gr-compactness in topological spaces

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Abstract

In this paper, we introduce the new concepts gr-compactness in topological spaces and obtain some of their properties using gr-closed sets.

Keywords: gr-closed sets, gr-continuous maps and gr-compactness.

1 Introduction

The notions of compactness is useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers [1-7] have analyzed the basic properties of compactness. The notions of compactness resulted in motivating mathematicians to generalize these notions further.

Bhattacharya S. [8] introduced and studied the properties of gr-closed sets in topological spaces. The aim of this paper is to study gr-compactness using gr-closed set and also discuss some of their properties

2 Preliminaries

Throughout this paper (X, τ), (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ), cl(A) and Int(A) denote the closure of A and interior of A respectively.

Definition 2.1. Let (X, τ) be a topological space. Then, a subset A of (X, τ) is called gr-closed set [8] if rcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

The complement of the above mentioned gr-closed set is gr-open set.

Definition 2.2. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(i) gr-continuous [9] if the inverse image of every closed set in (Y, σ) is gr-closed in (X, τ).

(ii) gr-irresolute [9] if the inverse image of every gr-closed set in (Y, σ) is gr-closed in (X, τ).

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3 gr-compactness

Definition 3.1. A collection $\{A_i : i \in I\}$ of gr-open sets in a topological space X is called a gr-open cover of a subset B of X if $B \subset \cup\{A_i : i \in I\}$ holds.

Definition 3.2. A topological space X is gr-compact if every gr-open cover of X has a finite subcover.

Definition 3.3. A subset B of a topological space X is said to be gr-compact relative to X if, for every collection $\{A_i : i \in I\}$ of gr-open subsets of X such that $B \subset \cup\{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \cup\{A_i : i \in I_0\}$.

Definition 3.4. A subset B of a topological space X is said to be gr-compact if B is gr-compact as a subspace of X.

Theorem 3.1. *Every gr-closed subset of a gr-compact space X is gr-compact relative to X.*

Proof. Let A be gr-closed subset of gr-compact space X. Then, A^c is gr-open in X. Let $M = \{G_\alpha : \alpha \in I\}$ be a cover of A by gr-open sets in X. Then, $M^* = M \cup A^c$ is a gr-open cover of X. Since X is gr-compact, M^* is reducible to a finite subcover of X, say $X = G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^c$, $G_{\alpha_k} \in M$. But, A and A^c are disjoint hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m}$, $G_{\alpha_k} \in M$, which implies that any gr-open cover M of A contains a finite subcover. Therefore, A is gr-compact relative to X. Thus, every gr-closed subset of gr-compact space X is gr-compact. □

Theorem 3.2. *Every gr-compact space is compact.*

Proof. Let X be a gr-compact space. Let $\{A_i : i \in I\}$ be an open cover of X. Then $\{A_i : i \in I\}$ is a gr-open cover of X as every open set is gr-open set. Since X is gr-compact, the gr-open cover $\{A_i : i \in I\}$ of X has a finite subcover, say $\{A_i : i = 1, ..., n\}$ for X. Hence X is compact. □

Definition 3.5. A function $f : X \longrightarrow Y$ is said to be gr-continuous [9] if $f^{-1}(F)$ is gr-closed in X for every closed set F of Y.

Definition 3.6. A function $f : X \longrightarrow Y$ is said to be gr-irresolute [9] if $f^{-1}(F)$ is gr-closed in X for every gr-closed set F of Y.

Theorem 3.3. *Let $f : X \to Y$ be surjective, gr-continuous function. If X is gr-compact, then Y is compact.*

Proof. Let $\{A_i : i \in I\}$ be an open cover of Y. Since f is gr-continuous function, then $\{f^{-1}(A_i) : i \in I\}$ is gr-open cover of X has a finite subcover, say $\{f^{-1}(A_i) : i = 1, ..., n\}$. Therefore, $X = \cup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \cup_{i=1}^{n} f(A_i)$. Since f is surjective, $Y = \cup_{i=1}^{n} f(A_i)$. Thus, $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for Y. Hence Y is compact. □

Theorem 3.4. *If a map $f : X \to Y$ is gr-irresolute and a subset B of X is gr-compact relative to X, then the image $f(B)$ is gr-compact relative to Y.*

Proof. Let $\{A_\alpha : \alpha \in I\}$ be any collection of gr-open subsets of Y such that $f(B) \subset \cup\{A_\alpha : \alpha \in I\}$. Then, $B \subset \{f^{-1}(A_\alpha) : \alpha \in I\}$ holds. From the hypothesis, B is gr-compact relative to X. Then, there exists a finite subset I_0 of I such that $B \subset \{f^{-1}(A_\alpha) : \alpha \in I_0\}$. Therefore, we have $f(B) \subset \cup\{A_\alpha : \alpha \in I_0\}$, which shows that $f(B)$ is gr-compact relative to Y. □

4 Conclusion

In this paper, we have introduced gr-compactness in the topological spaces by using gr-closed sets and their properties were studied.

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