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Branch and Bound Technique for Single Machine Scheduling Problem Using Type-2 Trapezoidal Fuzzy Numbers.

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Abstract- This paper deals branch and bound technique to solve single machine scheduling problem involving two processing times along with due date using Type-2 Trapezoidal fuzzy numbers. Our aim is to obtained optimal sequence of jobs and to minimize the total tardiness. The working of the algorithm has been illustrated by numerical example.

Index Terms- Branch and Bound Technique, Processing times P_1 and P_2 , Optimal sequence, Type-2 Trapezoidal fuzzy numbers.

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I. Introduction

Job scheduling is a useful tool in decision making problem. The scheduling problems are common occurrence in our daily life. The aim of this technique is used to determine an optimal job scheduling problem and minimizing the total tardiness.

In this paper,We propose a new concept in single machine scheduling problem. For many years,Scheduling research focused on single performance measure. Recent development of new technology, We are consider the single machine having capacity to do two different works to complete a job. Each work having separate processing times (ie) P_1 and P_2 addition to the due date (d_j). The most obvious objective is to scheduling the job and minimizing the total tardeness using Branch and Bound technique. This method is basically a stage wise search method of optimization problems whose solutions may be viewed as the result of a sequence of decisions that will help the decision maker in determining a best schedule for a given set of jobs effectively. This method is become lucrative to make decision. In most of the real life problem, There are elements of uncertainty in process. In practical situation processing times and due date are not always deterministic. So,We have associated with fuzzy environment.

The concept of a type-2 fuzzy set, Which is an extension of the concept of an arbitrary fuzzy set, was introduced by Zadeh[13]. A fuzzy set is two dimensional and a type-2 fuzzy set is three dimensional Type-2 fuzzy sets can better improve certain kinds of inference than do fuzzy sets with increasing imprecision,Uncertainty and fuzziness in information. A type-2 fuzzy set is characterized by a membership function (ie) the membership value for each element of this set is a fuzzy set in [0.1], unlike an ordinary fuzzy set where the membership value is a crisp number in [0,1].

1.1 REVIEW OF LITERATURE:

Various researchers have done a lot of work in different directions. Ishii andTada [6] considered a single machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. Hong et.al., [5] introduced a single machine scheduling problem with fuzzy due date. Itoh and Ishii [7] proposed a single machine scheduling problem dealing with fuzzy processing times and due date. Gawiejnowicz et.al., [3] deals with a single machine time dependent scheduling problem. Emmons [2] developed several theorems and dominance rules that can be used to restrict the search effort of a branch and bound algorithm, Lawler [10] applied a dynamic programming approach to the single machine total tardiness problem. Vaiaraktarakis and Chung [12] proposed a branch and bound algorithm to minimize total tardiness subject to minimum number of tardy jobs. Azizogulu [1] used a branch and bound method to solve the total earliness and total tardiness problem for the single machine problem. Raymond [11] proposed a branch and bound approach to solve the problem for steel plant involving single machine bi-criteria problem.

The paper is organized as follows: In section-2, Some basic concepts are given. In section-3, Arithmetic operations on Type-2 Trapezoidal fuzzy numbers and algorithm are discussed. In section-4, a numerical example is given,To illustrate the proposed branch and bound technique to solve single machine scheduling problem involving two processing times along with due date .

II. PRELIMINARIES

2.1 Fuzzy Set: A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set $\tilde{A}$ is set of ordered pairs $\{x, \mu_A(x) / x \varepsilon R\}$ where $\mu_A(x) : R \rightarrow [0,1]$ is upper semi continuous. Function $\mu_A(x)$ is called membership function of the fuzzy set.

2.2 Fuzzy Number : A fuzzy number f in the real line R is a fuzzy set $f : R \rightarrow [0,1]$ that satisfies the following properties.

(i) f is piecewise continous

(ii) There exist an $x \varepsilon R$ such that f(x) = 1

(iii) f is convex (ie), if $x_1, x_2 \varepsilon R$, and $\alpha \varepsilon [0,1]$,then

$$f(\lambda x_1 + (1-\lambda)x_2) \geq f(x_1) \wedge f(x_2)$$

2.3 Type-2 Fuzzy Set: A Type-2 fuzzy set denoted $\tilde{A}$ is characterized by a Type-2 membership function $\mu_A(x,.u)$ where $x \varepsilon X$ and $u \varepsilon J_x \subseteq [0,1]$.

ie, $\tilde{A} = \{ ((x,u), \mu_A(x\ .u)) \ / \ \forall x \varepsilon X, \forall u \varepsilon J_x \subseteq [0,1] \}$ in which $0 \leq \mu_A(x\ .u) \leq 1$. $\tilde{A}$ can be expressed as $= \int_{x\varepsilon X} \int_{u\varepsilon J} \mu_A(x\ .u) / (x,u) \quad J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible

and u. For discrete universe of discourse $\int$ is replaced by Σ.

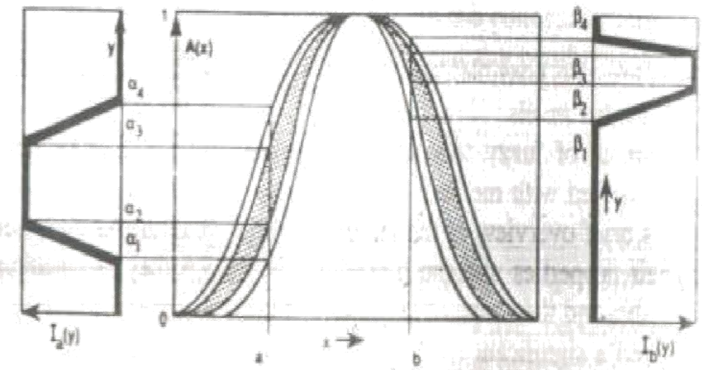


Illustration of the concept of a fuzzy set of type 2.

2.4 Type-2 Fuzzy Number: Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied.

1. $\tilde{A}$ normal,
2. $\tilde{\tilde{A}}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then is called a type-2 fuzzy number.

2.5 Type-2 Trapezoidal Fuzzy Number : A type-2 trapezoidal fuzzy number $\tilde{A}$ on R is given by $\tilde{A} = \{(x, \mu_A^1(x), \mu_A^2(x), \mu_A^3(x), \mu_A^4(x)) ; x \varepsilon R\}$ and $\mu_A^1(x) \leq \mu_A^2(x) \leq \mu_A^3(x) \leq \mu_A^4(x)$, for all $x \varepsilon R$. Denote $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4)$, where $\tilde{A}_1 = (A_1^L, A_1^M, A_1^N, A_1^U)$, $\tilde{A}_2 = (A_2^L, A_2^M, A_2^N, A_2^U)$, $\tilde{A}_3 = (A_3^L, A_3^M, A_3^N, A_3^U)$ and $\tilde{A}_4 = (A_4^L, A_4^M, A_4^N, A_4^U)$ are same type of fuzzy numbers.

2.6 Branch and Bound: Branching is the process of partitioning a large problem into two or more subproblems and Bounding is the process of calculating a lower bound on the optimal solution of a given subproblems.

2.7 Dominance Property: While subdividing a subproblem P^k_σ into (n-k) subproblems, a careful analysis would help us to create only one subproblem instead of n-k subproblems. This is called dominance property. This will reduce the computational effort to a greater extent. In a subproblem P^k_σ, if there exists a job $i \varepsilon \sigma^1$ such that $d_i \geq q_\sigma$, then it is sufficient to create only one subproblem $P_{i\sigma}^{k+1}$. The remaining subproblems under P^k_σ can be ignored. In the bounding process, $V_{i\sigma} = V_\sigma$.

III. Arithmetic Operations On Type-2 Trapezoidal Fuzzy Number:

3.1 Arithmetic Operations:

Let $\tilde{\tilde{a}} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4)$
$= ((a_1^L, a_1^M, a_1^N, a_1^U), (a_2^L, a_2^M, a_2^N, a_2^U), (a_3^L, a_3^M, a_3^N, a_3^U), (a_4^L, a_4^M, a_4^N, a_4^U))$

& $\tilde{\tilde{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4)$
$= ((b_1^L, b_1^M, b_1^N, b_1^U), (b_2^L, b_2^M, b_2^N, b_2^U), (b_3^L, b_3^M, b_3^N, b_3^U), (b_4^L, b_4^M, b_4^N, b_4^U))$

be two type-2 trapezoidal fuzzy numbers.

Then we define $\tilde{\tilde{a}} + \tilde{\tilde{b}} = (a_1^L + b_1^L, a_1^M + b_1^M, a_1^N + b_1^N, a_1^U + b_1^U)$

(i)**Addition**

$\tilde{\tilde{a}} + \tilde{\tilde{b}} = \{(a_1^L + b_1^L, a_1^M + b_1^M, a_1^N + b_1^N, a_1^U + b_1^U), (a_2^L + b_2^L, a_2^M + b_2^M, a_2^N + b_2^N, a_2^U + b_2^U), (a_3^L + b_3^L, a_3^M + b_3^M, a_3^N + b_3^N, a_3^U + b_3^U), (a_4^L + b_4^L, a_4^M + b_4^M, a_4^N + b_4^N, a_4^U + b_4^U)\}$.

(ii)**Subtraction:**

$\tilde{\tilde{a}} - \tilde{\tilde{b}} = \{(a_1^L - b_4^U, a_1^M - b_4^N, a_1^N - b_4^M, a_1^U - b_4^L), (a_2^L - b_3^U, a_2^M - b_3^N, a_2^N - b_3^M, a_2^U - b_3^L), (a_3^L - b_2^U, a_3^M - b_2^N, a_3^N - b_2^M, a_3^U - b_2^L), (a_4^L - b_1^U, a_4^M - b_1^N, a_4^N - b_1^M, a_4^U - b_1^L)\}$.

(iii)**Multiplication:**

$$\tilde{\tilde{a}} \times \tilde{\tilde{b}} = \left(\left(\left(\frac{a_1^L \sigma b}{16}, \frac{a_1^M \sigma b}{16}, \frac{a_1^N \sigma b}{16}, \frac{a_1^U \sigma b}{16}\right), \left(\frac{a_2^L \sigma b}{16}, \frac{a_2^M \sigma b}{16}, \frac{a_2^N \sigma b}{16}, \frac{a_2^U \sigma b}{16}\right) \right), \left(\left(\frac{a_3^L \sigma b}{16}, \frac{a_3^M \sigma b}{16}, \frac{a_3^N \sigma b}{16}, \frac{a_3^U \sigma b}{16}\right) \left(\frac{a_4^L \sigma b}{16}, \frac{a_4^M \sigma b}{16}, \frac{a_4^N \sigma b}{16}, \frac{a_4^U \sigma b}{16}\right) \right) \right)$$

Where $\sigma b = b_1^L + b_1^M + b_1^N + b_1^U + b_2^L + b_2^M + b_2^N + b_2^U + b_3^L + b_3^M + b_3^N + b_3^U + b_4^L + b_4^M + b_4^N + b_4^U$.

(iv) Division:

$$\frac{\tilde{\tilde{a}}}{\tilde{\tilde{b}}} = \left\{\left(\left(\frac{16a_1^{\,L}}{\sigma b},\frac{16a_1^{\,M}}{\sigma b},\frac{16a_1^{\,N}}{\sigma b},\frac{16a_1^{\,U}}{\sigma b}\right),\left(\frac{16a_2^{\,L}}{\sigma b},\frac{16a_2^{\,M}}{\sigma b},\frac{16a_2^{\,N}}{\sigma b},\frac{16a_2^{\,U}}{\sigma b}\right),\right.\right.$$

$$\left.\left.\left(\frac{16a_3^{\,L}}{\sigma b},\frac{16a_3^{\,M}}{\sigma b},\frac{16a_3^{\,N}}{\sigma b},\frac{16a_3^{\,U}}{\sigma b}\right),\left(\frac{16a_4^{\,L}}{\sigma b},\frac{16a_4^{\,M}}{\sigma b},\frac{16a_4^{\,N}}{\sigma b},\frac{16a_4^{\,U}}{\sigma b}\right)\right.\right.$$

3.2. Ranking on Type-2 Trapezoidal Fuzzy number:

Let F (R) be the set of all type-2 normal trapezoidal fuzzy number on convenient approach for solving numerical value problem is based on the concept of comparison of fuzzy number by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function $\hat{R}$: F (R) →R which maps each fuzzy number into R.

Suppose if $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4)$,
$= ((A_1^L, A_1^M, A_1^N, A_1^U), (A_2^L, A_2^M, A_2^N, A_2^U), (A_3^L, A_3^M, A_3^N, A_3^U) , (A_4^L, A_4^M, A_4^N, A_4^U)$)

Then we define $R(\tilde{A}) = (A_1^L + A_1^M + A_1^N + A_1^U + A_2^L + A_2^M + A_2^N + A_2^U + A_3^L + A_3^M + A_3^N + A_3^U + A_4^L + A_4^M + A_4^N + A_4^U) / 16$.

Also, we define orders on F(R) by

$$R\left(\tilde{\tilde{A}}\right) \geq R\left(\tilde{\tilde{B}}\right) \text{ if and only if } \tilde{\tilde{A}} \geq \tilde{\tilde{B}}$$

$$R\left(\tilde{\tilde{A}}\right) \leq R\left(\tilde{\tilde{B}}\right) \text{ if and only if } \tilde{\tilde{A}} \leq \tilde{\tilde{B}}$$

$$R\left(\tilde{\tilde{A}}\right) = R\left(\tilde{\tilde{B}}\right) \text{ if and only if } \tilde{\tilde{A}} = \tilde{\tilde{B}}$$

3.3 Notations:

n : The total number of independent jobs.
j : Represents the jth job, j =1,2,….N.
t_j : The Processing time of the job j.
d_j : The due date of the job j.
C_j : The Completion time of the job j

$$T_j : \begin{cases} c_j - d_j & if \quad c_j > d_j \\ 0, & otherwise \end{cases}$$

NT_j : Number of the tardy jobs.
T_{max} : Maximum tardiness.
J : Location of ith job on machine k.
N : Total number of jobs to be scheduled.
K : Machine on which ith job is assigned at position j.
σ : The set of Scheduled jobs at the end of the sequence.
σ' : The set of unscheduled jobs. (or) complement of σ .
$q_\sigma = q_\varphi$: The sum of the processing times of unscheduled jobs in σ'.
iσ : Partial sequence in which σ is immediately proceded by job i.
P_σ^K : A subproblem at level k in the branching tree. In this subproblem the last k positions in the sequence are assigned some jobs.

V_σ : Value associated with p_σ^k which is combination of jobs to total tardiness.

3.4 Algorithm:

The processing times of jobs and due date are uncertain. This leads to the use of Type-2 trapezoidal fuzzy numbers for representing these imprecise values. First we convert the Type-2 trapezoidal fuzzy numbers into crisp number described in the processing times and due date.

Step-1:

Place P_φ^0 on the active list; its associated values are $V_\varphi = 0$ and $q_\varphi = \sum_{j=1}^{n} t_j$. At a given stage of the algorithm, the active list consists of all the terminal nodes of the partial tree created up to that stage.

Step-2 ;

Remove the first subproblem P_σ^k from the active list. If k is equal to n-1, stop. Prefix the missing job with σ and treat it as the optimal sequence. Otherwise, check the dominance property for P_σ^k . If the property holds, go to step 3; otherwise go to step 4.

Step -3:

Let the job j be the job with the largest due date in σ^1. Create the subproblem $P_{j\sigma}^{k+1}$ with $q_{j\sigma} = q_\sigma - t_j$, $V_{j\sigma} = V_\sigma$, $bj_\sigma = V\sigma$. Place $P_{j\sigma}^{k+1}$ on the active list, ranked by its lower bound. Return to step 2.

Step-4 :

Create (n-k) subproblems, one for each i € σ^1. For $P_{i\sigma}^{k+1}$, let, $q_{j\sigma} = q_\sigma - t_j$,
$V_{i\sigma} = V_\sigma + \max (0, q_\sigma - d_i)$, $b_{i\sigma} = V_{i\sigma}$. Now place each $P_{i\sigma}^{k+1}$ on the active list, ranked by its lower bound. Return to step -2.

IV. Numerical Illustration

In milk producing factory, they required two processes to produce using single machine.

Initially, put the soya in the machine there are two processes done by the machine. (i) Crushed the soya to produce milk is the first process made by the machine. (ii) That milk will be packed by respective quantities is the second process made by the same machine. These two processes are having separate processing times (P_1, P_2). Here, we consider the processing time and due time are not always deterministic. So, we have associated this times with type-2 trapezoidal fuzzy numbers.

Job j	Processing time P_1	Processing time P_2	Processing time tj	Due date d_j
1	(2,4,6,8) (2,6,10,14) (4,8,12,16) (0,6,12,18)	(2,4,6,8) (4,5,6,7) (6,7,8,9) (4,8,12,16)	(4,8,12,16) (6,11,16,21) (10,15,20,25) (4,14,24,34)	(8,16,24,32) (13,16,19,22) (14,18,22,26) (15,20,25,30)
2	(1,2,3,4) (1,3,5,7) (2,4,6,8) (0,3,6,9)	(1,2,3,4) (1,3,5,7) (2,4,6,8) (0,3,6,9)	(2,4,6,8) (2,6,10,14) (4,8,12,16) (0,6,12,18)	(4,8,12,16) (6,12,18,24) (8,16,24,32) (12,14,16,18)
3	(1,2,3,4) (3,6,9,12) (6,10,14,18) (8,12,16,20)	(2,4,6,8) (2,6,10,14) (4,8,12,16) (0,6,12,18)	(3,6,9,12) (5,12,19,26) (10,18,26,34) (8,18,28,38)	(8,16,24,32) (12,24,36,48) (16,32,48,64) (24,28,32,36)
4	(-2,-1,1,2) (1,2,3,4) (2,4,6,8) (5,10,15,20)	(1,2,3,4) (1,3,5,7) (2,4,6,8) (0,3,6,9)	(-1,1,4,6) (2,5,8,11) (4,8,12,16) (5,13,21,29)	(8,12,16,20) (6,12,18,24) (13,17,21,25) (8,16,24,32)
5	(1,2,3,4) (2,4,6,8) (3,6,9,12) (0,6,12,18)	(1,2,3,4) (2,4,6,8) (3,6,9,12) (0,6,12,18)	(2,4,6,8) (4,8,12,16) (6,12,18,24) (0,12,24,36)	(5,10,15,20) (10,20,30,40) (15,20,25,30) (25,35,45,55)

Step -1:

Active list at level 0 = $\{P_\varphi^0\}$. $\sigma = \{\varphi\}$ $\sigma^1 = \{1,2,3,4,5\}$, $V_\varphi = 0$ & $q_\varphi = 61$. Since the current level k(0) is not equal to n-1(4). Check the dominance property. Also $\max_{i\in\sigma'} d_i = 30$. Since this maximum is not greater than q_φ. The details of computations of the lower bound for each of the node is

$Pi\sigma^1$	$V_{i\sigma} = V_\sigma + \max(0, q_\sigma - d_i)$	$b_{i\sigma} = v_{i\sigma}$
P_1^1	0 + max (0, 61-20) = 41	41
P_2^1	0 + max (0, 61-15) = 46	46
P_3^1	0 + max (0, 61- 30) = 31	31
P_4^1	0 + max (0, 61- 17) = 44	44
P_5^1	0 + max(0,61- 25) = 36	36

Active list = $\{P_3^1(31), P_5^1(36), P_1^1(41), P_4^1(44), P_2^1(46)\}$

Step-2:

check the dominance property, $\sigma = \{3\}$, $\sigma' = \{1,2,4,5\}$, $q_\sigma = 61-17 = 44$. $\max_{i \in \sigma'} d_i = 25$.
Since, this maximum value is not greater than q_σ(25). The details of computations of the lower bound for each of the node is

$Pi\sigma^2$	$V_{i\sigma} = V_\sigma + \max(0, q_\sigma - d_i)$	$b_{i\sigma} = v_{i\sigma}$
P_{13}^2	31 + max (0, 44-20) = 55	55
P_{23}^2	31 + max (0, 44-10) = 60	60
P_{43}^2	31+ max (0,44-17) = 58	58
P_{53}^2	31 + max (0,44-25) = 50	50

Active list = $\{P_5^1(36), P_1^1(41), P_4^1(44), P_2^1(46), P_{53}^2(50), P_{13}^2(55), P_{43}^2(58), P_{23}^2(60)\}$.

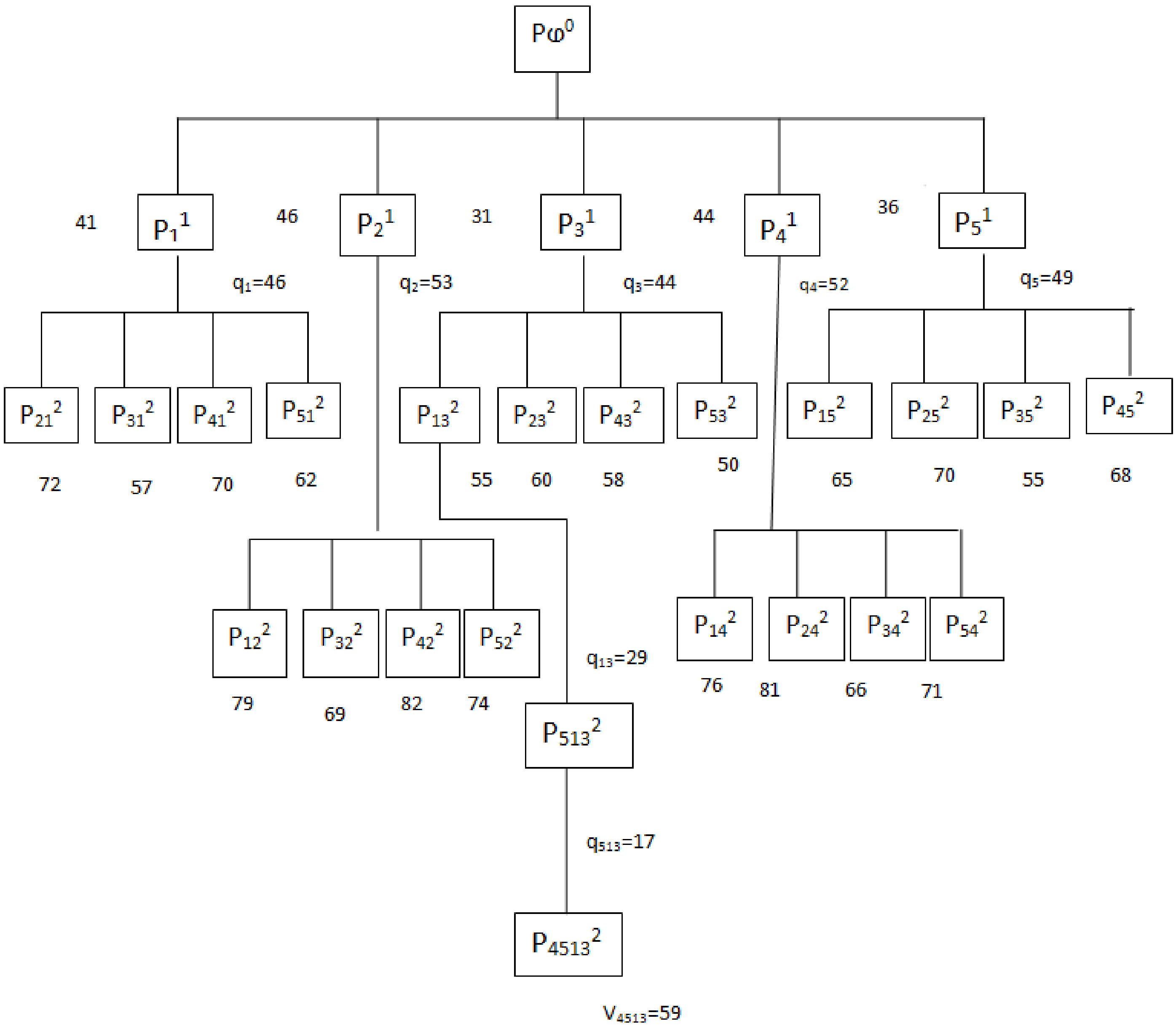
Step-3:

This subproblem occurs at level 3 which is not equal to (4). Hence, check the dominance property, $\sigma = \{5,1,3)$ $\sigma' = \{2.4\}$ $q_{513} = q_{13} - t_5$ 29-12 = 17. $\max_{i \in \sigma'} d_i = 17$. Since the maximum value is equal to q_σ (17)..

Step-4:

Job 4 has an element in σ' which has the highest due date. Hence based on the dominance property the subproblem P_{513}^3 is further partitioned with a single branch P_{4513}^4.
$\sigma = \{5,1,3\}$ & j = 4, $q_\sigma = 17$. $V_{j\sigma} = V_\sigma + \max(0, q_\sigma - d_j) = 59 + \max(0, 17-17) = 59$.

TREE DIAGRAM



The minimum total tardiness value is 59.

V. CONCLUSION

We considered single machine scheduling problem (SMSP) with fuzzy processing time and fuzzy due date to minimize the total tardiness. This method is very easy to understand each stage that will help the decision maker in determining a best schedule for a given set of jobs effectively. This method has significant use of practical results in industries.

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