Power of Mean Chart under Second Order Auto-Correlation with Known Coefficient of Variation

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Abstract**-** In this paper we investigate the effect of auto-correlation on the power function of mean chart with known coefficient of variation. We synthesize the second order auto-correlation process by its three different roots. In particular, the shift in the auto-correlation structure from independent data to a random walk, this is a special case of the structural shift occurring in the process. For various values of roots the values of power functions are tabulated with known coefficient of variation.

Index Terms- control chart, power function, auto-correlation, coefficient of variation.

I. Introduction

Control charts are effective for on-line process monitoring, and recent research in statistical process control (SPC) has focused on the development of advanced monitoring and control techniques. Traditionally, control charts were developed under the assumption that observations of quality characteristics of interest are statistically independent both within and between samples. Furthermore, successful implementation of traditional control charts relies on the assumption of process stability. However, these assumptions are usually violated in practice and new approaches need to be adopted. Many processes show certain intrinsic trend pattern. Usually as long as this is within a certain range, it is considered acceptable. A common example is the tool wear process, which may exhibit a trend in the process mean level because of the physical nature. For such trended processes, the absolute level is of great concern and it should not exceed a certain specification; otherwise they should be replaced. However, we also need to monitor the process in a traditional way in the sense that any abnormal changes can be detected quickly. In general, the traditional Shewhart control charts are not suitable for monitoring auto correlated process directly, as the independence and stability assumptions are violated. The purpose of the present study is to develop a suitable control charting and decision making procedure to monitor auto correlated processes.

In particular, the shift in the autocorrelation structure from independent data to a random Walk, which is considered by Box and Luceno (1997), Box et al. (1994), AstrÖm and Wittenmark (1989), Page (1955), Roberts (1958), Bissel (1978), Wheeler (1987), Champ and Woodall (1987), Nelson (1984) Ishikawa (1976), ASQC (1986), Davis and Woodall (1988) Aerne et al. (1991), Knoth et al.. (1998), and Montgomery (1985). Detection and elimination of parameter shifts are the typical task of statistical control charting. These techniques in be applied for the detection of shifts from the in-control to the out-of-control state in the random components. To detect shifts, samples are taken from the process. Since the power function will play main role in hypothesis testing, we used sample data for repeated statistical test of simple hypothesis versus alternative hypothesis in case of autocorrelation as follows :
(i)Null hypothesis (Ho) : process is in control (in our case : H_o : $\mu_1 = 0$, $\alpha_1 = \alpha_2 = 0$) against the
(ii)Alternative Hypothesis (H_1) : production is out of control (in our case: H_1 : $\mu_1 \neq 0$, or α_1 and $\alpha_2 \neq 1$).

The rejection of Hypothesis H_o is interpreted as an alarm or out-of- control signal : the process is stopped for investigative and corrective action. Acceptance of hypothesis H_o (no alarm) means that the process continuous without intervention. The alternative H_1 : $\mu_1 \neq 0$ or α_1 and $\alpha_2 \neq 1$ is often restricted to more specific alternatives by additional prior information (knowledge), e.g., that the autoregressive parameters (α_1 and α_2) is known to be in control, known particular alternatives $\mu_1 \neq 0$. The center line in control charts are fixed at μ_0 and power functions are calculated against alternative hypothesis.

In this paper we investigate the effect of autocorrelation on the power function of $\overline{X}$ -chart is studied with known coefficient of variation(cv). We synthesize the second order auto correlated process (AR (2)) by its three different roots. In particular, the shift in the autocorrelation structure from independent data to a random walk, which is a special case of the structural shift occurring in an AR(2) process. For various values of roots the values of power functions are tabulated and compared with those the independent case with known *cv* case.

II. Second order Autoregressive model

Consider a manufacturing process where a quality characteristic is measured at equidistance time points 1, 2, 3, ... n. This situation may occur in a discrete manufacturing process which produces discrete time 1, 2, 3, ... n, with one quality characteristic of interest. It may also occur in a continuous manufacturing process where the quality characteristic of interest is measured at discrete equidistant time points. We denote the behavior of the quality characteristic as x_1, x_2, ... x_n. It will assumed that on EPC control action can be represented by some controllable variable or factor x_t, such that

$$x_t = \mu + \xi_t, \qquad (1)$$

where μ is a constant, and ξ_t is a stationary time series with zero mean and standard deviation σ. A Durbin and Watson (1950) "d" statistic can be used to detect the presence or absence of serial correlation. The problem, however, is that to do once the suspicion of dependence via the serial correlation test is confirmed. If serial correlation exist we use identification techniques to define the nature of ξ_t. When identification is complete, the likelihood function can provide maximum likelihood estimate of the parameters of the identified model.

Suppose that a correlation test revealed the presence of data dependence and identification technique suggested autoregressive model of order two AR(2) say, then we can express ξ_t of equation (1) as

$$\xi_t = \alpha_1 \xi_{t-1} + \alpha_2 \xi_{t-2} + \epsilon_k,\ t = 1,2, \ldots. n \quad (2)$$

where

(i) $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$

(ii) $$\operatorname{cov}\left(\epsilon_t, \epsilon_\gamma\right) = \begin{cases} \sigma_\epsilon^2 & t = \gamma \\ 0 & t \neq \gamma \end{cases} \quad (3)$$

The class of stationary models that assume the process to remain in equilibrium about a constant mean level μ. The variance of AR (2) process is given by:

$$\sigma^2 = \left(\frac{1-\alpha_2}{1+\alpha_2}\right) \frac{\sigma_\epsilon^2}{\left[(1-\alpha_2)^2 - \alpha_1^2\right]}. \quad (4)$$

Following Kendall and Stuart (1976) it can be shown that for stationarity, the roots of the characteristic equation of the process in equation (2)

$$\phi(B) = 1 - \alpha_1 B - \alpha_2 B^2 \quad (5)$$

must lies outside the unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions :

$$\alpha_2 + \alpha_1 < 1$$
$$\alpha_2 - \alpha_1 < 1$$
$$-1 < \alpha_2 < 1 \quad (6)$$

Now If G_1^{-1} and G_2^{-1} are the roots of the characteristic equation of the process given by equation (5) then

$$G_1 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \quad (7)$$

$$G_2 = \frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \quad (8)$$

For stationary we require that $|G_i| < 1,\ i = 1, 2.$ Thus, three situations can theoretically arise:

(i) Roots G_1 and G_2 are real and distinct $\left(i.e.,\ \alpha_1^2 + 4\alpha_2 > 0\right)$

(ii) Roots G_1 and G_2 are real and equal $\left(i.e.,\ \alpha_1^2 + 4\alpha_2 = 0\right)$

(iii) Roots G_1 and G_2 are complex conjugate $\left(i.e.,\ \alpha_1^2 + 4\alpha_2 < 0\right)$.

When the serial correlation is present in the data, we have for the distribution of the sample mean $\bar{x}$, its mean and variance is given by,

$$E(\bar{x}) = \mu$$

$$\operatorname{Var}(\bar{x}) = \frac{\sigma^2}{n} \lambda_{ap}(\alpha_1, \alpha_2, n), \quad (9)$$

where $\lambda_{ap}(\alpha_1, \alpha_2, n)$ depends on the nature of the roots G_1 and G_2 , and for different situations is given as follows :

(i) If G_1 and G_2 are real and distinct,

$$\lambda_{ap}(\alpha_1, \alpha_2, n) = \left[\frac{G_1(1-G_2^2)}{(G_1-G_2)(1+G_1G_2)} \lambda(G_1, n) - \frac{G_2(1-G_1^2)}{(G_1-G_2)(1+G_1G_2)} \lambda(G_2, n)\right]$$

$$= \lambda_{rd}(\alpha_1, \alpha_2, n), \quad (10)$$

Where,

$$\lambda(G, n) = \left[\frac{1+G}{1-G} - \frac{2G}{n} \frac{(1-G^n)}{(1-G)^2}\right]$$

(ii) If G_1 and G_2 are real and equal

$$\lambda_{ap}(\alpha_1, \alpha_2, n) = \left(\frac{1+G}{1-G}\right) - \frac{2G(1-G^n)}{n(1-G)^2}\left[1 + \frac{(1+G)^2(1-G^n) - n(1-G^2)(1+G^n)}{(1+G^2)(1-G^n)}\right]$$

$$= \lambda_{re}(\alpha_1, \alpha_2, n) \quad (11)$$

(iii) If G_1 and G_2 and complex conjugate

$$\lambda_{ap}(\alpha_1, \alpha_2, n) = \left[\gamma(d, u) + \frac{2d}{n}\left(W(d,u,n) + z(d,u,n)\right)\right]$$

$$= \lambda_{cc}(\alpha_1, \alpha_2, n) \quad (12)$$

Where $\gamma(d,u) = \dfrac{1-d^4+2d\left(1-d^2\right)\cos u}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)}$,

$$W(d,u,n) = \frac{2d\left(1+d^2\right)\sin u - \left(1+d^4\right)\sin 2u - d^{n+4}\sin((n-2)u)}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)^2 \sin u}$$

,

$$Z(d,u,n) = \frac{2d^{n+3}\sin(n-1)u - 2d^{n+1}\sin(n+1)u + d^n\sin((n+2)u)}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)^2 \sin u},$$

$d^2 = -\alpha_2$,

and $u = \cos^{-1}\left(\dfrac{\alpha_1}{2d}\right)$.

The x_t denote the change in the level of the compensating variable model at the time t, i.e., the adjustment made at the time point t. The ε_t is Gaussian white noise with variance $\sigma_\in^2$. Throughout, we suppose that the noise variance is known. In practice, this is justified if reliable estimates of $\sigma_\in^2$ are available from the evaluation of a large number of previous values of the process, e.g., during the setup phase. The real - valued parameters α_1 and α_2 (the autoregressive parameters) determines the influence of the preceding time point (t - 1) and (t - 2) on the present time point t. We assume an in-control value $\alpha_1 = \alpha_2 = O$ for the autoregression parameters. It is possible that the autoregression parameters may shift to an out-of-control value $(\alpha_1,\alpha_2) \neq 0$.

III. Power of the $\bar{X}$ Chart under Second Order Autocorrelation with Known Coefficient of Variation(CV)

From a random sample of n observations $x_1, x_2, ... x_n$, an estimator $\bar{x}'$ has been constructed by Searls (1964), where

$$\bar{x}' = w\sum_{j=1}^{n} x_j \tag{13}$$

where, w is a scalar and is chosen so that the MSE, $E(\bar{x}'-\mu)^2$ is minimum. Searls (1964) has shown that

$w = \dfrac{1}{(n+\lambda(\alpha_1,\alpha_2)v^2)}$ and

hence

$$MSE\left(\bar{x}'\right) = \sigma^2 / \left(\frac{n}{\lambda(\alpha_1,\alpha_2)} + v^2\right) \tag{14}$$

The distribution of sample mean $\bar{x}'$ under second order autocorrelation with known coefficient of variation is given by

$$f\left(\bar{x}'\right) = \frac{\sqrt{\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2}}{\sigma}\left[\phi\left(\frac{\bar{x}'-\mu}{\sigma / \sqrt{\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2}}\right)\right], \tag{15}$$

where $\phi(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\left(x^2/2\right)}$

In this develop it is assumed that the process has a normal distribution with mean μ and $MSE\left(\bar{x}\right) = \sigma^2 / \left(\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2\right)$. It is further assumed that at the time of determining the control limits the process is in a state of statistical control, and the same device is used as will be employed for known *cv*. The distributional behavior of $\bar{x}'$ depends only on $\lambda(\alpha_1,\alpha_2)$, *v* and *n*. Thus the data used for establishing the limits on the control charts comes from a process that is $N\left(\mu, \sigma^2/n\right)$. When the process shifts, the data is assumed to come from an $N\left(\mu', \sigma^2 / \left(\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2\right)\right)$ population. If samples of size n are taken from the population $N\left(\mu', \sigma^2 / \left(\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2\right)\right)$ and the value of $\bar{x}'$ is plotted with control limits of $\mu \pm 3\dfrac{\sigma}{\sqrt{\dfrac{n}{\lambda(\alpha_1,\alpha_2)} + v^2}}$, the power of detecting the change of process is given by the following formula.

$$P_{\bar{x}} = P_r\{\bar{x} \geq \mu + 3\frac{\sigma}{\sqrt{n}}\} + p_r\{\bar{x} \leq \mu - 3\frac{\sigma}{\sqrt{n}}\}, \tag{16}$$

under second order autocorrelation with known cv we have power function is,

$$P_{\bar{x}}' = P_r\{\bar{x}' \geq \mu + 3\frac{\sigma}{\sqrt{n}}\} + p_r\{\bar{x}' \leq \mu - 3\frac{\sigma}{\sqrt{n}}\}, \tag{17}$$

$$P_{\bar{x}}' = P_r\left\{\frac{\bar{x}'-\mu'}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}} \geq \frac{\mu-\mu'}{\sqrt{\frac{\sigma^2}{(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}}} + \frac{3\sqrt{\sigma^2/n}}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}}\right\}$$

$$+P_r\left\{\frac{\bar{x}'-\mu'}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}} \leq \frac{\mu-\mu'}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}} - \frac{3\sqrt{\sigma^2/n}}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}}\right\} \quad (18)$$

$$P_{\bar{x}}' = P_r\left\{z \geq -d\sqrt{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2} + 3\sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}\right\}$$

$$+P_r\left\{z \leq -d\sqrt{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2} - 3\sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}\right\} \quad (19)$$

Where,

$$z = \frac{\bar{x}'-\mu'}{\sqrt{\sigma^2/(\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2)}} \quad and \quad d = \frac{\mu'-\mu}{\sigma}$$

$$P\bar{x}' = \Pr\left\{z \geq \sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}\,(3-d\sqrt{n})\right\} + \Pr\left\{z \leq \sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}(3-3d\sqrt{n})\right\} \quad (20)$$

$$= \Phi\left\{\sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}(-3+d\sqrt{n})\right\} + \Phi\left\{\sqrt{\frac{\frac{n}{\lambda(\alpha_1,\alpha_2)}+\nu^2}{n}}(-3-d\sqrt{n})\right\} \quad (21)$$

Where,

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} \exp\left(-u^2/2\right) du$$

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IV. Illustrative Numerical Comparison

The values of power function for some chosen values of v and three different roots along with independence case have been worked out using equation (21) and given in Table1 and Table 2 for n = 5 and 7.To give a visual comparison, the power curves have been drawn for n=5 in figure 1 to figure 4 attach. A comparison of various curves for AR(2) process with classical control chart having independent observations shows that complex conjugate roots have the tendency to bring the power curve for auto correlated data very close to the power curve for independent observation. However, there is marked difference in the power curves for the other two situations e.g., when the roots are (i) real and equal and (ii) real and distinct. In both the situations there is a large deviation in the power curves from the independent observation with known cv. In practical situations, however, the case of real and equal roots hardly arise. Moreover, this is too simple a case for calculations and included only for the sake of completion. It is seen from the table that possibility of shifts not only the mean parameters but also in the auto regression parameters □1 and □2 can take place. A shift in the auto regression parameters may result from assignable causes occurring over production time, but often also from misidentification of the auto regression model. e.g., a biased estimate of the auto regression parameters □1, □2. Hence we recommend to use of power curves for -chart under the process model where the shifts in the mean and the auto regression parameters are possible. When cv increases the values of power function in all situations close to each other.

TABLE:1 Power Function of $\overline{X}$ -Chart under AR(2) Process with Known cv (n=5)

d	Independent Observations				Roots are Real and Distinct (α_1=0.3,α_2=0.6)				Roots are Real and Equal (α_1=0.8,α_2=-0.16)				Roots are Complex Conjugate (α_1=0.8,α_2=-0.6)			
	v=0	v=2	v=4	v=6	v=0	v=2	v=4	v=6	v=0	v=2	v=4	v=6	v=0	v=2	v=4	v=6
0.00	0.0027	0.0001	0.0000	0.0000	0.1350	0.0021	0.0000	0.0000	0.0718	0.0012	0.0000	0.0000	0.0114	0.0002	0.0000	0.0000
0.10	0.0034	0.0001	0.0000	0.0000	0.1374	0.0027	0.0000	0.0000	0.0743	0.0016	0.0000	0.0000	0.0129	0.0004	0.0000	0.0000
0.20	0.0056	0.0003	0.0000	0.0000	0.1447	0.0047	0.0000	0.0000	0.0820	0.0031	0.0000	0.0000	0.0175	0.0009	0.0000	0.0000
0.30	0.0100	0.0009	0.0000	0.0000	0.1567	0.0086	0.0000	0.0000	0.0948	0.0061	0.0000	0.0000	0.0257	0.0021	0.0000	0.0000
0.40	0.0177	0.0024	0.0000	0.0000	0.1733	0.0156	0.0000	0.0000	0.1129	0.0117	0.0000	0.0000	0.0384	0.0048	0.0000	0.0000
0.50	0.0299	0.0058	0.0001	0.0000	0.1943	0.0270	0.0002	0.0000	0.1361	0.0213	0.0002	0.0000	0.0565	0.0103	0.0001	0.0000
0.60	0.0486	0.0130	0.0003	0.0000	0.2196	0.0448	0.0010	0.0000	0.1644	0.0370	0.0010	0.0000	0.0811	0.0207	0.0005	0.0000
0.70	0.0757	0.0271	0.0016	0.0000	0.2488	0.0709	0.0039	0.0000	0.1977	0.0611	0.0037	0.0000	0.1132	0.0389	0.0023	0.0000
0.80	0.1129	0.0521	0.0065	0.0003	0.2816	0.1075	0.0123	0.0005	0.2357	0.0960	0.0119	0.0004	0.1536	0.0683	0.0083	0.0003
0.90	0.1617	0.0926	0.0215	0.0023	0.3176	0.1560	0.0333	0.0035	0.2780	0.1437	0.0326	0.0033	0.2025	0.1124	0.0254	0.0027
1.00	0.2225	0.1527	0.0587	0.0143	0.3563	0.2171	0.0780	0.0185	0.3241	0.2053	0.0769	0.0178	0.2597	0.1738	0.0654	0.0158
1.10	0.2945	0.2343	0.1341	0.0609	0.3972	0.2901	0.1579	0.0702	0.3734	0.2803	0.1566	0.0687	0.3243	0.2533	0.1426	0.0643
1.20	0.3757	0.3355	0.2581	0.1822	0.4396	0.3729	0.2782	0.1937	0.4249	0.3665	0.2772	0.1919	0.3947	0.3485	0.2655	0.1865
1.30	0.4629	0.4503	0.4243	0.3949	0.4831	0.4620	0.4314	0.3997	0.4779	0.4601	0.4310	0.3990	0.4687	0.4544	0.4269	0.3967
1.40	0.5519	0.5695	0.6054	0.6457	0.5270	0.5531	0.5957	0.6391	0.5313	0.5559	0.5962	0.6401	0.5438	0.5637	0.6018	0.6432
1.50	0.6384	0.6826	0.7660	0.8447	0.5708	0.6415	0.7446	0.8331	0.5842	0.6486	0.7457	0.8349	0.6174	0.6683	0.7581	0.8404
1.60	0.7183	0.7808	0.8818	0.9510	0.6138	0.7229	0.8583	0.9426	0.6356	0.7331	0.8596	0.9439	0.6869	0.7612	0.8734	0.9479
1.70	0.7885	0.8588	0.9497	0.9891	0.6555	0.7940	0.9316	0.9856	0.6847	0.8060	0.9327	0.9862	0.7504	0.8377	0.9435	0.9879
1.80	0.8473	0.9154	0.9822	0.9983	0.6954	0.8530	0.9715	0.9974	0.7308	0.8652	0.9722	0.9976	0.8063	0.8962	0.9787	0.9980
1.90	0.8941	0.9530	0.9947	0.9998	0.7332	0.8994	0.9898	0.9997	0.7732	0.9107	0.9901	0.9997	0.8538	0.9376	0.9932	0.9998
2.00	0.9295	0.9759	0.9987	1.0000	0.7684	0.9341	0.9969	1.0000	0.8115	0.9436	0.9970	1.0000	0.8928	0.9648	0.9982	1.0000
2.10	0.9550	0.9885	0.9997	1.0000	0.8010	0.9587	0.9992	1.0000	0.8456	0.9661	0.9992	1.0000	0.9236	0.9814	0.9996	1.0000
2.20	0.9725	0.9950	1.0000	1.0000	0.8306	0.9753	0.9998	1.0000	0.8753	0.9807	0.9998	1.0000	0.9472	0.9908	0.9999	1.0000

2.30	0.9839	0.9980	1.0000	1.0000	0.8572	0.9859	1.0000	1.0000	0.9008	0.9895	1.0000	1.0000	0.9646	0.9958	1.0000	1.0000

TABLE:2 Power Function of $\overline{X}$ -Chart under AR(2) Process with Known cv (n=7).

d	**Independent Observations**				**Roots are Real and Distinct (α_1=0.3,α_2=0.6)**				**Roots are Real and Equal (α_1=0.8,α_2=-0.16)**				**Roots are Complex Conjugate (α_1=0.8,α_2=-0.6)**			
	v=0	**v=2**	**v=4**	**v=6**	**v=0**	**v=2**	**v=4**	**v=6**	**v=0**	**v=2**	**v=4**	**v=6**	**v=0**	**v=2**	**v=4**	**v=6**
0.00	0.0027	0.0002	0.0000	0.0000	0.1967	0.0091	0.0000	0.0000	0.0880	0.0045	0.0000	0.0000	0.0053	0.0003	0.0000	0.0000
0.10	0.0037	0.0003	0.0000	0.0000	0.1996	0.0109	0.0000	0.0000	0.0916	0.0058	0.0000	0.0000	0.0068	0.0006	0.0000	0.0000
0.20	0.0069	0.0010	0.0000	0.0000	0.2083	0.0169	0.0001	0.0000	0.1024	0.0101	0.0000	0.0000	0.0114	0.0016	0.0000	0.0000
0.30	0.0138	0.0028	0.0000	0.0000	0.2225	0.0280	0.0003	0.0000	0.1203	0.0186	0.0002	0.0000	0.0205	0.0041	0.0000	0.0000
0.40	0.0261	0.0075	0.0002	0.0000	0.2421	0.0458	0.0011	0.0000	0.1453	0.0332	0.0009	0.0000	0.0358	0.0100	0.0003	0.0000
0.50	0.0468	0.0178	0.0012	0.0000	0.2667	0.0724	0.0042	0.0001	0.1771	0.0563	0.0034	0.0000	0.0597	0.0223	0.0015	0.0000
0.60	0.0789	0.0383	0.0052	0.0002	0.2958	0.1096	0.0132	0.0006	0.2155	0.0908	0.0113	0.0005	0.0948	0.0454	0.0061	0.0003
0.70	0.1255	0.0751	0.0187	0.0022	0.3291	0.1590	0.0356	0.0040	0.2599	0.1388	0.0318	0.0036	0.1432	0.0846	0.0208	0.0025
0.80	0.1885	0.1341	0.0547	0.0143	0.3658	0.2211	0.0825	0.0207	0.3095	0.2017	0.0768	0.0194	0.2060	0.1451	0.0585	0.0152
0.90	0.2680	0.2189	0.1310	0.0625	0.4053	0.2952	0.1653	0.0766	0.3636	0.2792	0.1588	0.0740	0.2828	0.2294	0.1361	0.0647
1.00	0.3616	0.3285	0.2604	0.1900	0.4470	0.3790	0.2888	0.2068	0.4208	0.3688	0.2836	0.2038	0.3711	0.3357	0.2648	0.1927
1.10	0.4643	0.4552	0.4354	0.4121	0.4901	0.4689	0.4440	0.4180	0.4801	0.4662	0.4424	0.4170	0.4668	0.4572	0.4368	0.4130
1.20	0.5694	0.5868	0.6244	0.6677	0.5339	0.5605	0.6083	0.6568	0.5398	0.5657	0.6112	0.6587	0.5645	0.5829	0.6218	0.6659
1.30	0.6698	0.7092	0.7872	0.8620	0.5778	0.6489	0.7552	0.8448	0.5988	0.6612	0.7611	0.8479	0.6584	0.7006	0.7822	0.8592
1.40	0.7593	0.8113	0.8991	0.9595	0.6210	0.7299	0.8658	0.9479	0.6556	0.7473	0.8723	0.9501	0.7434	0.8004	0.8942	0.9578
1.50	0.8336	0.8877	0.9604	0.9918	0.6629	0.8002	0.9361	0.9873	0.7091	0.8202	0.9412	0.9882	0.8158	0.8769	0.9572	0.9912
1.60	0.8912	0.9389	0.9873	0.9989	0.7031	0.8583	0.9737	0.9978	0.7584	0.8783	0.9768	0.9980	0.8739	0.9301	0.9857	0.9987
1.70	0.9329	0.9698	0.9967	0.9999	0.7410	0.9037	0.9907	0.9997	0.8028	0.9217	0.9922	0.9998	0.9178	0.9635	0.9961	0.9999
1.80	0.9610	0.9864	0.9993	1.0000	0.7763	0.9373	0.9972	1.0000	0.8419	0.9522	0.9978	1.0000	0.9491	0.9826	0.9991	1.0000
1.90	0.9787	0.9945	0.9999	1.0000	0.8087	0.9611	0.9993	1.0000	0.8754	0.9724	0.9995	1.0000	0.9701	0.9924	0.9998	1.0000
2.00	0.9890	0.9980	1.0000	1.0000	0.8381	0.9769	0.9998	1.0000	0.9037	0.9849	0.9999	1.0000	0.9833	0.9970	1.0000	1.0000

2.10	0.9947	0.9993	1.0000	1.0000	0.8644	0.9869	1.0000	1.0000	0.9269	0.9922	1.0000	1.0000	0.9912	0.9989	1.0000	1.0000
2.20	0.9976	0.9998	1.0000	1.0000	0.8876	0.9929	1.0000	1.0000	0.9456	0.9962	1.0000	1.0000	0.9956	0.9996	1.0000	1.0000
2.30	0.9990	0.9999	1.0000	1.0000	0.9079	0.9964	1.0000	1.0000	0.9603	0.9982	1.0000	1.0000	0.9979	0.9999	1.0000	1.0000

Fig.1: Power Curve of $\overline{X}$ chart under AR(2) Process with cv = 0.

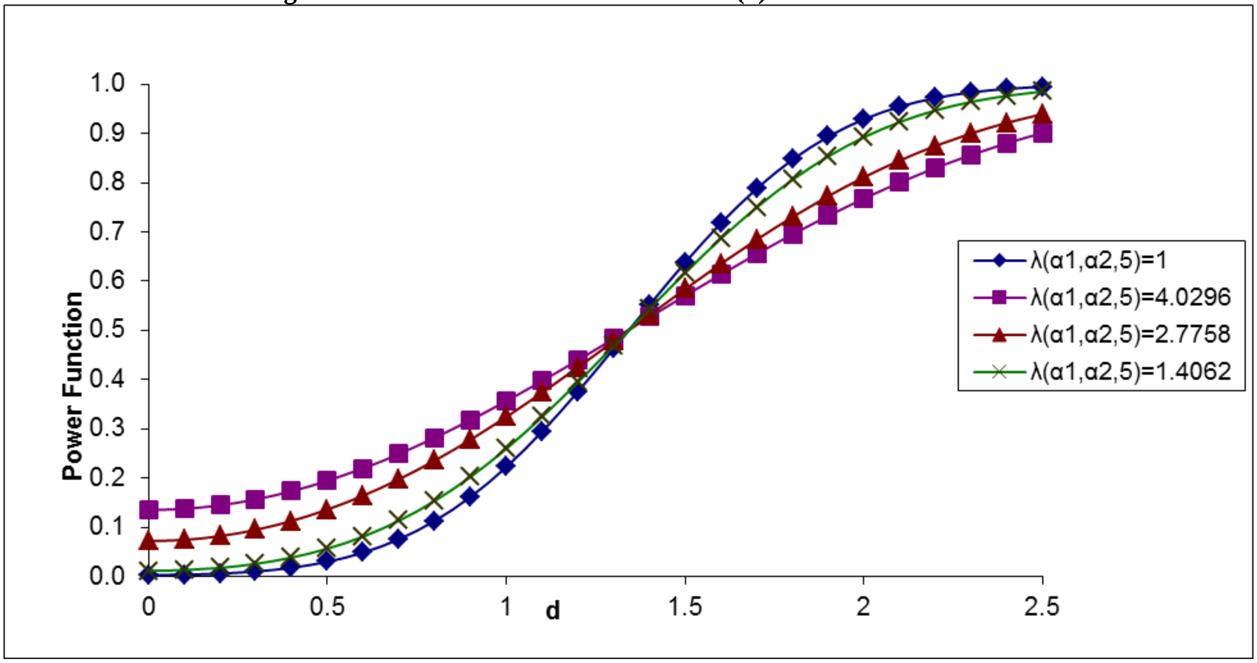


Fig.2: Power Curve of $\overline{X}$ chart under AR(2) Process with cv = 2.

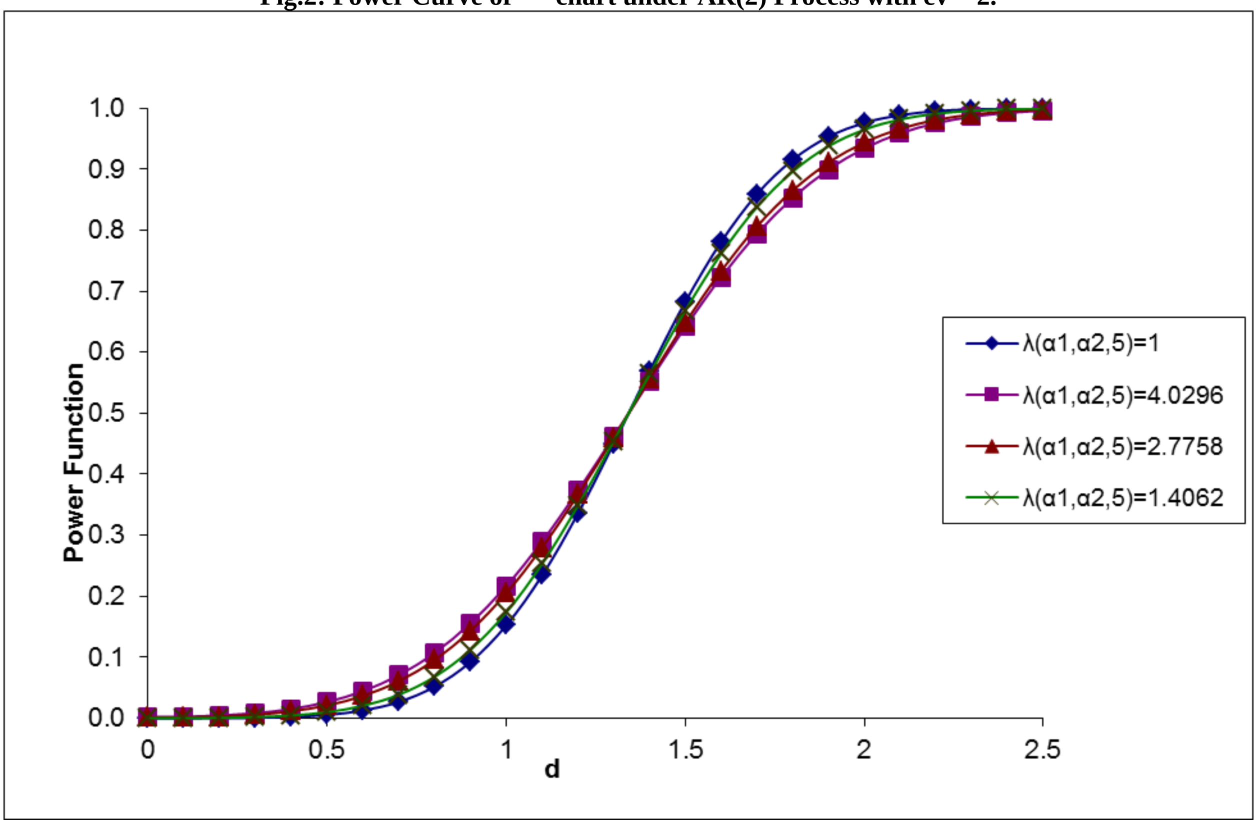


Fig.3: Power Curve of $\overline{X}$ chart under AR(2) Process with cv = 4.

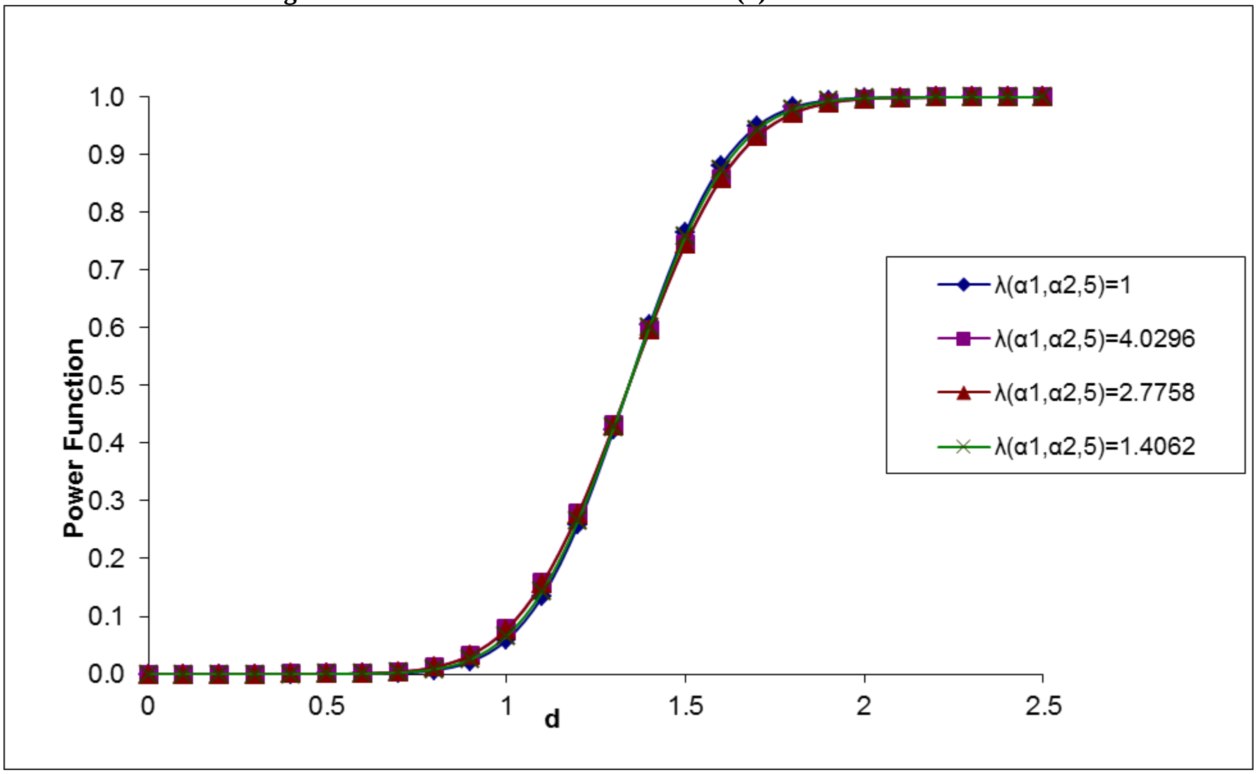
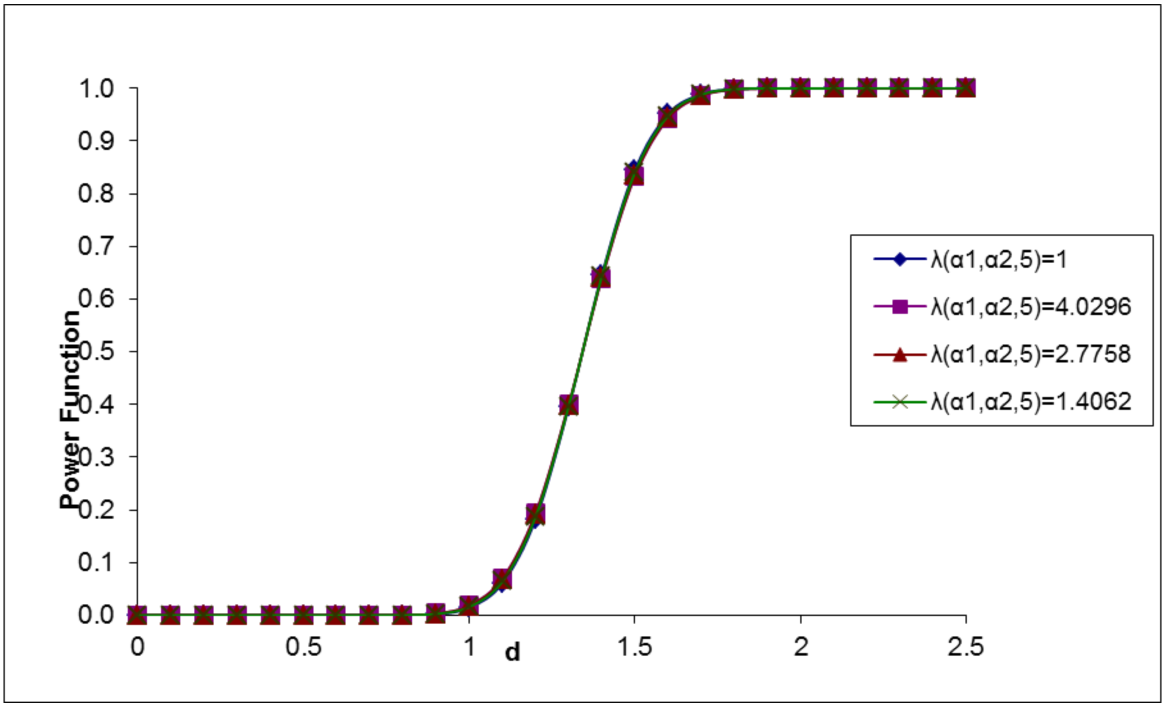


Fig.4: Power Curve of $\overline{X}$ chart under AR(2) Process with cv = 6.



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