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Frequentist Comparison of the Bayesian Credible and Maximum Likelihood Confidence interval for the Median of the Lognormal Distribution for the Censored Data

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Abstract- Lognormal Distribution is widely used in scientific investigation. Rao and D'Cunha (2016) reported that the Bayes credible intervals are also confidence intervals when the sample size is moderate to large. In this paper we have investigated whether the same conclusion holds for the censored data under random censoring. Extensive Monte Carlo simulation indicates that the result does not hold under random censoring. Leukemia free survival time for Allogeneic transplant patients reported in Klein and Moeschberger (2003) is reanalyzed and the results indicate that Bayes estimate of the median survival time is close to the Kaplan Meier estimator.

Keywords- Lognormal distribution, complete sample, credible interval, random censoring, Monte Carlo simulation.

I. INTRODUCTION

For the last 50 years, lognormal distribution is widely used in scientific investigation. In the reliability studies, lognormal distribution is one of the life time distributions that are widely used. Standard textbooks on analysis of failure time data (Kalbfleisch & Prentice (2002), Lawless (2003)) discuss the properties of the lognormal distribution. A seminal paper on the use of lognormal distribution is due to Nelson (1980) who used the distribution to develop step stress reliability model. Mullen (1998) used lognormal distribution to study software reliability. Lognormal distribution is also used in the analysis of stock market data (Antoniou et al. (2004), D'Cunha & Rao (2014)). Length biased lognormal distribution is used in the analysis of data from oil field exploration studies (Ratnaparkhi & Naik-Nimbalkar (2012) and also see the reference cited therein). Although Cox's (1972) Proportional Hazard model is widely used for the analysis of survival time in clinical studies, a recent application of the lognormal distribution for the cancer patients is given by Royston (2001). Textbooks and monographs on lognormal distributions are due to Kalbfleisch and Prentice (2002), Lawless (2003) and Aitchison and Brown (1957).

Although Kendall and Stuart (1989) discuss the maximum likelihood and Bayesian estimation of parameters of the mean and median of the lognormal distribution, a rigorous investigation of this on the parameter estimation is due to Nelson (1980). The recent applications on the Bayes estimators of the parameters of the lognormal distribution are due to Zellner (1971), Padgett and Wei (1977), Padgett and Johnson (1983), Sarabia et al. (2005) and Harvey and Merwe (2012), D'Cunha and Rao (2014 a, 2014 b, 2015, 2016 a, 2016 b), Rao and D'Cunha (2016).

In the analysis of failure time data, censoring is very common. For the censored observations, median can be obtained easily rather than mean, and in reliability and clinical studies, median survival time is often reported. Bayesian procedures are computationally tedious. Barring the computational difficulty, from the frequentist view point Bayesian credible intervals are acceptable when they are also confidence intervals. In the past, several papers have appeared to check whether Bayesian credible intervals are also confidence intervals; not necessarily for lognormal distribution. Some of the references in this area are Bartholomew (1965), Woodroofe (1976), Hulting and Harville (1991), Severini (1993), Sweeting (2001), Genovese and Wasserman (2002), Stern and Zacks (2002), Agresti and Min (2005) and Moon and Schorfheide (2012). The focus of this paper is to check whether Bayesian credible intervals for the median of the lognormal distribution are also confidence interval. For the censored data, analytic computation of the coverage probability of the credible interval is algebraically prohibitive and is not attempted in this paper. On the other hand, extensive Monte Carlo simulation is used to compute this coverage probability. The simulation is extensive in terms of objective priors, sample size and the values of the coefficient of variation (CV) of the lognormal distribution. The results indicate that Bayesian credible intervals do not maintain the confidence level and thus are not the confidence interval from the frequentist view point. The conclusion differs from the uncensored case where Bayesian credible intervals are also confidence intervals (Rao and D'Cunha (2016)).

The paper unfolds in six sections. Section 2 discusses the various priors and the associated Bayes estimator for the median of the lognormal distribution. The details of the simulation experiment are given in section 3. Section 4 presents the numerical results. A real life data is analysed in section 5. And the paper concludes in section 6.

II. CREDIBLE AND CONFIDENCE INTERVAL FOR THE MEDIAN OF THE LOGNORMAL DISTRIBUTION

Given a random sample $X = \{X_1, X_2, \ldots, X_n\}$ from a lognormal distribution with density

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma x} e^{\frac{-(\log x-\mu)^2}{2\sigma^2}}, \; x > 0, \; -\infty < \mu < \infty, \sigma > 0 \tag{1}$$

Then likelihood $L(\mu,\sigma|x)$ under random censoring is given by

$$L(\mu,\sigma|x_i) = \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}\sigma x_i} e^{\frac{-(\log x_i-\mu)^2}{2\sigma^2}} \right)^{\delta_i} \left(S(x_i;\mu,\sigma)\right)^{1-\delta_i}, x > 0, -\infty < \mu < \infty, \sigma > 0 \tag{2}$$

where, $S(.)$ is the survival function of the lognormal distribution. The function normfit in Matlab version 7.0 gives maximum likelihood estimator of μ and σ along with $(1-\alpha)\%$ confidence interval. For this purpose, the transformed variable $Y = \log X$ is used, where Y follows normal distribution with parameters μ and σ^2. Since maximum likelihood estimator of μ is invariant (Kale (1999)) for a continuous function $g(\mu)$, the confidence interval for e^{μ} is given by $\left(e^{\hat{\mu}_L}, e^{\hat{\mu}_U}\right)$, where $\hat{\mu}_L$ and $\hat{\mu}_U$ denote the lower and upper confidence interval for μ.

The Bayes estimator of e^{μ} exists under the following mild regularity conditions; $0 < \mu < B$, where B is some positive real number. For any prior $\pi(\mu,\sigma)$, the Bayes estimator of e^{μ} is given by

$$BE\left(e^{\hat{\mu}^B}\right) = \frac{\int e^{\mu} L(\mu,\sigma|x)\pi(\mu,\sigma)}{\int L(\mu,\sigma|x)\pi(\mu,\sigma)} d\sigma d\mu \tag{3}$$

Analytical expression for the above integral is not tractable and we have used importance sampling approach to compute the numerical value of the Bayes estimator. Equitailed credible interval for e^{μ} corresponds to ${\alpha/2}^{th}$ and $\left(1-\alpha/2\right)^{th}$ percentile value of the simulated posterior distribution of e^{μ}. Four objective priors are used in the investigation. They are (1) Uniform prior $\pi(\mu,\sigma) = 1$ (2) Right invariant prior $\pi(\mu,\sigma) = \frac{1}{\sigma}$ (3) Left invariant Jeffreys prior $\pi(\mu,\sigma) = \frac{1}{\sigma^2}$ and (4) Jeffreys rule prior $\pi(\mu,\sigma) = \frac{1}{\sigma^3}$ (Harvey and Merwe (2012)).

III. SIMULATION EXPERIMENT

In order to compare the coverage probability and the length of the confidence/credible interval, a simulation experiment is carried out. For each sample size *n*, observations X are generated from lognormal distribution with parameter μ and σ. The censoring distribution is assumed to be Uniform $U(0,\theta)$. Let δ denote the censoring indicator, which takes the value $\delta = 1$ if $x \le u$ and $\delta = 0$ if $x > u$. For each sample size *n*, the confidence interval for the median of the normal distribution is computed using the function normfit in Matlab software and from which the confidence interval for the lognormal distribution is obtained. The coverage probability and length of the confidence interval is obtained using 1000 simulations.

The Bayes estimator of the median of the lognormal distribution and the credible interval for e^{μ} is computed using the procedure developed by Chen and Shao (1999). A detailed algorithm in another context is given in Kundu and Howlader (2010). The procedure involves the derivation of the posterior density $\pi(\mu,\sigma|data)$ for the uncensored observations. The posterior density is the product of independent gamma distribution for $\eta = \frac{1}{\sigma^2}$ and conditional normal distribution for μ. For details see D'Cunha and Rao (2016). Using 10,000 observations of (μ,σ), the Bayes estimator of e^{μ} is given by

$$\frac{\sum_i e^{\mu_i} S(x_i,\mu_i,\sigma_i)}{\sum_i S(x_i,\mu_i,\sigma_i)} \tag{4}$$

For estimating the CDF of the posterior density $\pi(\mu,data)$, the duplet (μ,σ) is arranged in ascending order of magnitude of μ along with the values of σ. The estimated ${\alpha/2}\,th$ and $\left(1-\alpha/2\right)th$ percentile values corresponds to the lower and upper credible interval for μ, from which the lower and upper credible interval for e^{μ} is obtained. Using 1000 simulations we determine the proportion of times the median of the lognormal distribution lies in this interval. This gives us the estimated coverage probability.

The value θ in Uniform $U(0,\theta)$ distribution is determined such that the percentage of censoring corresponds to 10% and 20%. Since closed form solution does not exist for the survival probability of the lognormal distribution, we have used Monte Carlo integration to determine the value of θ.

The simulation is extensive in the sense that it covers 128 configurations. When the sample size is moderate to large, the average computational time for the Bayesian credible interval exceeds 6 hours using a PC with Intel core i5 processor.

IV. NUMERICAL RESULTS

Table 1a) and 1b) presents the number of times coverage probability is maintained by the credible/confidence interval for 8 combinations of CV across sample sizes for 10% and 20% censoring, respectively. We say that a credible/confidence interval maintains credible/confidence level of $(1-\alpha) = 0.95$ if the coverage probability is in the interval of 0.940 to 0.960, such a criterion has been used in Guddattu and Rao (2010).

Table 1a). Coverage probability of the credible and confidence interval for the Median across sample sizes for 8 combinations of specified values of CV for 10% censoring

n	Bayes Procedure (Equitailed)								MLE(Equitailed)	
	No. of times Cov prob is maintained				Average Length				No. of times Cov prob is maintained	Average length
	U	R	L	JR	U	R	L	JR		
10	0	0	0	0	*	*	*	*	0	*
20	0	0	0	0	*	*	*	*	4	731.66
40	0	0	0	1	*	*	*	*	3	683.48
60	0	0	0	0	*	*	*	*	8	398.81
80	0	0	0	0	*	*	*	*	8	344.91
100	0	0	0	0	*	*	*	*	5	380.39
150	0	0	0	0	*	*	*	*	7	247.49
200	0	0	0	0	*	*	*	*	6	192.81
overall	0	0	0	0	*	*	*	*	41	2979.54

Note: Whenever coverage probability is not maintained average length has not been calculated. U-Uniform prior, R-Right invariant prior, L-Left invariant prior, JR-Jeffreys rule prior.

Table 1b). Coverage probability of the credible and confidence interval for the Median across sample sizes for 8 combinations of specified values of CV for 20% censoring

n	Bayes Procedure (Equitailed)								MLE(Equitailed)	
	No. of times Cov prob is maintained				Average Length				No. of times Cov prob is maintained	Average length
	U	R	L	JR	U	R	L	JR		
10	0	0	0	0	*	*	*	*	0	*
20	0	0	0	0	*	*	*	*	1	1433.80
40	0	0	0	1	*	*	*	*	5	551.19
60	0	0	0	0	*	*	*	*	5	423.95
80	0	0	0	0	*	*	*	*	5	488.10
100	0	0	0	0	*	*	*	*	5	375.87
150	0	0	0	0	*	*	*	*	5	212.95
200	0	0	0	0	*	*	*	*	8	224.06
overall	0	0	0	0	*	*	*	*	34	3709.91

Note: Whenever coverage probability is not maintained average length has not been calculated. U-Uniform prior, R-Right invariant prior, L-Left invariant prior, JR-Jeffreys rule prior.

Rao and D'Cunha (2016) have compared the confidence and credible intervals for the median of the lognormal distribution for the same set of configurations for the complete sample. Their results indicate that the credible interval maintains confidence level $(1-\alpha) = 0.95$ for the sample size $n \geq 80$ and for the sample size $n \geq 60$ for the confidence interval based on MLE. For 20% censoring the confidence level is maintained for the confidence interval based on MLE for smaller samples of size n=40. We have checked the numerical computation and it is not clear why the confidence interval maintains confidence level for smaller sample size of n=40 for the case of 20% censoring. The conclusion is the same for all the 4 priors and thus the choice of the prior distribution does not affect the coverage probability (Table not shown here).

Table 2a) and 2b) presents the coverage probability for the confidence interval as well as the credible interval for various values of CV across different priors for sample size n=100, under 10% and 20% censoring, respectively.

Table 2 a). Length of the confidence/credible interval for various values of CV when sample size=100, under 10% censoring.

V.

Sample size	Conf/cred interval based on	Length(Coverage probability) when CV equal to							
		0.1	**0.3**	**0.5**	**0.7**	**1**	**1.5**	**2**	**2.5**
100	**MLE**	40.69 (0.937)	119.24 (0.942)	190.82 (0.938)	255.50 (0.951)	335.77 (0.937)	438.24 (0.943)	513.49 (0.943)	575.48 (0.944)
	Uniform	39.14 (1)	113.63 (1)	178.79 (1)	232.63 (0.999)	292.86 (1)	356.76 (1)	396.88 (1)	419.95 (0.987)
	Right	38.94 (1)	113.05 (1)	177.87 (1)	231.27 (0.999)	291.37 (1)	354.94 (1)	394.89 (1)	417.44 (0.985)
	Left	38.75 (1)	112.49 (1)	176.99 (1)	230.05 (0.999)	289.89 (1)	353.12 (1)	392.84 (1)	415.21 (0.985)
	Jeffreys Rule	38.55 (1)	111.92 (1)	176.09 (1)	228.97 (0.999)	288.34 (1)	351.08 (1)	390.46 (1)	412.85 (0.985)

Table 2 b). Length of the confidence/credible interval for various values of CV when sample size=100, under 20% censoring.

VI.

Sample size	Conf/cred interval based on	Length(Coverage probability) when CV equal to							
		0.1	**0.3**	**0.5**	**0.7**	**1**	**1.5**	**2**	**2.5**
100	**MLE**	42.94 (0.951)	124.24 (0.935)	198.49 (0.939)	264.44 (0.952)	347.05 (0.936)	451.58 (0.950)	534.69 (0.954)	585.72 (0.950)
	Uniform	39.12 (1)	11.54 (1)	169.73 (1)	211.20 (0.984)	254.60 (0.088)	293.01 (0.002)	316.03 (0.002)	323.36 (0)
	Right	38.89 (1)	110.99 (1)	168.89 (1)	209.95 (0.984)	253.11 (0.080)	291.57 (0.002)	314.46 (0.002)	321.68 (0)
	Left	38.69 (1)	110.43 (1)	168.04 (1)	208.84 (0.984)	252.02 (0.081)	289.98 (0.003)	312.70 (0.002)	319.90 (0)
	Jeffreys Rule	38.55 (1)	109.87 (1)	167.15 (1)	207.87 (0.984)	250.29 (0.048)	287.86 (0.001)	310.20 (0.002)	317.14 (0)

When we compare the results for 10% and 20% censoring the coverage probability is closer to the nominal level $(1-\alpha) = 0.95$ for 20% censoring rather than 10% censoring. The result is true for various values of CV.

VII. EXAMPLE

We have reanalyzed the data set on Leukemia free survival times (in months), for the 50 Allogeneic transplant patients available in the text book authored by Klein and Moeschberger (2003). The original data consists of 28 censored and 22 uncensored observations. From this data set we have randomly selected the censored and uncensored observations for the 2 scenarios of 10% and 20% censoring. The data set for 10%censored observations consists of 2 censored observations and the data set for 20% censored observations consists of 5 censored observations. The data is given below.

10% censored data: 0.030, 0.493, 0.855, 1.184, 1.480, 1.776, 2.138, 2.763, 2.993, 3.224, 3.421, 4.178, 5.691, 6.941, 8.882, 11.480, 12.105^{+}, 12.796, 20.066, 34.211^{+}.

20% censored data: 0.030, 0.493, 0.855, 1.184, 1.283, 1.480, 1.776, 2.138, 2.500, 2.993, 3.224, 3.421, 4.178, 5.691, 6.941, 8.882, 9.145^{+}, 11.480, 11.513, 12.796, 20.066, 20.329^{+}, 28.717^{+}, 34.211^{+}, 46.941^{+}.

Table 3 a) and 3 b) gives the maximum likelihood estimator (MLE) and Bayes estimator of the median of Leukemia free survival times along with 95% confidence/credible interval for 10% and 20% censoring, respectively.

Table 3 a). Credible/confidence interval and length of the credible/confidence interval for 4 priors under Bayes and Maximum Likelihood estimation for Leukemia data with 10% censoring.

Procedure	Prior	Estimate	Credible/confidence interval	Length of the Credible/confidence interval
Bayes	**Uniform**	3.07	(1.58,5.49)	3.91
	Right	3.07	(1.61,5.42)	3.81
	Left	3.05	(1.64,5.26)	3.62
	Jeffreys Rule	2.95	(1.63,5.01)	3.38

MLE	**-**	3.50	(1.69,7.24)	5.55
Kaplan Meier	**-**	2.99	(1.52,4.47)	2.95

Table 3 b). Credible/confidence interval and length of the credible/confidence interval for 4 priors under Bayes and Maximum Likelihood estimation for Leukemia data with 20% censoring.

Procedure	Prior	Estimate	Credible/confidence interval	Length of the Credible/confidence interval
Bayes	**Uniform**	3.50	(2.00,5.91)	3.91
	Right	3.47	(1.99,5.68)	3.68
	Left	3.48	(2.04,5.77)	3.74
	Jeffreys Rule	3.40	(1.98,5.64)	3.67
MLE	**-**	5.19	(2.44,11.06)	8.62
Kaplan Meier	**-**	4.18	(0.15,8.20)	8.05

For 10% censoring, the Bayes estimator of the median disease free survival time ranges from 2.95 to 3.07 months. The MLE of the median of disease free survival time is 3.50 months, while the Kaplan Meier estimate is 2.99 months. The Kaplan Meier estimator is close to the Bayes estimator for the Jeffreys rule prior. The length of the confidence interval based on MLE is 5.55 months while the length of the credible interval ranges from 3.38 to 3.91 months. And the length of the Kaplan Meier confidence interval is 2.95 months.

Under 20% censoring although the pattern is same the values of the median survival time and length of the confidence interval increases for the Bayes estimator, MLE and Kaplan Meier estimator. In the presence of large number of censored observations, it is difficult to check whether the underlined distribution is lognormal. For comparison of the MLE and Bayes estimator, the Kaplan Meier estimator may be treated as standard. It is worthy to note that Bayes estimators are close to the Kaplan Meier estimator.

VIII. CONCLUSION

In this paper we have compared the performance of the Bayesian credible interval and the confidence interval based on maximum likelihood estimator for the censored data for estimating the median of the lognormal distribution. The performance is measured in terms of coverage probability and length of the interval. Frequentist accept the credible interval if they maintain confidence level. Rao and D'Cunha (2016) made this comparison for the complete data and arrived at the conclusion that Bayesian credible interval is also a confidence interval for moderate to large sample sizes. From the present investigation it follows that this conclusion does not hold for the censored data. Therefore we advocate the use of confidence interval for analyzing data in industrial engineering and clinical studies when lognormal distribution is considered. The program is written in Matlab software version 7.0 for the computation of credible and confidence interval and interested readers can obtain the same from the first author.

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