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [y]$$

Matlab program can be used to analyze seat mass and sprung mass displacement of a three degree of freedom quarter car model.

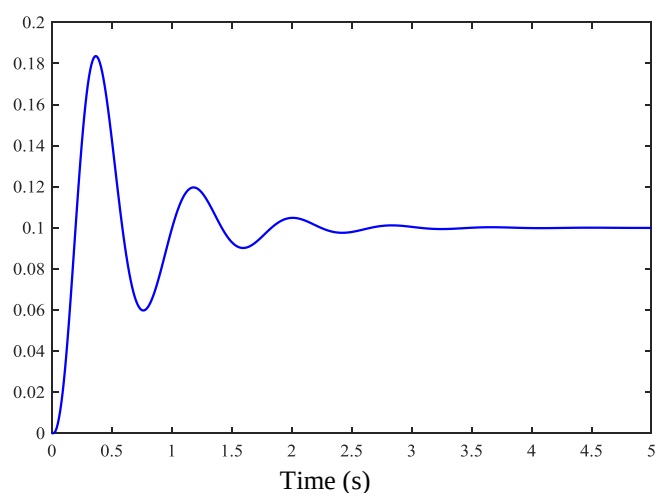

Figure 12. Seat Displacement of Front suspension without Tire Damping

The seat mass displacement of front suspension without tire damping is shown in Figure 12. In this figure, maximum overshoot is 0.1836 m and settling time occurs at 3 sec.

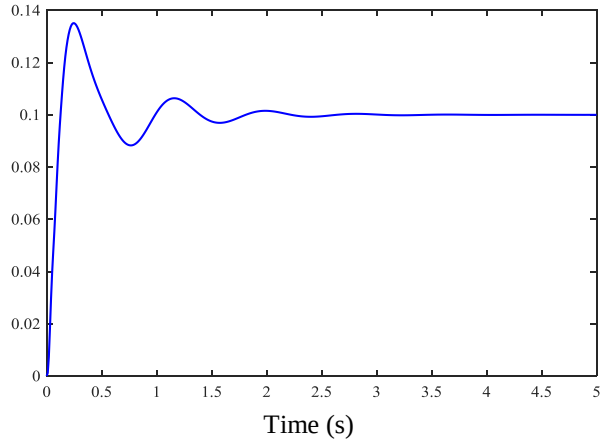

Figure 13. Sprung Mass Displacement of Front suspension without Tire Damping

The sprung mass displacement of front suspension without tire damping is shown in Figure 13. In this figure, maximum overshoot is 0.1351 m and settling time occurs at 2 sec.

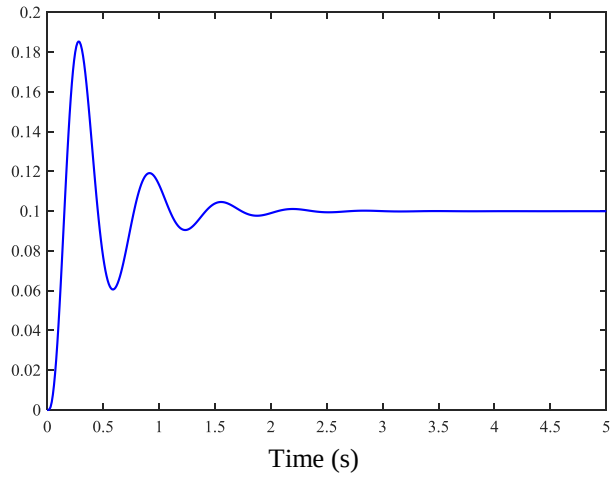

Figure 14. Seat Mass Displacement of Rear suspension without Tire Damping

The seat mass displacement of rear suspension without tire damping is shown in Figure 14. In this figure, maximum overshoot is 0.1854 m and settling time occurs at 3 sec.

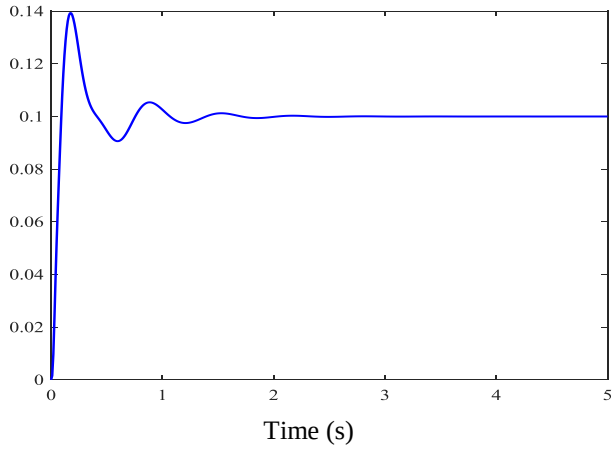

Figure 15. Sprung Mass Displacement of Rear suspension without Tire Damping

The sprung mass displacement of rear suspension without tire damping is shown in Figure 15. In this figure, maximum overshoot is 0.1393 m and settling time occurs at 2 sec.

Seat mass and sprung mass displacement for front and rear suspension are analyzed by using different damping ratio from 0.2 to 0.4.

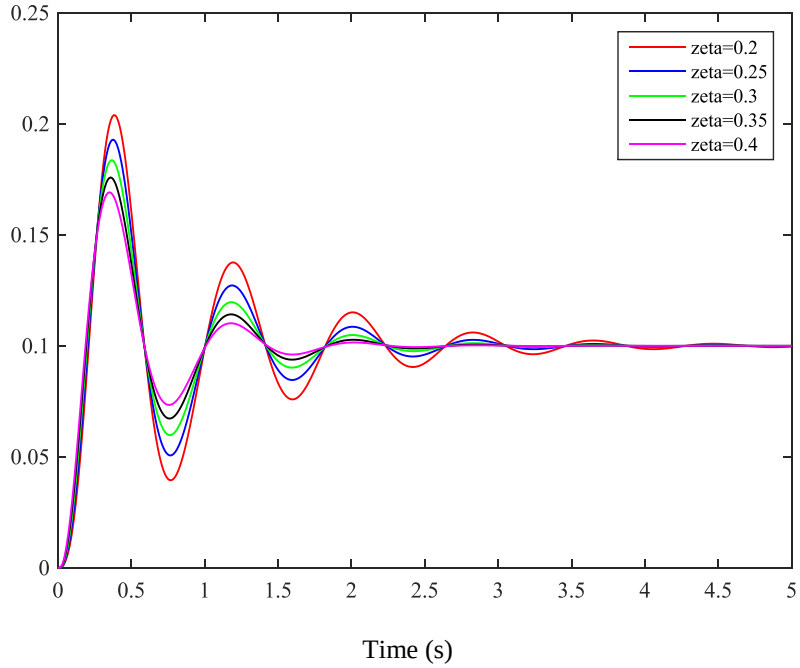

Figure 16. Seat Mass Displacement of Different Damping Ratio for Front Suspension without Tire Damping

. Figure 16 shows seat mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.2039 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time. Therefore, the greater the damping ratio, the more comfortable the passenger.

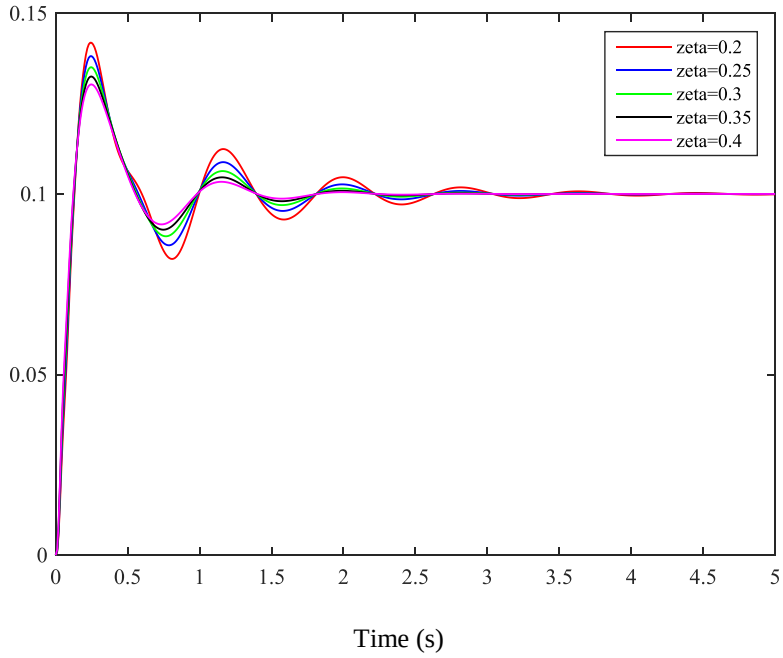

Figure 17. Sprung Mass Displacement of Different Damping Ratio for Front Suspension without Tire Damping

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 17 shows sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1419m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

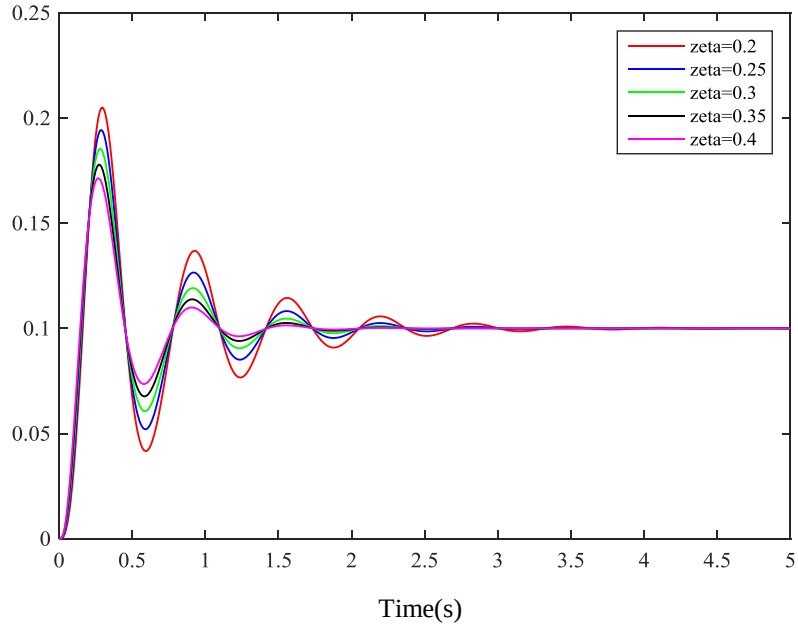

Figure 18. Seat Mass Displacement of Different Damping Ratio for Rear Suspension without Tire Damping

Seat mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 18 shows seat mass displacement using various damping ratio for rear suspension without tire damping. As a result, maximum displacement is 0.2049 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

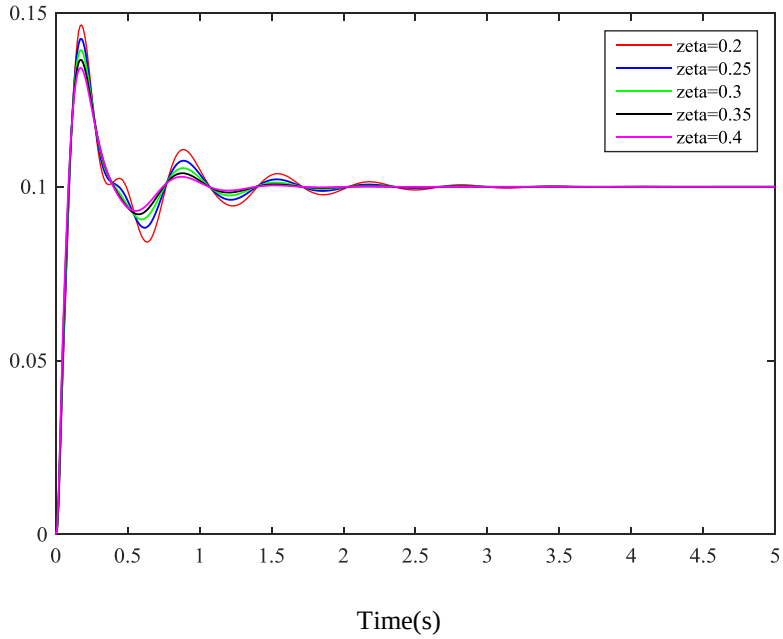

Figure 19. Sprung Mass Displacement of Different Damping ratio for Rear Suspension without Tire Damping

Sprung mass displacement is also analyzed by using different damping ratios from 0.2 to 0.4. Figure 17 shows sprung mass displacement using various damping ratio for front suspension without tire damping. As a result, maximum displacement is 0.1466 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

B. Three Degree of Freedom Suspension System with Tire Damping

Seat displacement and sprung mass displacement are also considered depend on three degree of freedom quarter car model. Modeling of quarter car with tire damping is shown in Figure 18.

Let $x_1 \rangle x_2 \rangle x_3 \rangle y$

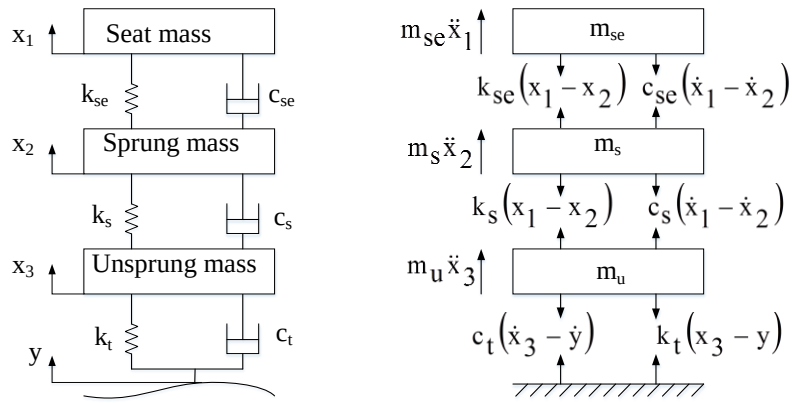

Figure 18. Modeling of Three Degree of Freedom with Tire Damping

Equation of motion can be written from the free body diagram;

For passenger and seat mass:

$$m_{se}\ddot{x}_1 = -k_{se}(x_1 - x_2) - c_{se}(\dot{x}_1 - \dot{x}_2)$$

For sprung mass:

$$m_s\ddot{x}_2 = k_{se}(x_1 - x_2) + c_{se}(\dot{x}_1 - \dot{x}_2) - k_s(x_2 - x_3) - c_s(\dot{x}_2 - \dot{x}_3)$$

For unsprung mass:

$$m_u\ddot{x}_3 = k_s(x_2 - x_3) + c_s(\dot{x}_2 - \dot{x}_3) - k_t(x_3 - y) - c_t(\dot{x}_3 - \dot{y})$$

Let $\dot{x}_1 = V_1$,

$\dot{x}_2 = V_2$, $\dot{x}_3 = V_3$

$\Delta_1 = (x_1 - x_2)$, $\dot{\Delta}_1 = (V_1 - V_2)$

$\Delta_2 = (x_2 - x_3)$, $\dot{\Delta}_2 = (V_2 - V_3)$

From seat and person equation,

$$m_{se}\dot{V}_1 = -k_{se}\Delta_1 - c_{se}(V_1 - V_2)$$

$$\dot{V}_1 = -\frac{k_{se}}{m_{se}}\Delta_1 - \frac{c_{se}}{m_{se}}(V_1 - V_2) \tag{12}$$

For sprung mass equation

$$m_s\dot{V}_2 = k_{se}\Delta_1 + c_{se}(V_1 - V_2) - k_s\Delta_2 - c_s(V_2 - V_3)$$

$$\dot{V}_2 = \frac{k_{se}}{m_s}\Delta_1 + \frac{c_{se}}{m_s}(V_1 - V_2) - \frac{k_s}{m_s}\Delta_2 - \frac{c_s}{m_s}(V_2 - V_3) \tag{13}$$

For unsprung mass:

$$m_u\dot{V}_3 = k_s\Delta_2 + c_s(V_2 - V_3) - k_t(x_3 - y) - c_t(V_3 - \dot{y})$$

$$\dot{V}_3 = \frac{k_s}{m_u}\Delta_2 + \frac{c_s}{m_u}V_2 - \frac{c_s}{m_u}V_3 - \frac{k_t}{m_u}x_3 + \frac{k_t}{m_u}y - \frac{c_t}{m_u}V_3 + \frac{c_t}{m_u}\dot{y}$$

$$\dot{V}_3 - \frac{c_t}{m_u}\dot{y} = \frac{k_s}{m_u}\Delta_2 + \frac{c_s}{m_u}V_2 - \left[\frac{c_s}{m_u} + \frac{c_t}{m_u}\right]V_3 - \frac{k_t}{m_u}x_3 + \frac{k_t}{m_u}y \tag{14}$$

$$\text{Let } \dot{T} = \dot{V}_3 - \frac{c_t}{m_u}\dot{y}$$

$$T = V_3 - \frac{c_t}{m_u} y$$

$$V_3 = T + \frac{c_t}{m_u} y \tag{15}$$

Equation (15) substitute in equation (14)

$$\dot{T} = \frac{k_s}{m_u}\Delta_2 + \frac{c_s}{m_u}V_2 - \left[\frac{c_s}{m_u} + \frac{c_t}{m_u}\right]\left[T + \frac{c_t}{m_u} y\right] - \frac{k_t}{m_u}x_3 + \frac{k_t}{m_u} y$$

$$= \frac{k_s}{m_u}\Delta_2 + \frac{c_s}{m_u}V_2 - \left[\frac{(c_s + c_t)}{m_u}\right]T + \left[-\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2}\right]y - \frac{k_t}{m_u}x_3 + \frac{k_t}{m_u} y$$

$$\dot{T} = \frac{k_s}{m_u}\Delta_2 + \frac{c_s}{m_u}V_2 - \left[\frac{(c_s + c_t)}{m_u}\right]T + \left[-\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2} + \frac{k_t}{m_u}\right]y - \frac{k_t}{m_u}x_3 \tag{16}$$

$$\dot{\Delta}_2 = V_2 - V_3$$

$$= V_2 - \left[T + \frac{c_t}{m_u} y\right] \tag{17}$$

Equation (15) substitute in equation (13)

$$\dot{V}_2 = \frac{k_{se}}{m_s}\Delta_1 + \frac{c_{se}}{m_s}(V_1 - V_2) - \frac{k_s}{m_s}\Delta_2 - \frac{c_s}{m_s}(V_2 - V_3)$$

$$= \frac{k_{se}}{m_s}\Delta_1 + \frac{c_{se}}{m_s}V_1 - \frac{c_{se}}{m_s}V_2 - \frac{k_s}{m_s}\Delta_2 - \frac{c_s}{m_s}V_2 + \frac{c_s}{m_s}\left[T + \frac{c_t}{m_u} y\right]$$

$$= \frac{k_{se}}{m_s}\Delta_1 + \frac{c_{se}}{m_s}V_1 - \left[\frac{c_{se}}{m_s} + \frac{c_s}{m_s}\right]V_2 - \frac{k_s}{m_s}\Delta_2 - \frac{c_s}{m_s}T + \frac{c_s c_t}{m_s m_u} y$$

State variable are x_3, Δ_1, Δ_2, V_1, V_2, T

State space vector matrix can be written as

$$\begin{bmatrix} \dot{x}_3 \\ \dot{\Delta}_1 \\ \dot{\Delta}_2 \\ \dot{V}_1 \\ \dot{V}_2 \\ \dot{T} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -\frac{k_{se}}{m_{se}} & 0 & -\frac{c_{se}}{m_{se}} & \frac{c_{se}}{m_{se}} & 0 \\ 0 & \frac{k_s}{m_s} & -\frac{k_s}{m_s} & \frac{c_{se}}{m_s} & -\left[\frac{(c_{se} + c_s)}{m_s}\right] & \frac{c_s}{m_s} \\ -\frac{k_t}{m_u} & 0 & \frac{k_s}{m_u} & 0 & \frac{c_s}{m_u} & -\left[\frac{(c_s + c_t)}{m_u}\right] \end{bmatrix} \begin{bmatrix} x_3 \\ \Delta_1 \\ \Delta_2 \\ V_1 \\ V_2 \\ T \end{bmatrix} + \begin{bmatrix} \frac{c_t}{m_u} \\ 0 \\ -\frac{c_t}{m_u} \\ 0 \\ \frac{c_s c_t}{m_s m_u} \\ -\frac{c_s c_t}{m_u^2} - \frac{c_t^2}{m_u^2} + \frac{k_t}{m_u} \end{bmatrix}$$

Output matrix is

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ \Delta_1 \\ \Delta_2 \\ V_1 \\ V_2 \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [y]$$

Matlab program can be used to analyze seat mass and sprung mass displacement of a three degree of freedom quarter car model.

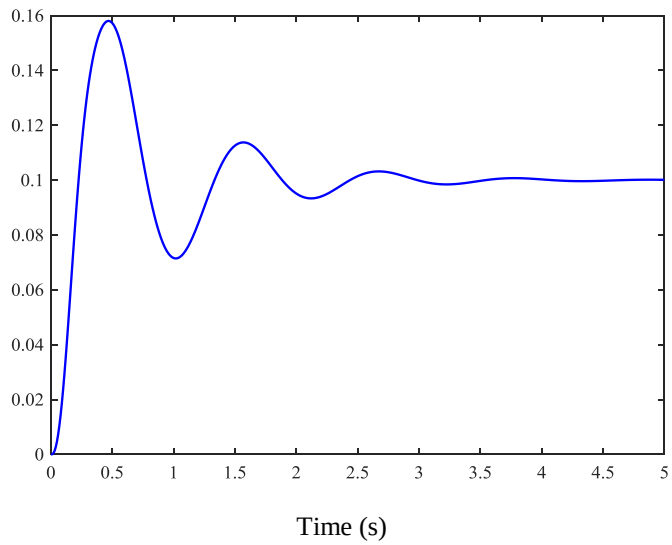

Figure 19. Seat Mass Displacement of Front Suspension for with Tire Damping

The seat mass displacement of front suspension without tire damping is shown in Figure 19. In this figure, maximum overshoot is 0.1581 m and settling time occurs at 3 sec.

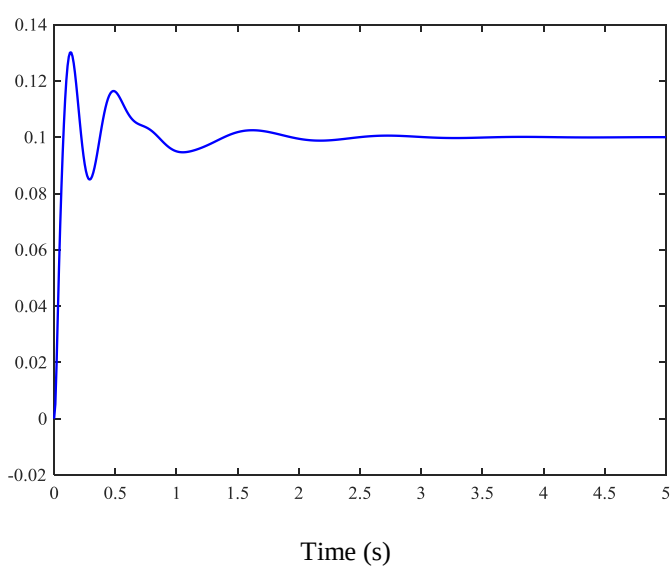

Figure 20. Sprung Mass Displacement of Front Suspension for with Tire Damping

The sprung mass displacement of front suspension with tire damping is shown in Figure 20. In this figure, maximum overshoot is 0.1301 m and settling time occurs at 2 sec.

The seat mass displacement of rear suspension with tire damping is shown in Figure 21. In this figure, maximum overshoot is 0.1590 m and settling time occurs at 3 sec.

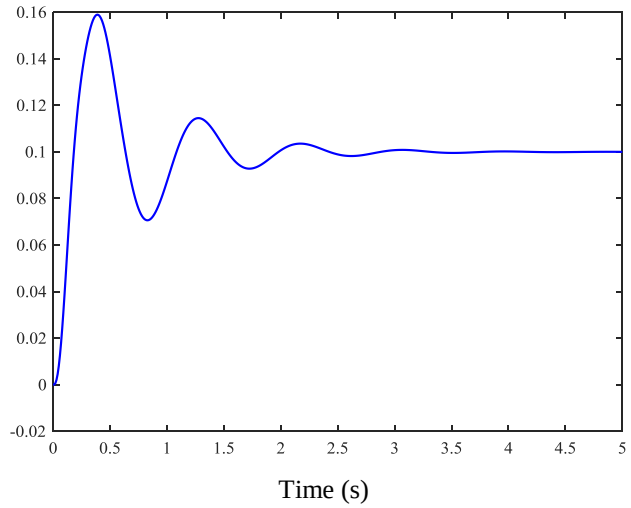

Figure 21. Seat Mass Displacement of Rear Suspension for with Tire Damping

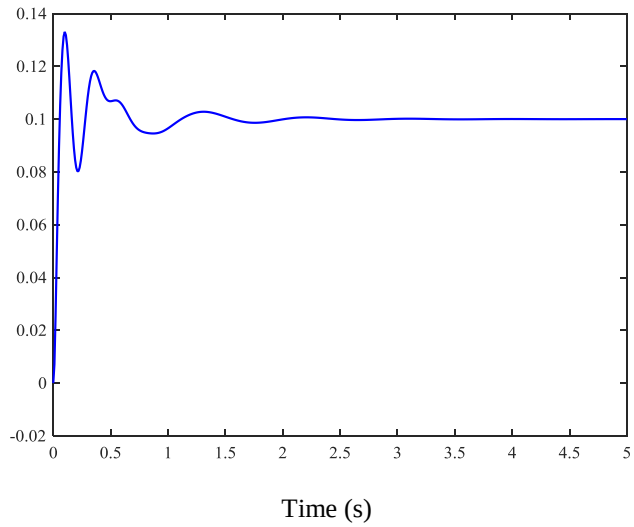

Figure 22. Sprung Mass Displacement of Rear Suspension for with Tire Damping

The sprung mass displacement of rear suspension with tire damping is shown in Figure 22. In this figure, maximum overshoot is 0.1331 m and settling time occurs at 2 sec.

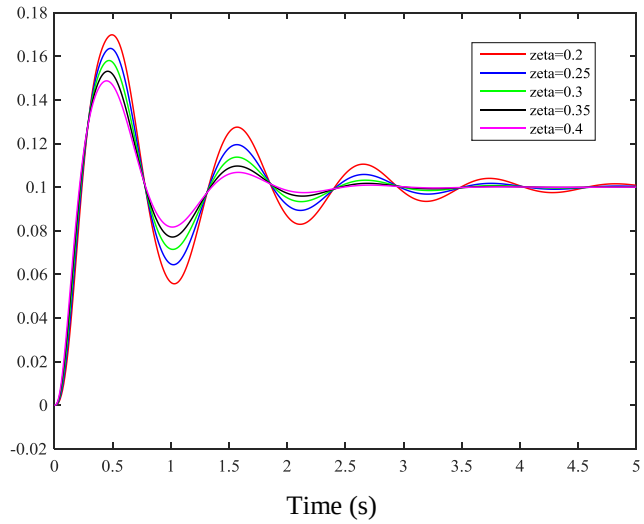

Figure 23. Seat Mass Displacement of Different Damping Ratio for Front Suspension with Tire Damping

Figure 23 shows seat mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1698 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

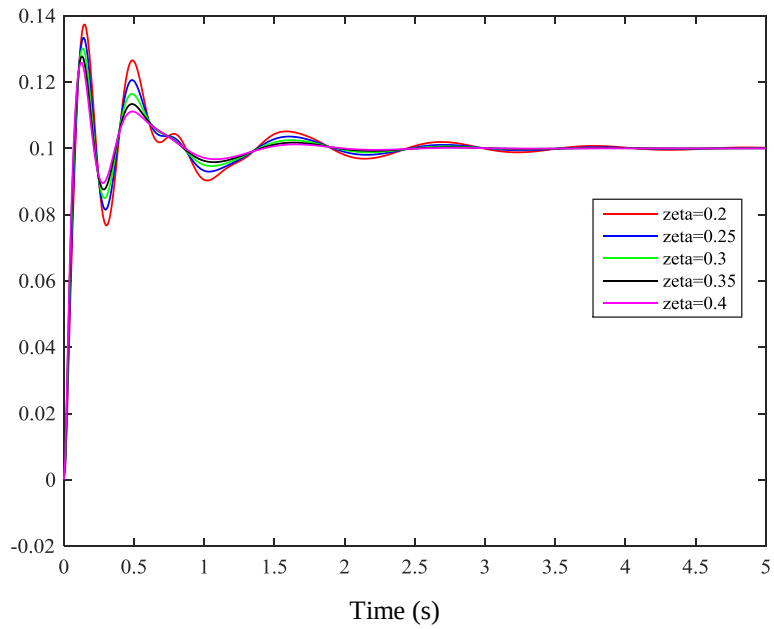

Figure 24. Sprung Mass Displacement of Different Damping Ratio for Front suspension with Tire Damping

Figure 24 shows sprung mass displacement using various damping ratio for front suspension with tire damping. As a result, maximum displacement is 0.1375 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

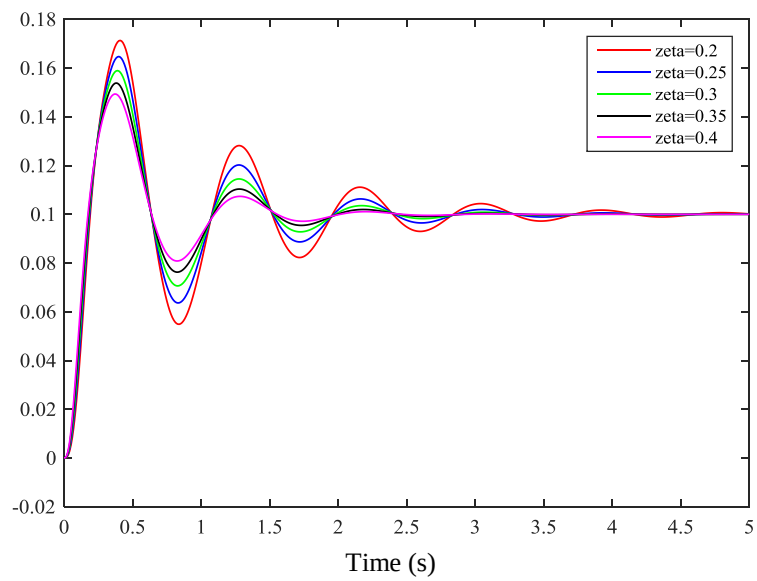

Figure 25. Seat Mass Displacement of Different Damping Ratio for Rear Suspension with Tire Damping

Figure 25 shows seat mass displacement using various damping ratio for rear suspension with tire damping. As a result, maximum displacement is 0.1714 m and maximum settling time is 4s which occur at the smallest damping ratio. The more the damping ratio, the less the sprung mass displacement and settling time.

Sprung mass displacement using various damping ratio for rear suspension with tire damping is shown in Figure 26. According from this figure, the maximum displacement is0.1392 m and maximum settling time is 4s which occur at the smallest damping ratio. The minimum displacement is 0.1296m and minimum settling time is 2 s which occurs at the greatest damping ratio. Therefore, the greater the damping ratio, the more comfortable the passenger.

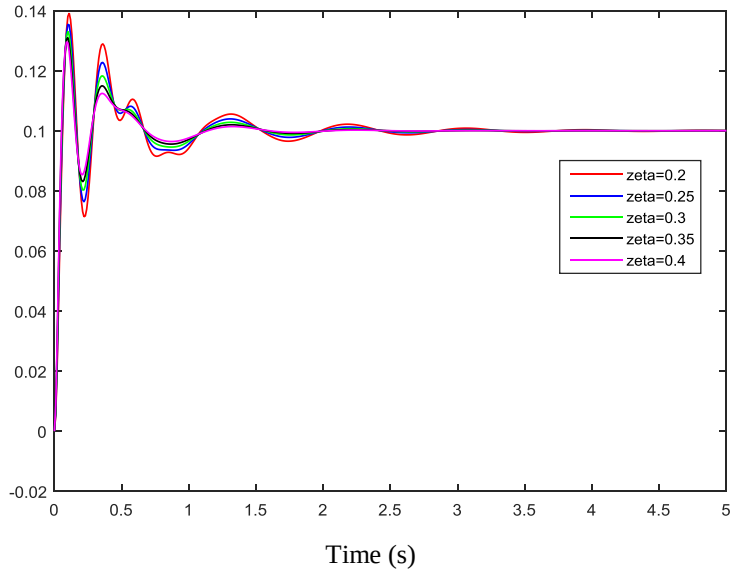

Figure 26. Sprung Mass Displacement of Different Damping ratio for Rear Suspension with Tire Damping

C. Comparison of Three Degree of Suspension System without and with Tire Damping

Seat mass and sprung mass displacement of three DOF suspension system without tire damping is compared with suspension system with tire damping for front and rear suspension. The comparison of seat mass displacement for front suspension is shown in Table VI.

TABLE VI
COMPARISON OF SEAT MASS DISPLACEMENT FOR FRONT SUSPENSION

Damping ratio ξ	Seat mass displacement with tire damping , X_1 , m	Seat mass displacement without tire damping, X_1 , m
ξ = 0.2	0.1698	0.2039
ξ = 0.25	0.1636	0.1928
ξ = 0.3	0.1581	0.1836
ξ = 0.35	0.1531	0.1758
ξ = 0.4	0.1488	0.1692

In Table VI, the seat mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest seat mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest seat mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

TABLE VII
COMPARISON OF SPRUNG MASS DISPLACEMENT FOR FRONT SUSPENSION

Damping ratio ξ	Sprung mass displacement with tire damping , X_2 , m	Sprung mass displacement without tire damping, X_2 , m
ξ = 0.2	0.1375	0.1419
ξ = 0.25	0.1335	0.1381
ξ = 0.3	0.1301	0.1351
ξ = 0.35	0.1134	0.1325
ξ = 0.4	0.1258	0.1303

The comparison of sprung mass displacement for front suspension is shown in Table VII. In this table, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

TABLE VIII
COMPARISON OF SEAT MASS DISPLACEMENT FOR REAR SUSPENSION

Damping ratio ξ	Seat mass displacement with tire damping , X_1 , m	Seat mass displacement without tire damping, X_1 , m
ξ = 0.2	0.1714	0.2049
ξ = 0.25	0.1647	0.1943
ξ = 0.3	0.1590	0.1854
ξ = 0.35	0.1539	0.1778
ξ = 0.4	0.1494	0.1695

The comparison of seat mass displacement for rear suspension is shown in Table VIII. In this table, the seat mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest seat mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest seat mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

TABLE VII
COMPARISON OF SPRUNG MASS DISPLACEMENT FOR REAR SUSPENSION

Damping ratio ξ	Sprung mass displacement with tire damping , X_2, m	Sprung mass displacement without tire damping, X_2, m
ξ = 0.2	0.1392	0.1466
ξ = 0.25	0.1354	0.1426
ξ = 0.3	0.1331	0.1393
ξ = 0.35	0.1310	0.1365
ξ = 0.4	0.1296	0.1342

The comparison of sprung mass displacement for rear suspension is shown in Table VII. In this table, the sprung mass displacement is analysed by using various damping ratio range from 0.2 to 0.4. The greatest sprung mass displacement for without and with tire damping occur at damping ratio 0.2 and the smallest sprung mass displacement occurs at damping ratio 0.4. It is observed that suspension system with tire damping is more comfortable than without tire damping for all different damping ratios.

V. CONCLUSION

In the present work, vibration analysis of two degree of freedom and three degree of freedom quarter car model are performed. Sprung mass displacement and seat mass displacement of both type are analyze depend on two types of conditions such as without tire damping and with tire damping. Matlab software is used to analyze the two types of suspension system. As a results, the sprung mass displacement of two degree of freedom suspension system with and without tire damping for front suspension are 0.1503 m and 0.1509 m respectively. Two degree of freedom rear suspension with and without tire damping are 0.1548 m and 0.1564 m. For three degree of freedom, the sprung mass displacement of front suspension for with and without tire damping are 0.1301 m and 0.1351 m. Sprung mass displacement for rear suspension for with and without tire damping are0.1331 m and 0.1393 m. Seat mass displacement of front suspension for with and without tire damping are 0.1581 m and 0.1836 m. Seat mass displacement for rear suspension for with and without tire damping are 0.1590 m and 0.1854 m. The results show that suspension system of both types with tire damping is more passenger comfortable than suspension system without tire damping. For front suspension, the sprung mass displacement of three degree of freedom suspension system with tire damping is 13% more comfortable than two degree of freedom suspension system. For rear suspension, the sprung mass displacement of three degree of freedom suspension system with tire damping is 14% more comfortable than two degree of freedom suspension system. Moreover, it is observed that the greater the damping effect, the more the ride handling and ride comfort of passenger.

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