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Numerical Study of Flow of Walter's Liquid B over an Exponentially Stretching Sheet

Hymavathi Talla

Department of Mathematics, Adikavi Nannaya University
Rajahmundry, Andhra Pradesh, India
E-mail: talla.hymavathianur@gmail.com

Abstract: The paper deals with the study of steady laminar flow of an incompressible visco-elastic fluid referred to as Walter's liquid B over an exponentially stretching sheet. The flow is assumed to be generated by applying two equal and opposite forces along the x-axis in such a way that the stretching of the boundary surface is of exponential order in x. Using quasi-linearization technique with a numerical approach, the numerical solution is obtained. The velocity profiles are obtained numerically and these are displayed through graphs for diverse values of visco-elastic parameter. The results are compared with those available in literature obtained through analytical procedures and are seen to be in good agreement.

Index Terms: Heat Transfer, Visco-elastic fluid, Stretching sheet, Visco-elastic parameter, Quasi-linearization

INTRODUCTION

Most of the fluids which are used in industry are non-Newtonian and particularly visco-elastic in nature. So, in recent years, the study of visco-elastic fluids gains the attention of researchers. In polymer processing applications, it is essential to consider flow over a stretching sheet as mentioned by Rajagopal et.al. [1]. Several Mathematicians and the Scientists obtain the analytical solutions similar to [2]. However, the constitutive equations of this type of fluid flows are highly non linear in nature. So, getting analytical solution may not be always possible. This paper deals with the numerical study of visco-elastic fluid flow over an exponentially stretching sheet [3] using the method of quasi-linearization [4] [5].

MATHEMATICAL FORMULATION

Consider the steady laminar flow of an incompressible visco-elastic fluid caused by a stretching sheet, which is placed in a fluid at rest. The flat sheet issues from a thin slit at x = y = 0 and subsequently being stretched. It is assumed that the boundary sheet is moving axially with a velocity of exponential order in the axial direction and generating a boundary layer type of flow. The boundary layer equations governing the such flow are (given in Rajagopal et.al.[1])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} \qquad (2)$$

The boundary sheet is assumed to be stretched with a large force in such a way that stretching velocity along the axis direction x is of exponential order of the directional coordinate. Hence, the boundary conditions on velocity we impose are:

$$u=U_w(x)=U_0\exp(\frac{x}{l}), \qquad v=0 \qquad at \qquad y=0 \qquad (3)$$

$$u=0 \qquad as \qquad y\to\infty$$

Where U_0 is a constant and 'l' is the reference length.

Solution of the momentum boundary layer equation:

Equation (2) can be rewritten in terms of the stream function ψ(x,y), which satisfies the equation (1), by writing

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \qquad (4)$$

where $\psi=(2\gamma l U_0)^{1/2}\, f(\eta)\exp\left(\frac{x}{2l}\right) \quad ; \quad \eta=y\left(\frac{U_0}{2l\gamma}\right)^{1/2}\exp\left(\frac{x}{2l}\right) \qquad (5)$

Here f is the dimensionless stream function and η is the similarity variable. Using the equations (4) and (5) we obtain a fourth order non-linear ordinary differential equation of the form:

$$2f_\eta^2-f\ f_{\eta\eta}=f_{\eta\eta\eta}-k_1\left\{3f_\eta f_{\eta\eta\eta}-\frac{1}{2}f\ f_{\eta\eta\eta\eta}-\frac{3}{2}f_{\eta\eta}^2\right\} \qquad (6)$$

where $k_1=\frac{k_0U_w}{\gamma}$, is the dimensionless visco-elastic parameter.

Boundary conditions are:

$$\begin{aligned} &f = 0,\ f_{\eta} = 1 && at \quad \eta = 0 \\ &f_{\eta} = 0,\ f_{\eta\eta} = 0 && as \quad \eta \to \infty \end{aligned} \tag{7}$$

Numerical solution of the problem:

The quasi-linearization method applied for solving the equations (6) subject to the boundary conditions given by (7) is as follows :

Rearrangement of (6) gives

$$f'''' = \frac{2}{k_1 f}\left[2(f')^2 - f\,f'' - \frac{3}{2}k_1(f'')^2 + 3k_1 f'\,f''' - f'''\right] \tag{8}$$

The boundary conditions are:

$$\begin{aligned} &f = 0, \quad f' = 1 && at \quad \eta = 0 \\ &f' = f'' = 0 \quad as && \eta \to \infty \end{aligned} \tag{9}$$

Defining new variables by the equations:

$$x_1 = f, \quad x_2 = f', \quad x_3 = f'', \quad x_4 = f'''$$

The higher order non-linear differential equation (8) and boundary conditions (9) may be transformed to four equivalent first order differential equations (10) and boundary conditions (11) respectively as given below:

$$x_1^1 = x_2 \tag{10a}$$

$$x_2^1 = x_3 \tag{10b}$$

$$x_3^1 = x_4 \tag{10c}$$

$$x_4^1 = \frac{4}{k_1}(x_1)^{-1}x_2^2 - \frac{2}{k_1}x_3 - 3x_1^{-1}x_3^2 + 6x_1^{-1}x_2x_4 - \frac{2}{k_1}x_1^{-1}x_4 \tag{10d}$$

and

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$$\begin{aligned} &x_1 = 0, \quad x_2 = 1 \qquad at \qquad \eta = 0 \\ &x_2 = x_3 = 0 \qquad as \qquad \eta \to \infty \end{aligned} \tag{11}$$

The above equations (10) are in the form:

$$\overline{x'} = \bar{f}(\bar{x}, \eta)$$

By performing Taylor's series expansion of differential equations (10) we obtain the following equations expressed in matrix form:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \equiv \overline{X'} = A\overline{X} + \bar{e} \tag{12}$$

where,

$$a_{ij} = \left[\frac{\partial f_i}{\partial x_j} \right]_{\overline{x^0}} \quad \text{for} \quad i = 1 \text{ to } 4 \ \text{ and } \ j = 1 \text{ to } 4, \tag{13a}$$

$$e_i = f_i\left(\overline{x^0}, \eta\right) - \sum_{j=1}^{r} \left[\frac{\partial f_i}{\partial x_j} \right]_{\overline{x^0}} x_j^0 \quad \text{for} \ \ i = 1 \text{ to } 4 \tag{13b}$$

The elements of the matrices $A_{4 \times 4}$ and $e_{4 \times 1}$ are thus obtained as given below:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{14}$$

where ,

$$a_{41} = -\frac{4}{k_1}(x_1^o)^{-2}(x_2^o)^2 + 3(x_1^o)^{-2}(x_3^o)^2 - 6(x_1^o)^{-2}(x_2^o)(x_4^o) + \frac{2}{k_1}(x_1^o)^{-2}(x_4^o)$$

$$a_{42} = \frac{8}{k_1}(x_1^o)^{-1}(x_2^o) + 6(x_1^o)^{-1}(x_4^o)$$

$$a_{43} = -\left[\frac{2}{k_1} + 6(x_1^o)^{-1}(x_3^o)\right]$$

$$a_{44} = 6(x_1^o)^{-1}(x_2^0) - \frac{2}{k_1}(x_1^0)^{-1}$$

and

$$\bar{e} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{2}{k_1}(x_1^o)^{-2}(x_1^o)(x_4^o) \end{bmatrix} \qquad (15)$$

The coefficients a_{ij} and e_i are dependent on the nominal trajectories $\overline{X^o}(\eta)$. Hence, depending on the initial guess of nominal trajectories $\overline{X^o}(\eta)$, the solution of (12) yields the neighbouring trajectories $\overline{X^1}(\eta)$. The neighbouring trajectories so obtained are treated as nominal trajectories and next neighbouring trajectories are obtained, and this process is continued until the assumed convergence criteria is attained. The solution is said to be convergent when the following stopping criteria norm as satisfied:

where,

$$\in = \sum_{j=1}^{m} \max_{\eta} \left\| x_j^1(\eta) - x_j^o(\eta) \right\| < \delta \qquad (16)$$

It may be noted that the $\in$ defined in the equation (16) is the sum of maximum absolute differences between the nominal trajectories and neighbouring trajectories of all the dependent variables. If the value of $\in$ falls below a prescribed number δ, the iterative procedure may be stopped and the sequence of solutions of the approximately linear differential equations may be considered to have converged. The value of δ = 0.000001 is chosen in the current solution. The equations (12) can be solved by numerical integration scheme following the quasi-linearization procedure with the help of computer.

Results and Discussions

Numerical solution to the equation (8) subjected to the boundary conditions (9) are obtained by using quasi-linearization technique with a numerical approach. We have studied the effect of visco-elastic parameter on velocity and these are displayed graphically in fig.1.

In figure(1) velocity profiles were given for different values of visco-elastic parameter k_1 for increasing values of η. It is clear that for any fixed η velocity is decreasing with the increase of k_1.

CONCLUSIONS

It is observed that with very approximate initial guesses, only in six to seven iterations good accuracy up to five decimal places is obtained. So, we may apply this method even for solving higher order coupled system of equations for which finding analytical solution may not be possible.

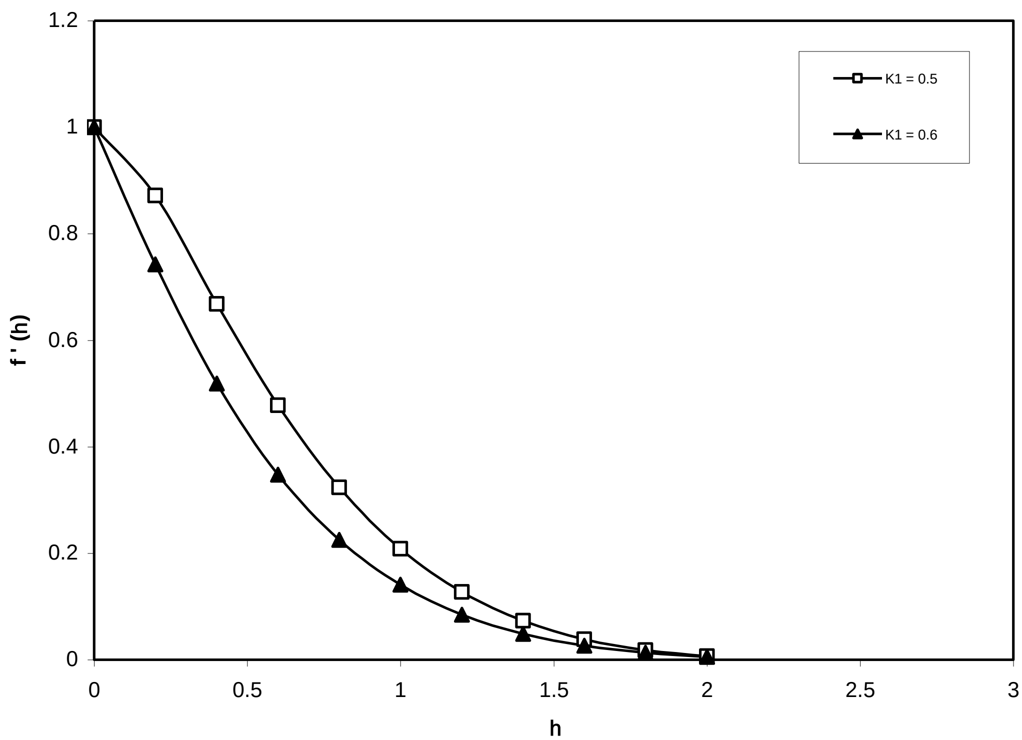

Fig. 1: Velocity profiles for different values of visco-elastic parameter

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