Finite Element Analysis and Vibration Aspects of Rotating Turbine Blade with Known Stress Concentration Factors

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Abstract- Modal analysis has been important to the designers and operators since the inception of the Turbo machinery. Blades typically fail because of low-cycle fatigue (LCF), and high-cycle fatigue (HCF), environmental attack, Creep, Oxidation, erosion, embrittlement. In the present work the turbine blade is modeled and analyzed using finite element method the eigen value extraction for flexural and torsional modes is obtained. Foreign object damage is simulated by considering a circular crack along the leading edge of the blade. It was found that the stretching induced by the centrifugal inertia force due to the rotational motion of the blades was the cause for the increment of the bending stiffness of the structure. This resulted in maximum operational Vonmises stresses; the object is to provide understanding and information for designers to improve the life and efficiency of gas turbines.

Index Terms- Turbine blade, Eigen value, Low cycle fatigue, High cycle fatigue

I. Introduction

Resonance is an important failure mechanism that arises when a periodic force acts at a frequency corresponding to a blade natural frequency. If the damping is inadequate for absorption of the periodic input energy, amplitude and stresses grow until failure occurs by overstress or by propagation of fatigue crack. A complex and irregular distribution of minor indentations covers the surface, including leading edge impacts. The damage caused by foreign objects often in the form of a geometric discontinuity like a notch. However the presence of residual stress and sub-structural damage in regions adjacent to the notch prohibit the use of simple notch analyses. FE analysis is used to estimate stress concentration effect of the geometry of the notch. For this purpose, the compressor aerofoil blade is idealized into simple rectangular cantilever plate for FEM study. The complexity of the problem is reduced to a simplex problem by assuming the aerofoil section to a flat rectangular plate. Because of the non-presence of the aerofoil section, analysis is so simple for finding the stress concentration effect of the geometrical notch made. To analyze the stress concentration effect of foreign object damage (FOD), different notches are made by varying the notch dimensions viz., depth, radius of the notch and the location of the notch on the trailing edge of the rectangular plate.

Figure 1: Circular shaped impacts

Foreign object damaged compressor blades with nicks, dents and cracks can be ground into the smooth curved cut-outs to reduce the stress intensification and stress concentration with a high speed grinding wheel. In doing so, depending on the extent and type of damage, designer has a requirement to estimate size and shape of the cut-out to be made on the aerofoil that generates the known SCF. With the estimated values of SCF for different geometry of cut using FEM further a modal analysis has been conducted to assess the variation of frequencies for different geometry of cut-outs with known values of SCF to aster the correctness of the solution, the finite element analysis is carried out on a rectangular cantilever plate for which literature is available. Further the study is extended to an idealized problem of cantilever rectangular plate both in static and rotating conditions with and without the notches.

II. THEORETICAL BACKGROUND

In a normal mode, each element of the beam oscillates up and down at the same frequency. The amplitude of oscillation varies along the beam as shown below for each of the first four normal modes (with Y(x) exaggerated). In figure 2 Y(x) shows the shape of the beam at the extreme of the oscillation when all points on the beam are instantaneously at rest. All the points also go through zero displacement at the same time. Y(x) is the vertical displacement relative to the fixed end, and the horizontal scale is expressed as a fraction of the full length, ℓ, of the beam. In formal terms, a cantilever is a beam that is constrained to have

$Y(x) = 0$ and $\frac{dY(x)}{dx} = 0$ at x = 0 and the other end free.

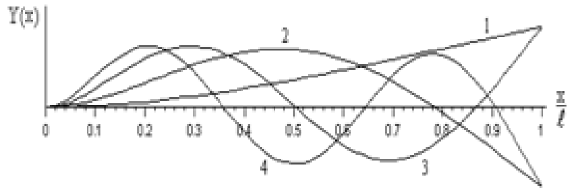


Figure 2: Mode Shapes

The parameters that determine the shape, Y(x), are as follows:

ℓ The length of the beam [m]
A The cross-sectional area of the beam [m^2]
ρ The mass density of the material [kg m^{-3}]
E Young's modulus for the material [kg m^{-1} s^{-2}]
I A geometrical property called the second moment of area of the cross-section. For a rectangular cross-section of width, w, and thickness, d, this is given by $I = \frac{wd^3}{12}$

The theoretical expression for the displacement of the free end of a static cantilever is:

$$Y(\ell) = \frac{\rho A g}{8EI}\ell^4$$

[g is the acceleration due to gravity]

If, in addition, a load of mass m is suspended from the free end the displacement is increased to

$$Y(\ell) = \frac{\rho A g}{8EI}\ell^4 + \frac{mg}{3EI}\ell^3$$

Measurements on the static beam can therefore give information about groups of parameters,

such as $\frac{\rho A}{EI}$ or EI. We shall see that these parameters are relevant to its dynamic behaviour.

In mode 1, all parts of the beam move, except the fixed end. In mode 2 there is a stationary point, or *node*, away from the end (at $x/\ell = 0.784$). In mode 3 there are two nodes, and so on.

2.1 For Flexural Modes

The Natural frequency of flexural modes
$\omega_n = \beta^2_n \surd (EI / \boldsymbol{\rho} L)$

The vibration of beam with constant mass and stiffness is given by

$$EI\frac{\partial^4 \bar{y}}{\partial \bar{x}^4} + m\frac{\partial^2 \bar{y}}{\partial \bar{t}^2} = \bar{f}(\bar{x},\bar{t}), \qquad 0 < x < l,$$

Substituting the above expression into the homogeneous form

$$Y^{(4)}T + Y\ddot{T} = 0,$$

$$\frac{Y^{(4)}}{Y} = -\frac{\ddot{T}}{T} = \beta^4,$$

where β_j is the jth positive root of the equation

$$\cos\beta_j \cosh\beta_j = -1, \quad \text{and} \quad \alpha_j = \frac{\cosh\beta_j + \cos\beta_j}{\sinh\beta_j + \sin\beta_j}$$

2.2 For Torsional Modes

The angular frequencies, ω_n, of the normal modes, are given by

$$\omega_n = 2\pi\upsilon_n = \frac{\theta_n^2}{\ell^2}\sqrt{\frac{EI}{\rho A}} = \theta_n^2\sqrt{\frac{EI}{\rho A \ell^4}}$$

θ_n is a number. The general solution Y(x) of the form

$$Y_n(x) = \cosh\left(\theta_n \frac{x}{\ell}\right) - \cos\left(\theta_n \frac{x}{\ell}\right) - \sigma_n\left[\sinh\left(\theta_n \frac{x}{\ell}\right) - \sin\left(\theta_n \frac{x}{\ell}\right)\right]$$

In which the θ_n are the solutions of $\cosh(\theta_n)\cos(\theta_n) = -1$

And the □$_n$ are given by $\sigma_n = \frac{\sinh(\theta_n) - \sin(\theta_n)}{\cosh(\theta_n) + \cos(\theta_n)}$

III. FINITE ELEMENT FORMULATION

The plate geometry shown in figure is modeled and meshed by using ANSYS *macros* which is to be entered in the command prompt box of ANSYS. The dimensions of the plate are length L=200mm, width D=50mm and thickness t=5mm and modeled by using the RECTNG or BLC4/ BLC5 *macro* commands under /PREP7 pre-processor by giving the equalent dimensions of the plate in the respective working plane coordinates.

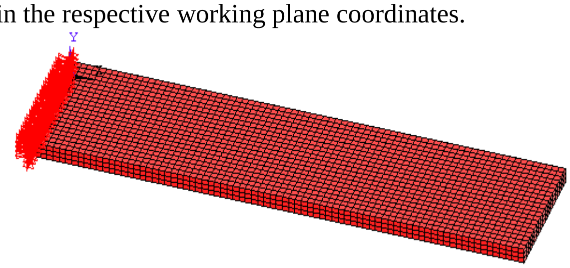


Figure 3: Finite Element model of a Rectangular Blade

The material taken for the plate is steel, have properties
Young's Modulus = 2.1e5 MPa.
Density = 7850 Kg/m^3
Poisson's ratio = 0.3

The material is assumed to be in linear isotropic elastic condition.

The blade is assumed to be rotating at a speed of 15000 rpm and the angular velocity of the blade is calculated from the available data as

$$\omega = \frac{2\pi N}{60} = \frac{2\pi * 15000}{60} = 1570.796 rad/sec$$

The model is marked into areas by lines for map meshing around the notch geometry and the macros used issued for material properties are entered by the commands MP, DENS-for density, MP-EX for Young's modulus and MP-PRXY-for Poisson's ratio in this context. The element type for this model taken is PLANE42 and SOLID45and written in macro commands by using ET-for element type. The model is fine meshed and coarse meshed the LESIZE-line element size and divisions and AMESH-area mesh as shown in figure .after meshing the plane 42 element, then the plane 42 element type is extruded to SOLID45 by using the EXT command. After extruding the PLANE42 elements are deleted and the nodes along z=0 are selected. The model is arrested in X, Y and Z direction by selecting the nodes to be fixed with D. The node selection is made by the NSEL with S or R command.

The solution phase begins with /SOLU command and the modal analysis type is switch on by writing ANTYPE, 1. The problem is solved by using the SOLVE *macro* command. The Post processing of results can be carried by the sequence of /POST1. The model consists of 4800 SOLID45 elements.

IV. RESULTS AND DISCUSSIONS

The Free and Forced Vibration Analysis of Rectangular Cantilever beam were done for various notch parameters to obtain the Natural frequencies and Dynamic responses of the beam. The average natural frequency for First Flexural (1F) and First Torsion(1T) for notch dimension h=20mm is shown in figure 4 and notch radius ranging from 1 to 10mm increased in steps of 1mm are 371.928 Hz and 892.041 Hz respectively. It is observed that the 1 F frequency decreases. The centrifugal force is applied along the leading edge where the notch is present.

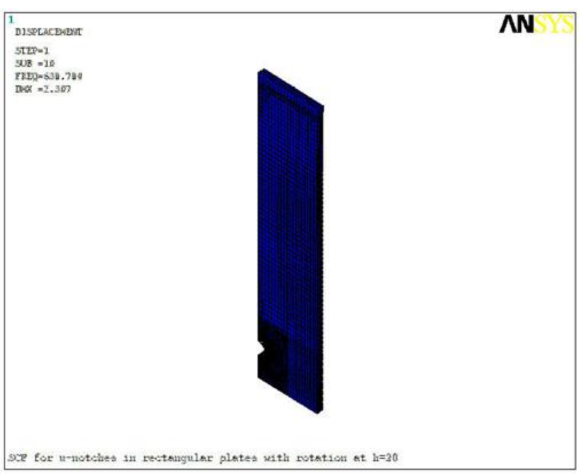


Figure 4: FEA model of rectangular blade with semicircular notch at h = 20mm

The centrifugal force is obtained from the angular velocity which is calculated from the constant engine speed of 15000 rpm. The notch radius is varied from 2 mm to 10 mm in steps of 2 mm with the various location heights of 20 mm, 80 mm, 100 mm, 120 mm, and 150 mm from the root to tip of the blade along the leading edge It is observed that for 2 mm, radius of notch and for the locations mentioned, frequency decreases from the root to the tip of the blade. The percentage decrease in 1F frequency for 2 mm notch radius, from the initial height of 20 mm to the final height of 150 mm is 0.145%. as shown in figure 5.

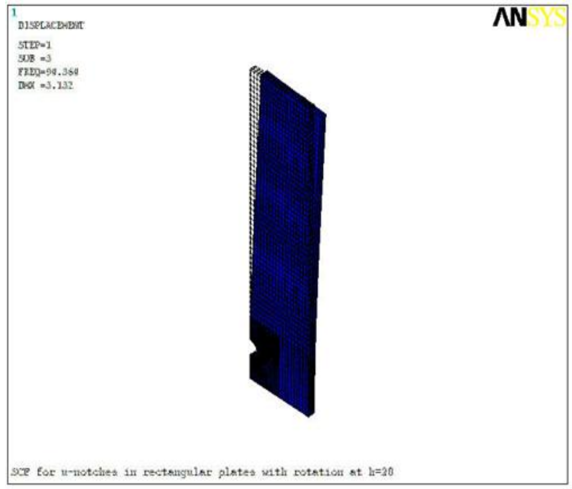
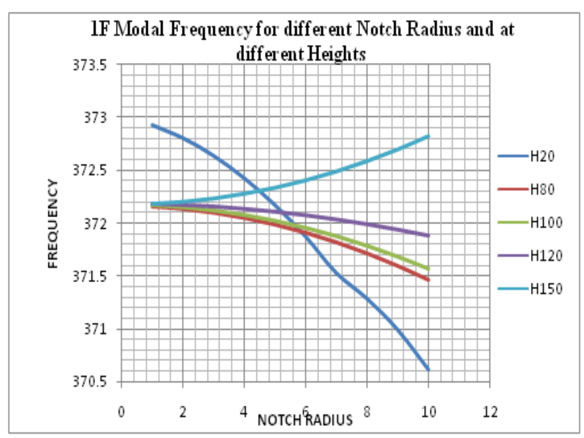


Figure 5: 1F mode of rectangular blade with semicircular notch at h = 20mm

For further increase of the notch radius in steps of 2 mm for the locations mentioned, frequency increases from the root to the tip of the blade. The percentage increase in 1F frequency for 4 mm notch radius, from the initial height of 20 mm to the final height of 150 mm is 0.02%. For further increase of the notch radius for 6 mm, and for mentioned height the percentage increase in frequency for 1F frequency is 0.3%, as shown in Graph 1.



Graph 1. 1F Modal Frequency for different Notch Radius and at different Heights

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First Torsional frequency (1T) for notch location at a distance of 20 mm from the root and for the notch radius of 1 mm, the 1T frequency is 906.605 Hz shown in figure 6. As the notch radius is increased from 1 mm to 10 mm in steps of 1 mm, the 1T frequency decreases, the decrease in the frequency is marginal. For 10 mm notch radius 1T frequency obtained is 868.439 Hz, the maximum percentage decrease in frequency is around 4.20%.

Comparison of First Torsional (1T) frequency for different notch radius varied from 2 mm to 6 mm in steps of 2 mm with varying location of the notch height of 20 mm, with the varying location of notch height 80 mm, 100 mm, 120 mm and 150 mm from the root to the tip of the blade along the leading edge variation in the frequency is marginal as shown in graph 2. For the initial parameters the Second

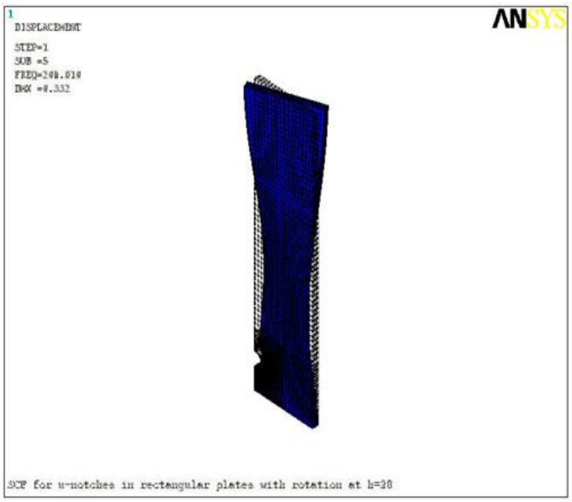
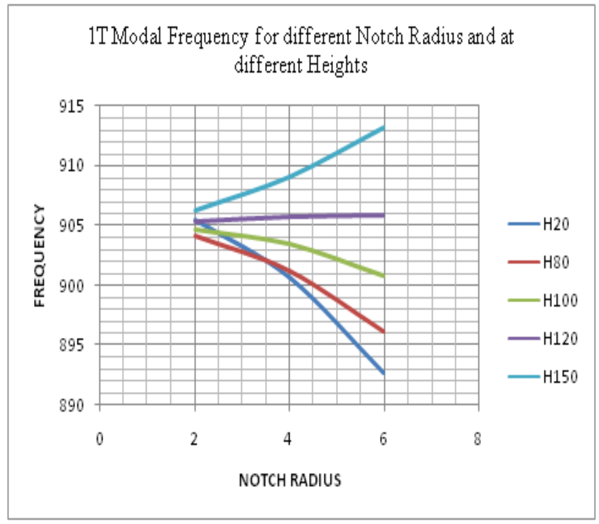


Figure 6: 1T mode of rectangular blade with semicircular notch at h = 20mm

Flexural (2F) frequency is 1201 Hz, as the notch radius is increased from 1 mm to 10 mm in steps of 1 mm, with constant location height of 20 mm, the 2F frequency decreases. The (2F) frequency for 10 mm notch radius is 1195 Hz. The maximum percentage decrease in 2F frequency is around 0.499%.

Second Torsional frequency (2T) of 1 mm radius notch at a distance of 20 mm from the root of the blade is 2783 Hz as shown in figure 7. As the notch radius is increased from 1 mm to 10 mm in steps of 1 mm, at constant location height of 20 mm from the root, the frequency obtained for 10 mm notch radius is 2698 Hz,



Graph 2: 1t Modal Frequency Vs Notch Radius

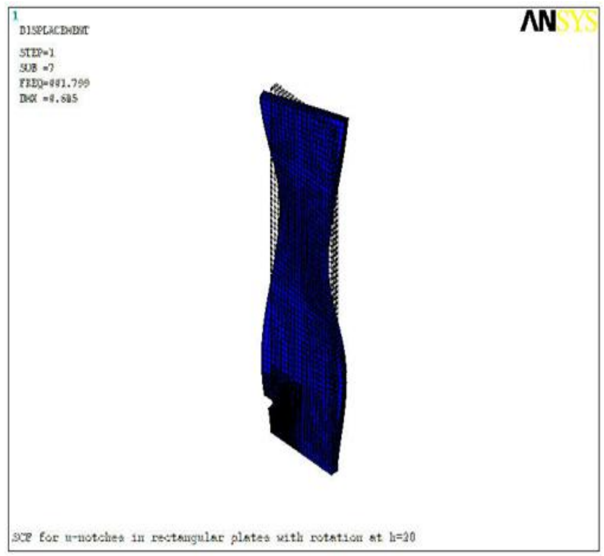


Figure 7: 2T mode of rectangular blade with semicircular notch at h = 20mm

the frequency is decreases and the maximum percentage decrease in the frequency (2T) is 3.05%. Similarly for various location heights of notches (like 50 mm, 80 mm, 100 mm, 120 mm and 150 mm from the root on the leading edge of the blade) and also with radius of semicircular notches varied from 1 mm to 10 mm in steps of 1 mm, are analyzed and it is observed that 1F, 2F, 1T and 2T frequencies shows the similar trend as observed and discussed above. When the notch is at the tip of the beam the Natural frequency is slightly greater than those obtained when the notch is at the root of the beam. The Natural Frequencies of the beam increase for the notch location far from the root of the cantilever beam.

V. CONCLUSIONS

The natural frequency and mode shapes of rectangular cantilever beam rotating at constant speed (15000 rpm) has been carried out for various possibilities of notch parameters used for converting the nicks and dents to notch of known geometry and thereby predicting the modal values. An attempt has been made to address the effect of notches on cantilever beam which is an idealization of blades subjected to FOD. As the notch location moves from the root to tip, the frequency increases. The frequency of the cantilever plate under rotation increases as notch diameter increase, in case of increase in location height from the root frequency decreases. Large notch sizes are not possible at the mid section and tip of the aerofoil blade because of the high increase in the modal frequency. Large notches are not possible at root of the blade as the centrifugal field get altered drastically which can affect the overall strength of aerofoil adversely. This methodology can be extended to Aerofoil as the Finite Element Method results are in good agreement with the closed form solution.

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