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FUZZY w- CONTINUOUS MAPPINGS

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Abstract: The purpose of this paper to introduce and study the concepts of fuzzy w-closed sets and fuzzy w-continuous mappings in fuzzy topological spaces.

Index Terms: fuzzy open set , fuzzy closed set , fuzzy semi open set, fuzzy closed set , fuzzy semi closed set , fuzzy g-closed set, fuzzy g-open set, fuzzy sg-closed set,fuzzy sg-open set,fuzzy gs-closed set, fuzzy gs-open set, fuzzy continuous mapping,fuzzy semi continuous mapping,fuzzy irresolute mapping,fuzzy g-continuous mapping, fuzzy sg-continuous mapping, fuzzy gs-continuous mapping

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1. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y)=\beta$ for y=x and x(y) =0 for $y \ne x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \le B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.

Definition 1.1[1]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) fuzzy semi open if there exists an open set O such that $O\le A\le cl(O)$.
(b) fuzzy semi closed if its complement 1-A is fuzzy semi open.

Remark 1.1[1]: Every fuzzy open (resp. fuzzy closed) set is fuzzy semi open (resp. fuzzy semi closed) but the converse may not be true .

Definition 1.2[13]: The intersection of all fuzzy closed sets which contains A is called the semi closure of a fuzzy set A of a fuzzy topological space (X,τ) . It is denoted by scl(A).

Definition 1.3[3,11,8,9,10]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) fuzzy g-closed if $cl(A) \le G$ whenever $A \le G$ and G is open.
(b) fuzzy g-open if its complement 1-A is fuzzy g-closed.
(c) fuzzy sg-closed if $scl(A) \le O$ whenever $A \le O$ and O is fuzzy semi open.
(d) fuzzy sg-open if if its complement 1-A is sg-closed.
(e) fuzzy gs-closed if $scl(A) \le O$ whenever $A \le O$ and O is fuzzy open.
(f) fuzzy gs-open if if its complement 1-A is gs-closed.

Remark 1.2[10 , 11]: Every fuzzy closed (resp. fuzzy open) set is fuzzy g-closed (resp. fuzzy g-open) and every fuzzy g-closed (resp. fuzzy g-open) set is fuzzy gs-closed (resp. gs –open) but the converses may not be true.

Remark 1.3[10, 11]: Every fuzzy semi closed (resp. fuzzy semi open) set is fuzzy sg-closed (resp. fuzzy sg-open) and every fuzzy sg-closed (resp. fuzzy sg-open) set is fuzzy gs-closed (resp. gs – open) but the converses may not be true .

Definition 1.4 [1,3,9,10,11] :A mapping from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,Γ) is said to be :

(a) fuzzy continuous if $f^{-1}(G)$ is fuzzy open set in X for every fuzzy open set G in Y.
(b) Fuzzy closed if the inverse image of every fuzzy closed set of Y is fuzzy closed in X.
(c) fuzzy semi continuous if $f^{-1}(G)$ is fuzzy semi open set in X for every fuzzy open set G in Y.
(d) fuzzy irresolute if the inverse image of every fuzzy semi open set of Y is fuzzy semi open in X.
(e) fuzzy g-continuous if $f^{-1}(G)$ is fuzzy g-closed set in X for every fuzzy closed set G in Y
(f) fuzzy sg-continuous if $f^{-1}(G)$ is fuzzy sg-closed set in X for every fuzzy closed set G in Y.
(g) fuzzy gs-continuous if $f^{-1}(G)$ is fuzzy gs-closed set in X for every fuzzy closed set G in Y.

Remark 1.4[1, 11]: Every fuzzy continuous mapping is fuzzy g-continuous (resp. fuzzy semi continuous) but the converse may not be true.

Remark 1.5[10, 11]: Every fuzzy g-continuous mapping is fuzzy gs-continuous but the converses may not be true.

Remark 1.6[10, 11]: Every fuzzy semi continuous mapping is fuzzy sg-continuous and every sg-continuous mapping is gs-continuous but the converses may not be true.

Definition 1.5[10, 11]: A fuzzy topological space (X,τ) is said to be fuzzy $T_{1/2}$ (resp. fuzzy semi $T_{1/2}$) if every fuzzy g-closed (resp. fuzzy sg-closed) set in X is fuzzy closed (resp. fuzzy semi closed).

2. Fuzzy w-closed sets

In this section we introduce and study a new class of fuzzy closed sets which contains the class of all fuzzy closed sets and contained in the class of all fuzzy generalized closed sets.

Definition 2.1: A fuzzy set A of a fuzzy topological space (X,τ) is called fuzzy w-closed if cl(A) ≤ U whenever A ≤ U and U is fuzzy semi open.

Remark2.1: Every fuzzy closed set is fuzzy w-closed but its converse may not be true. For,

Example2.1: Let X = {a, b} and the fuzzy sets A and U are defined as follows:

A(a)= 0.5, A(b)=0.5

U(a)= 0.5,U(b)=0.4

Let ℑ = {**0**, **1**, U} be a fuzzy topology on X. Then the fuzzy set A is fuzzy w-closed but it is not fuzzy closed.

Remark2.2: Every fuzzy w-closed set is fuzzy g-closed but the converse may not be true. For,

Example2.2: Let X = {a, b} and the fuzzy sets A and U are defined as follows

U(a)= 0.7 , U(b)=0.6

A(a)= 0.6 ,A(b)=0.7

Let ℑ = {0, 1, U} be a fuzzy topology on X. Then the fuzzy set A , is fuzzy g-closed but it is not fuzzy w-closed.

Remark2.3: Every fuzzy w-closed set is fuzzy sg-closed, but the converse may not be true. For,

Example2.3: Let X = {a, b} and the fuzzy sets A and U be defined as follows:

A (a) = 0.5, A(b)= 0.3

U (a) = 0.5, U(b)= 0.4.

Let ℑ = {0, **1**, U} be a fuzzy topology on X. Then the fuzzy set A is fuzzy sg-closed but it is not fuzzy w-closed.

Remark2.4: Remarks 1.1, 1.2, 1.3, 2.1, 2.2 and 2.3 reveals the following diagram of implications:

fuzzy closed ⇒ fuzzy w-closed ⇒ fuzzy g-closed

⇓ ⇓ ⇓

fuzzy semi closed ⇒ fuzzy sg-closed ⇒ fuzzy gs-closed

Theorem2.1: Let (X,τ) be a fuzzy topological space and A is fuzzy sub set of X .Then A is fuzzy w-closed if and only if $\rceil(A_qF) \Rightarrow \rceil(cl(A))$ for every fuzzy semi closed set F of X.

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Proof: Necessity: Let F be a fuzzy semi closed sub set of X, and $\rceil(A_q F)$ .Then $A \leq 1\text{-}F$ and 1-F is fuzzy semi open in X. Therefore $cl(A) \leq 1\text{-}F$ because A is fuzzy w-closed, Hence $\rceil(cl(A)_q F)$.

Sufficiency: Let U be a fuzzy semi open set of X such that $A \leq U$ then $\rceil(A_q(1\text{-}U))$ and 1-U is fuzzy semi closed in X. Hence by hypothesis $\rceil(cl(A)_q(1\text{-}U))$.Therefore $cl(A) \leq U$. Hence A is fuzzy w-closed in X.

Theorem2.2: Let A be a fuzzy w-closed sets in a fuzzy topological space (X,τ) and x_β be a fuzzy point of X such that $x_{\beta q} cl(A)$, then $cl(x_\beta)_q A$.

Proof: If $\rceil(cl(x_\beta)_q A)$.then $A \leq 1\text{-}cl(x_\beta)$ and so $cl(A) \leq 1\text{-}cl(x_\beta) \leq 1\text{-}x_\beta$ because A is fuzzy w-closed in X .Hence $\rceil((x_\beta)_q(cl(A))$,a contradiction.

Theorem2.3: If Aand B are fuzzy w-closed sets in a fuzzy topological space (X,τ) then $A \cup B$ is fuzzy w-closed.

Proof: Let U be a fuzzy open set in X such that $A \cup B \leq U$. Then $A \leq U$ and $B \leq U$. Since A and B are fuzzy w-closed, $cl(A) \leq U$ and $cl(B) \leq U$. Therefore $cl(A) \cup cl(B) = cl(A \cup B) \leq U$. Hence $A \cup B$ is fuzzy w-closed.

Remark2.4: The intersection of any two fuzzy w-closed sets in a fuzzy topological space (X,τ) may not be fuzzy w-closed for,

Example2.4: Let X = {a, b, c} and the fuzzy sets U, A and B of X are defined as follows:

U (a) =1, U (b) =0, U(c) =0

A (a) =1 , A (b) =1, A(c) =0

B (a) =1, B (b) =0, B(c) =1

Let $\tau = \{0, U, 1\}$ be a fuzzy topology on X. Then A and B are fuzzy w-closed set in (X,τ) but $A \cap B$ is not fuzzy w-closed.

Theorem2.4: Let $A \leq B \leq cl(B)$ and A is fuzzy w-closed set in a fuzzy topological space (X,τ), then B is fuzzy w-closed.

Proof: Let U be a fuzzy semi open set in X such that $B \leq U$. Then $A \leq U$ and since A is fuzzy w-closed $cl(A) \leq U$. Now $B \leq cl(A) \Rightarrow cl(B) \leq cl(A) \leq U$. Consequently B is fuzzy w-closed.

Defination2.2: A fuzzy set A of a fuzzy topological space (X,τ) is called fuzzy w-open if and only if 1-Ais fuzzy w-closed.

Remark2.5: Every fuzzy open (resp. fuzzy g-open) set is fuzzy w-open .But the converse may not be true. For the fuzzy set B defined by B(a) = 0.5, B(b) = 0.5 in the fuzzy topological space (X,τ) of example (2.1) is fuzzy w-open but it is not fuzzy open and the fuzzy set C defined by C(a) = 0.4, C(b)= 0.3 in the fuzzy topological space (X,τ) of example (2.2) is fuzzy w-open but it is not fuzzy g-open .

Theorem2.5: A fuzzy set A of a fuzzy topological space (X,τ) is fuzzy w-open if and only if $F \leq int(A)$ whenever $F \leq A$ and F is fuzzy semi closed .

Proof: Obvious

Theorem2.6: Let A be a fuzzy w-open subset of a fuzzy topological space (X,τ) and $int(A) \leq B \leq A$ then B is fuzzy w-open.

Proof: Suppose A is fuzzy w-open in X and $int(A) \leq B \leq A$ then 1-A is fuzzy w-closed and $1\text{-}A \leq 1\text{-}B \leq cl(1\text{-}A)$.Therefore by theorem (2.4) 1-B is fuzzy w-closed in X. Hence B is fuzzy w-open in X.

Theorem2.7: Let (X,τ) is fuzzy topological space and FSO(X)(resp. FC(X)) be the family of all fuzzy semi open (resp. fuzzy closed) sets of X. Then FSO(X) =FC(X) if and only if every fuzzy sub set of X is fuzzy w-closed.

Proof: Necessity: Suppose that FSO(X) = FC(X) and that $A \leq B \in FSO(X)$. Then $cl(A) \leq cl(U)=U$ .Hence A is fuzzy w-closed.

Sufficiency: Suppose that every fuzzy subset of X is fuzzy w-closed. Let $U \in FSO(X)$. Since $U \leq U$ and U is fuzzy w-closed, $cl(U) \leq U$ which implies $U \in FC(X)$.Thus $FSO(X) \subseteq FC(X)$. If $T \in FC(X)$ then $1\text{-}T \in FSO(X) \subseteq FC(X)$ and hence $T \in FO(X) \subseteq FSO(X)$. Consequently $FC(X) \subseteq FSO(X)$ and FSO(X) = FC(X).

Defination2.3:A fuzzy topological space (X,τ) is said to be fuzzy semi normal if for every pair of fuzzy semi closed sets A and B of X such that A ≤ (1-B), there exists fuzzy semi open sets U and V such that A ≤ U, B ≤ V and ⌉(U_qF).

Theorem 2.8: If F is fuzzy regular closed and A is fuzzy w-closed sub set of a fuzzy semi normal space (X,τ) and ⌉(A_q F).Then there exists fuzzy open set U and V such that cl(A) ≤ U, F ≤ V and ⌉(U_qV).

Proof: Since ⌉(F_qA), A ≤ 1-F and hence cl(A) ≤ 1-F . And so cl(A) ≤ 1-F. Therefore there exists fuzzy semi open set U and V in X such that cl(A) ≤ U,F ≤ V and Hence ⌉(U_qV).

Theorem 2.9: Let A be a fuzzy w-closed set in a fuzzy topological space (X,τ) and f: (X,τ)→(Y,σ) is a fuzzy irresolute and fuzzy closed mapping. Then f (A) is fuzzy w-closed in Y.

Proof: If f(A) ≤ G where G is fuzzy semi open in Y then A ≤ f^{-1}(G) and hence cl(A) ≤ f^{-1}(G).Thus f(cl(A)) ≤G and f(cl(A)) ≤ G and f(cl(A)) is a fuzzy semi closed set. It follows that cl(f(A)) ≤ cl(f(cl(A))) = f(cl(A)) ≤ G. Thus cl(f(A)) ≤ G and f(A) is a fuzzy w-closed in X.

Definition 2.4: A collection {A_i : i∈Λ) of fuzzy w-open sets in a fuzzy topological space (X,τ) is called a fuzzy w-open cover of a fuzzy set B of X if B ≤ ∪{A_i : i∈Λ).

Definition2.5: A fuzzy topological space (X,τ) is called w-compact if every fuzzy w-open cover of X has a finite sub cover.

Definition2.6: A fuzzy set B of B of a fuzzy topological space (X,τ) is said to be fuzzy w-compact relative to X if for every collection {A_i: i ∈ Λ} of fuzzy w-open sub set of X such that B ≤ ∪{A_i :I ∈ Λ) there exists a finite subset Λ_0 of Λ such that B≤ ∪{A_i: i ∈ Λ_0).

Definition2.7: A crisp subset B of a fuzzy topological space (X,τ) is said to be fuzzy w-compact if B is fuzzy w-compact as a fuzzy subspace of X.

Theorem2.10: A fuzzy w-closed crisp subset of a fuzzy w-compact space is fuzzy w-compact relative to X.

Proof: Let A be a fuzzy w-closed crisp set of fuzzy w-compact space (X,τ).Then 1-A is fuzzy w-open in X. Let M be a cover of A by fuzzy w-open sets in X. Let M be a cover of A by fussy w-open sets in X.Then the family {M,1-A} is a fuzzy w-open cover of X. Since X is fuzzy w-compact, it has a finite sub cover say {G_1,G_2,……G_n}.If this sub cover contains 1-A ,we discards it. Otherwise leave the sub cover as it is. Thus we have obtained a finite fuzzy w-open sub cover of A. Therefore A is fuzzy w-compact relative to X.

3. Fuzzy w-continuous mappings

The present section investigates and a new class of fuzzy mappings which contains the class of all fuzzy continuous mappings and contained in the class of all fuzzy g-continuous mappings.

Definition 3.1: A mapping f: (X,τ)→(Y,σ) is said to be fuzzy w-continuous if the inverse image of every fuzzy closed set of Y is fuzzy w-closed in X.

Theorem 3.1: A mapping f: (X,τ)→(Y,σ) is fuzzy w-continuous if and only if the inverse image of every fuzzy open set of Y is fuzzy w-open in X.

Proof: It is obvious because f^{-1}(1-U)=1-f^{-1}(U) for every fuzzy set U of Y.

Remark 3.1: Every fuzzy continuous mapping is fuzzy w-continuous, but the converse may not be true for,

Example 3.1: Let X = {a, b} and Y = {x, y} and the fuzzy sets U and V are defined as follows

U (a) = 0.5, U (b) = 0.4

V(x) = 0.5, V(y) = 0.5,

Let τ = {0,U,1} and σ = {0,V,1}, be fuzzy topologies on X and Y respectively .Then the mapping f: (X,τ)→(Y,σ) defined by f(a) = x and f(b) = y is fuzzy w-continuous but not fuzzy continuous.

Remark 3.2: Every fuzzy w-continuous mapping is fuzzy g-continuous, but the converse may not be true for,

Example 3.2: Let X = {a, b} and Y = {x, y} and the fuzzy sets U and V are defined as follows

U (a) = 0.7, U (b) = 0.6,V(x) = 0.6, V(y) = 0.7,

Let τ= {0,U,1} and σ = {0,V,1}, be fuzzy topologies on X and Y respectively. Then the mapping f: $(X,\tau)\rightarrow(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy g-continuous but not fuzzy w- continuous.

Remark 3.3: Every fuzzy w-continuous mapping is fuzzy sg-continuous, but the converse may not be true for,

Example 3.3: Let X = {a, b} and Y={x, y} and the fuzzy sets U and V are defined as follows

U (a) =0.5, U (b) = 0.4 ,V(x) = 0.5, V(y) = 0.3,

Let τ={0,U,1} and σ={0,V,1}, be fuzzy topologies on X and Y respectively. Then the mapping f: $(X,\tau)\rightarrow(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy sg-continuous but not fuzzy w-continuous.

Remark 3.4: Remarks 1.4, 1.5, 1.6, 3.1, 3.2 and 3.3 reveals the following diagram of implications:

fuzzy continuous $\Rightarrow$ fuzzy w-continuous $\Rightarrow$ fuzzy g-continuous

$\Downarrow$ $\Downarrow$ $\Downarrow$

fuzzy semi continuous $\Rightarrow$ fuzzy sg-continuous $\Rightarrow$ fuzzy gs-continuous

Theorem 3.2: If f: $(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy w-continuous then for each fuzzy point x_β of X and each fuzzy open set V of Y such that $f(x_\beta) \in V$ then there exists a fuzzy w-open set U of X such that $x_\beta \in U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V is fuzzy open set of Y such that $f(x_\beta) \in V$, put $U = f^{-1}(V)$.Then by hypothesis U is fuzzy w-open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Theorem 3.3: If f: $(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy w-continuous then for each point $x_\beta \in X$ and each fuzzy open set V of Y such that $f(x_\beta)_q V$ then there exists a fuzzy w-open set U of X such that $x_{\beta q}U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V is fuzzy open set such that $f(x_\beta)_q V$. Put $U= f^{-1}(V)$.Then by hypothesis U is fuzzy w-open set of X such that $x_{\beta q}U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Definition 3.5: Let (X,τ) be a fuzzy topological space. The w-closure of a fuzzy set A of X denoted by wcl(A) is defined as follows:

$wcl(A) = \wedge\{ B: B \geq A$, B is fuzzy w-closed set in X $\}$

Remark 3.3: It is clear that, $A \leq gcl(A) \leq wcl(A) \leq cl(A)$ for any fuzzy set A of X.

Theorem 3.4: A mapping f: $(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy w- continuous then $f(wcl(A)) \leq cl(f(A))$ for every fuzzy set A of X.

Proof: Let A be a fuzzy set of X. Then cl(f(A)) is a fuzzy closed set of Y. Since f is fuzzy w-continuous $f^{-1}(cl(f(A))$ is fuzzy w-closed in X. Clearly $A \leq f^{-1}(cl(f(A)))$. Therefore $wcl(A) \leq wcl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A)))$. Hence $f(wcl(A)) \leq cl(f(A))$.

Definition 3.6: A fuzzy topological space (X,τ) is said to be fuzzy w-$T_{1/2}$ if every fuzzy w-closed set in X is fuzzy semi closed.

Theorem 3.5: A mapping f from a fuzzy w-$T_{1/2}$ space (X,τ) to a fuzzy topological space (Y,σ) is fuzzy continuous if and only if it is fuzzy w-continuous.

Proof: Obvious.

Remark 3.4: The composition of two fuzzy w-continuous mappings may not be fuzzy w-continuous. For

Example 3.2: Let X = {a, b} , Y = {x, y} , Z= {p, q}and the fuzzy sets U ,V and W defined as follows:

U (a) =0.5 , U (b) = 0.4,V(x) = 0.5 , V(y) = 0.3 ,W (p) = 0.6 , W (q) = 0.4 Let τ = {0,U,1} and σ = {0,V,1} and μ ={0,W,1}be the fuzzy topologies on X ,Y and Z respectively. Then the mapping f: $(X,\tau)\rightarrow(Y,\sigma)$ defined by f(a) = x and f(b) = y and the mapping $g:(Y,\sigma)\to(Z,\mu)$ defined by g(x) = p, and g(y) = q. Then f and g are w-continuous but gof is not fuzzy w-continuous. However,

Theorem 3.6: If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-continuous and $g:(Y,\sigma)\to(Z,\mu)$ is fuzzy continuous . Then gof : $(X,\tau)\to(Z,\mu)$ is fuzzy w-continuous.

Proof: Let A be a fuzzy closed in Z then g^{-1} (A) is fuzzy closed in Y, because g is continuous .Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy w-closed in X. Hence gof is fuzzy w-continuous.

Theorem 3.7: If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-continuous and $g:(Y,\sigma)\to(Z,\mu)$ is fuzzy continuous is fuzzy g-continuous and (Y,σ) is fuzzy $T_{1/2,}$ then gof: $(X,\tau)\to(Z,\mu)$ is fuzzy w-continuous.

Proof: Let A be a fuzzy closed set in Z then $g^{-1}(A)$ is fuzzy g-closed set in Y because g is g-continuous. Since Y is $T_{1/2}$, $g^{-1}(A)$ is fuzzy closed in Y. And so, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy w-closed in X. Hence gof: $(X,\tau)\to(Z,\mu)$ is fuzzy w-continuous

Theorem 3.8: A fuzzy w-continuous image of a fuzzy w-compact space is fuzzy compact.

Proof: Let f: $(X,\tau)\to(Y,\sigma)$ is a fuzzy w-continuous mapping from a fuzzy w-compact space (X,τ) on to a fuzzy topological space (Y,σ). Let $\{A_i:i\in\Lambda\}$ be a fuzzy open cover of Y. Then $\{f^{-1}(A_i : i\in\Lambda)\}$ is a fuzzy w-open cover of X. Since X is fuzzy w-compact it has a finite fuzzy sub cover say $f^{-1}(A_1), f^{-1}(A_2),\ldots\ldots\ldots,f^{-1}(A_n)\}$. Since f is on to $\{A_1,A_2,\ldots..A_n\}$ is an open cover of Y. Hence (Y,σ) is fuzzy compact.

Definition 3.7: A fuzzy topological space X is said to be fuzzy w-connected if there is no proper fuzzy set of X which is both fuzzy w-open and fuzzy w-closed.

Remark 3.5: Every fuzzy w-connected space is fuzzy connected, but the converse may not be true for the fuzzy topological space (X,τ) in example 2.1 is fuzzy connected but not fuzzy w-connected.

Theorem 3.9: If f: $(X,\tau)\to(Y,\sigma)$ is a fuzzy w-continuous surjection and X is fuzzy w-connected then Y is fuzzy connected.

Proof: Suppose Y is not fuzzy connected .Then there exists a proper fuzzy set G of Y which is both fuzzy open and fuzzy closed .Therefore $f^{-1}(G)$ is a proper fuzzy set of X, which is both fuzzy w-open and fuzzy w-closed in X, because f is fuzzy w-continuous surjection. Hence X is not fuzzy w-connected, which is a contradiction.

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