Anti-Fuzzy Lattice Ordered M-Group

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Abstract- In this paper we introduce the notion of anti fuzzy lattice ordered m-groups and investigated some of its basic properties. We also study the homomorphic image, pre-image of anti fuzzy lattice ordered m-groups, arbitrary family of anti fuzzy lattice ordered m-groups and anti fuzzy lattice ordered m-groups using T-norms. We introduce the notion of sensible anti fuzzy lattice ordered m-groups in groups and some related properties of lattices are discussed.

***Index Terms*-** Lattice ordered group, anti fuzzy lattice ordered m-group, Sensible fuzzy lattice , pre-image, direct product.

I. INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh [5].Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, Rosenfield [8] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the roader frame work of fuzzy settings. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. In [2], Biswas introduced the concept of anti- fuzzy subgroups of groups. Palaniappan. N and Muthuraj**,** [7] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [10] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. Dr.M.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [11] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagrajan & Chellappa [9] extended the concept to m fuzzy groups with operator. In this paper we define a new algebraic structure of an anti fuzzy lattice ordered m-group and study some related properties.

II. SECTION-2 PRELIMINARIES

Definition 2.1: Let μ: X to [0, 1] be a fuzzy set & G ϵ þ(X) = Set of all fuzzy sets on X. A fuzzy set μ on G is called an anti fuzzy subgroup if i) $\mu(x\,y) \le \max\{\mu(x), \mu(y)\}$ ii) $\mu(x^{-1}) \le \mu(x)$, for all x, yϵ G .

Definition 2.2: Let μ: X to [0, 1] be a fuzzy set & G ϵ þ(X). A fuzzy set μ on G is called a normal fuzzy subgroup if $\mu(x^{-1}\,y\,x) \le \mu(y)$ for all x, yϵ G

Definition 2.3: An anti lattice ordered group is a system (G,.,≤) if i) (G, ·) is a group ii) (G,≤) is a lattice . iii) x ≤ y implies a x b ≤ a y b
(compatibility) for a, b, x, yϵ G

Definition 2.4: Let μ: X to [0, 1] be a fuzzy set & G is a lattice ordered group, G ϵ þ(X). A function μ on G is said to be an anti fuzzy lattice ordered group if
i) $\mu(x\,y) \le \max\{\mu(x), \mu(y)\}$ ii) $\mu(x^{-1}) \le \mu(x)$
for all x, yϵ G

Definition 2.5: Let G be a group, M be any set if
i)m x ϵ G. ii) m (x y) = (m x) y = x m y for all
x, yϵ G, m ϵ M. Then G is called a m group.

Definition 2.6: Let μ: X to [0, 1] be a fuzzy set & G be a M group G. A fuzzy set on G, G ϵ þ(X)) is called an anti fuzzy m group if i) $\mu(m(x\,y)) \le \max\{\mu(m\,x), \mu(my)\}$ ii) $\mu(mx^{-1}) \le \mu(m\,x)$
for all x, yϵ G, m ϵ M

Definition 2.7: μ: X to [0, 1], G ϵ þ(X), M C X.
A function μ on G is said to be an anti fuzzy lattice ordered m-group if
i) (G, ·) is a M-group.
ii) (G,·, ≤) is an anti lattice ordered group.
iii) $\mu(m(x\,y)) \le \max\{\mu(m\,x), \mu(my)\}$
iv) $\mu((m\,x)^{-1}) \le \mu(m\,x)$
v) $\mu(m\,x \vee my) \le \max\{\mu(m\,x), \mu(m\,y)\}$
vi) $\mu(m\,x \wedge my) \le \max\{\mu(m\,x), \mu(my)\}$
For all x, y ϵ G

III. SECTION-3 PROPERTIES OF ANTI FUZZY LATTICE ORDERED M-GROUP

Proposition 3.1: Let G and G' be two anti fuzzy lattice ordered m-groups and ɵ:G→G' be a m-homomorphism defined by ɵ (m x)= m ɵ(x) .If B is an anti fuzzy lattice ordered m-group of G' then the pre-image $ɵ^{-1}(B)$ is an anti fuzzy lattice ordered m-group of G.

Proof- Assume B is an anti fuzzy lattice ordered m-group of G'. Let x, y ϵ G
i) $\mu_{ɵ^{-1}(B)}(m(x\,y)) = \mu_B\, ɵ(m(x\,y))$
$= \mu_B(m\,ɵ(x\,y))$

ISSN 2250-3153

$= \mu_B(m\, \theta(x)\, \theta (y))$
$\leq \max \{ \mu_B(m\, \theta (x)), \mu_B(m\, \theta (y))\}$
$\leq \max \{ \mu_B (\theta (m x)), \mu_B(\theta (m y))\}$
$\leq \max \{\mu_{\theta^{-1}(B)} (m x), \mu_{\theta^{-1}(B)} (m y)\}$
ii) $\mu_{\theta^{-1}(B)} ((m x)^{-1}) = \mu_B\, \theta ((m x)^{-1})$
$= \mu_B (\theta (m x))^{-1}$
$= \mu_B (m\, \theta (x))^{-1}$
$\leq \mu_B (m\, \theta (x))$
$\leq \mu_B (\theta (m x))$
$\leq \mu_{\theta^{-1}(B)} (m x)$
iii) $\mu_{\theta^{-1}(B)} (m x \vee my) \quad = \mu_B\, \theta (m x \vee m y)$
$= \mu_B\, \theta(m x) \vee \theta(my)$
$\leq \max \{ \mu_B\, \theta(m x), \mu_B\, \theta(m y))$
$\leq \max \{\mu_{\theta^{-1}(B)} (m x), \mu_{\theta^{-1}(B)} (m y)\}$
iv) $\mu_{\theta^{-1}(B)} (m x \wedge my) \quad = \mu_B\, \theta (m x \wedge m y)$
$= \mu_B\, \theta(m x) \wedge \theta(my)$
$\leq \max \{ \mu_B\, \theta(m x), \mu_B\, \theta(m y))$
$\leq \max \{\mu_{\theta^{-1}(B)} (m x), \mu_{\theta^{-1}(B)} (m y)\}$
Therefore $\theta^{-1}(B)$ is anti fuzzy lattice ordered m-group of G.

Proposition 3.2: Let G and G' be two anti fuzzy lattice ordered m-groups and θ:G→G' be a m-epimorphism. B is a fuzzy set in G'. If $\theta^{-1}(B)$ is an anti fuzzy lattice ordered m-group of G then B is an anti fuzzy lattice ordered m group of G'.

Proof- Let x , y ϵ G', therefore there exist an element a, b ϵG such that $\theta(a) = x$ and $\theta(b) = y$.
i) $\mu_B (m(x y)) \quad = \mu_B (m(\theta(a)\, \theta(b))$
$= \mu_B (m\, \theta (a b))$
$= \mu_B\, \theta (m (a b))$
$= \mu_{\theta^{-1}(B)} (m(a b))$
$\leq \max \{ \mu_{\theta^{-1}(B)} (ma)), \mu_{\theta^{-1}(B)} (m b))\}$
$\leq \max \{ \mu_B\, \theta (m a), \mu_B\, \theta (m b)\}$
$\leq \max \{ \mu_B\, m\, \theta (a), \mu_B\, m\, \theta (b)\}$
$\leq \max \{ \mu_B (m x), \mu_B (my)\}$
ii) $\mu_B ((m x)^{-1}) \quad = \mu_B (m\, \theta (a))^{-1}$
$= \mu_B (\theta (m a))^{-1}$
$= \mu_B (\theta (m a)^{-1})$
$= \mu_{\theta^{-1}(B)} (m a)^{-1}$
$\leq \mu_{\theta^{-1}(B)} (m a)$
$\leq \mu_B\, \theta (m a)$
$\leq \mu_B\, m\, \theta(a)$
$\leq \mu_B (m x)$
iii) $\mu_B (m x \vee m y)= \mu_B (m\, \theta (a) \vee m\, \theta (b))$
$= \mu_B (\theta (m a) \vee \theta (m b))$
$= \mu_B (\theta (m a \vee m b))$
$= \mu_{\theta^{-1}(B)} (m a \vee m b)$
$\leq \max \{\mu_{\theta^{-1}(B)} (ma), \mu_{\theta^{-1}(B)} (m b)\}$
$\leq \max \{ \mu_B\, \theta(m a), \mu_B\, \theta(m b))$
$\leq \max \{ \mu_B\, m\, \theta(a), \mu_B\, m\, \theta(b))$
$\leq \max \{ \mu_B(m x), \mu_B(m y)\}$
iv) $\mu_B (m x \wedge m y)= \mu_B (m\, \theta (a) \wedge m\, \theta (b))$
$= \mu_B (\theta (m a) \wedge \theta (m b))$
$= \mu_B (\theta (m a \wedge m b))$
$= \mu_{\theta^{-1}(B)} (m a \wedge m b)$
$\leq \max \{\mu_{\theta^{-1}(B)} (ma), \mu_{\theta^{-1}(B)} (m b)\}$
$\leq \max \{ \mu_B\, \theta(m a), \mu_B\, \theta(m b))$
$\leq \max \{ \mu_B\, m\, \theta(a), \mu_B\, m\, \theta(b))$
$\leq \max \{ \mu_B(m x), \mu_B(m y)\}$
B is an anti fuzzy lattice ordered m group of G'.

Proposition 3.3: If $\{A_i\}$ is a family of an anti fuzzy lattice ordered m-group of G then $U A_i$ is an anti fuzzy lattice ordered m-group of G where $U A_i = \{ x , \vee \mu_{Ai} (x) / x \epsilon G \}$

Proof- x, y ϵ G
i) $(U \mu_{Ai})\, m(x y) = \vee\, \mu_{Ai}\, m (x y)$
$= \vee\, \mu_{Ai} (m x m y)$
$\leq \vee \max \{ \mu_{Ai} (m x), \mu_{Ai} (m y) \}$
$\leq \max \{ (U\mu_{Ai}) (m x), (U \mu_{Ai}) (m y) \}$
ii) $(U \mu_{Ai})\, (m x)^{-1} = \vee\, \mu_{Ai} (m x)^{-1}$
$\leq \vee\, \mu_{Ai} (m x)$
$\leq (U \mu_{Ai}) (m x)$
iii) $(U \mu_{Ai}) (m x \vee m y) = \vee\, \mu_{Ai} (m x \vee m y)$
$\leq \vee \max \{ \mu_{Ai} (m x), \mu_{Ai} (m y) \}$
$\leq \max \{ (U\mu_{Ai}) (m x), (U \mu_{Ai}) (m y) \}$
iv) $(U \mu_{Ai}) (m x \wedge m y) = \vee\, \mu_{Ai} (m x \wedge m y)$
$\leq \vee \max \{ \mu_{Ai} (m x), \mu_{Ai} (m y) \}$
$\leq \max \{ (U\mu_{Ai}) (m x), (U \mu_{Ai}) (m y) \}$

Proposition 3.4: If A is a fuzzy set in G such that all nonempty level subset U (A ; t) is an anti fuzzy lattice ordered m-group of G then A is an anti fuzzy lattice ordered m-group of G.

Proof- Let x, y ϵ U (A ; t) , we have $A(m x) \leq t$ and $A (m y) \leq t$. So that $A (m(x y)) \leq t$
i) $A (m(x y)) \leq t$
$\leq \max \{ t , t \}$
$\leq \max \{ A(m x), A(my)\}$
ii) $A ((m x)^{-1}) \leq t = A(m x)$
iii) $A(m x \vee my) \leq t$
$\leq \max \{ t , t \}$
$\leq \max \{ A(m x), A(my) \}$
iv) $A(m x \wedge my) \leq t$
$\leq \max \{ t , t \}$
$\leq \max \{ A(m x), A(my) \}$
Therefore A is an anti fuzzy lattice ordered m-group.

Proposition 3.5: Let A be an anti fuzzy lattice ordered m-group of G. Let A* be a fuzzy set in G defined by $A^* (x) = A(x) + 1 - A (e)$ for all x ϵG. Then A* is an anti fuzzy lattice ordered m-group of G containing A.

Proof – Let x, y ϵ G
i) $A^* (m(x y)) = A (m (x y)) +1- A (e)$
$\leq \max\{A(m x),A(my)\} +1 -A(e)$
$\leq \max \{A(m x) +1-A(e), A(m y)+1 -A(e)\}$
$\leq \max \{ A^*(m x),A^*(my) \}$
ii) $A^*((m x)^{-1})= A ((mx)^{-1}) + 1 - A(e)$

ISSN 2250-3153

$\leq A(mx) + 1 - A(e)$
$\leq A^*(mx)$
iii)$A^*(m x \vee my)) = A(m x \vee m y)) + 1 - A(e)$
$\leq \max\{A(m x), A(my)\} + 1 - A(e)$
$\leq \max\{A(m x) + 1 - A(e), A(m y) + 1 - A(e)\}$
$\leq \max\{A^*(m x), A^*(my)\}$
iv)$A^*(m x \wedge my)) = A(m x \wedge m y)) + 1 - A(e)$
$\leq \max\{A(m x), A(my)\} + 1 - A(e)$
$\leq \max\{A(m x) + 1 - A(e), A(m y) + 1 - A(e)\}$
$\leq \max\{A^*(m x), A^*(my)\}$
Also $A(x) \leq A^*(x)$ for all $x \in G$.
Therefore A* is a fuzzy lattice ordered m-group of G containing A.

Proposition 3.6: If A is an anti fuzzy lattice ordered m-group of G and θ is a m-homomorphism of G then the fuzzy set $A^\theta = \{ < m x ; \mu_{A^\theta}(m x) >, x \in G \}$ is an anti fuzzy lattice ordered m-group.

Proof- Let x, y ∈ G
i)$\mu_{A^\theta}(m(x y)) = \mu_A\, \theta(m(x y))$
$= \mu_A\, m\theta(x y)$
$= \mu_A\, m(\theta(x)\theta(y))$
$\leq \max\{\mu_A\, m\theta(x), \mu_A\, m\theta(y)\}$
$\leq \max\{\mu_A\, \theta(m x), \mu_A\, \theta(m y)\}$
$\leq \max\{\mu_{A^\theta}(m x), \mu_{A^\theta}(m y)\}$
ii)$\mu_{A^\theta}(m x)^{-1} = \mu_A\, \theta(m x)^{-1}$
$= \mu_A(\theta(m x))^{-1}$
$= \mu_A(m\theta(x))^{-1}$
$\leq \mu_A(m\theta(x))$
$\leq \mu_A\, \theta(m x)$
$\leq \mu_{A^\theta}(m x)$
iii)$\mu_{A^\theta}(m x \vee my)) = \mu_A\, \theta(m x \vee m y))$
$= \mu_A\, \theta(m x) \vee \theta(m x)$
$\leq \max\{\mu_A\, \theta(m x), \mu_A\, \theta(m y))$
$\leq \max\{\mu_{A^\theta}(m x), \mu_{A^\theta}(m y))$
iv)$\mu_{A^\theta}(m x \wedge my)) = \mu_A\, \theta(m x \wedge m y))$
$= \mu_A\, \theta(m x) \wedge \theta(m x)$
$\leq \max\{\mu_A\, \theta(m x), \mu_A\, \theta(m y))$
$\leq \max\{\mu_{A^\theta}(m x), \mu_{A^\theta}(m y))$
Therefore A^θ is an anti fuzzy lattice ordered m-group of G.

Proposition 3.7: Let T be a continuous t- norm and let f be a m-homomorphism on G. If μ is an anti fuzzy lattice ordered m-group on G then μ^f is an anti fuzzy lattice ordered m-group of f (G).

Proof- Let $A_1 = f^{-1}(m y_1)$, $A_2 = f^{-1}(m y_2)$, $A_{12} = f^{-1}(m (y_1 y_2))$
Consider $A_1A_2 = \{m x \in G / m x = m x_1\, m x_2$ for $m x_1 \in A_1$, $m x_2 \in A_2\}$
If $m x \in A_1A_2$ then $m x = m x_1\, m x_2$ &
$f(m x) = f(m x_1\, m x_2) = f(m x_1) f(m x_2)$
$= m y_1\, m y_2 = m(y_1 y_2)$
$m x \in f^{-1}(m(y_1 y_2))$ therefore $A_1A_2 \subset A_{12}$
i)$\mu^f(m(y_1y_2)) = \sup\{\mu(mx)/m x \in f^{-1}(m(y_1 y_2))\}$
$= \sup\{\mu(m x)/m x \in A_{12}\}$
$\leq \sup\{\mu(m x)/m x \in A_1 A_2\}$
$\leq \sup\{\mu(m x_1\, m x_2)/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq \sup\{T(\mu(m x_1), T(\mu(m x_2))/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq T[\sup\{\mu(mx_1)/mx_1 \in A_1\}, \sup\{\mu(m x_2)/m x_2 \in A_2\}]$
$\leq T[\sup\{\mu(m x_1)/m x_1 \in f^{-1}(m y_1)\}, \sup\{\mu(m x_2)/m x_2 \in f^{-1}(m y_2)\}]$
$\leq T\{\mu^f(m y_1), \mu^f(m y_2)\}$
ii)$\mu^f((m y)^{-1}) = \sup\{\mu(m x)^{-1}/(m x)^{-1} \in f^{-1}(my)^{-1}\}$
$= \sup\{\mu(m x)^{-1}/(m x) \in f^{-1}(my)\}$
$\leq \sup\{\mu(m x)/(m x) \in f^{-1}(my)\}$
$\leq \mu^f((m y))$
iii)$\mu^f(my_1 \vee m y_2)) = \sup\{\mu(mx)/mx \in f^{-1}(my_1 \vee m y_2)\}$
$= \sup\{\mu(m x)/m x \in A_{1\vee 2}\}$
$\leq \sup\{\mu(m x)/m x \in A_1 \vee A_2\}$
$\leq \sup\{\mu(m x_1 \vee m x_2)/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq \sup\{T(\mu(m x_1)), T(\mu(m x_2))/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq T[\sup\{\mu(mx_1)/m x_1 \in A_1\}, \sup\{\mu(m x_2)/m x_2 \in A_2\}]$
$\leq T[\sup\{\mu(m x_1)/m x_1 \in f^{-1}(m y_1)\}, \sup\{\mu(m x_2)/m x_2 \in f^{-1}(m y_2)\}]$
$\leq T\{\mu^f(m y_1), \mu^f(m y_2)\}$
iv)$\mu^f(m y_1 \wedge m y_2))$
$= \sup\{\mu(m x)/m x \in f^{-1}(m y_1 \wedge m y_2)\}$
$= \sup\{\mu(m x)/m x \in A_{1\wedge 2}\}$
$\leq \sup\{\mu(m x)/m x \in A_1 \wedge A_2\}$
$\leq \sup\{\mu(m x_1 \wedge m x_2)/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq \sup\{T(\mu(m x_1)), T(\mu(m x_2))/m x_1 \in A_1, m x_2 \in A_2\}$
$\leq T[\sup\{\mu(m x_1)/m x_1 \in A_1\}, \sup\{\mu(m x_2)/m x_2 \in A_2\}]$
$\leq T[\sup\{\mu(m x_1)/m x_1 \in f^{-1}(m y_1)\}, \sup\{\mu(m x_2)/m x_2 \in f^{-1}(m y_2)\}]$
$\leq T\{\mu^f(m y_1), \mu^f(m y_2)\}$
Therefore μ^f is an anti fuzzy lattice ordered m-group of f (G).

Proposition 3.8: LetT be a t-norm. Then every sensible anti fuzzy lattice ordered m-group is an anti fuzzy lattice ordered m-group of G.

Proof- A is sensible anti fuzzy lattice ordered m-group then we have
i)$A(m(x y)) \leq T[A(m x), A(m y)]$
ii)$A((m x)^{-1}) \leq A(m x)$
iii)$A(m x \vee m y) \leq T[A(m x), A(m y)]$
iv)$A(m x \wedge m y) \leq T[A(m x), A(m y)]$
i)$\max\{A(m x), A(my)\} =$
$T[\max\{A(m x), A(my)\}, \max\{A(m x), A(my)\}]$
$= \max\{T[A(m x), A(my)], T[A(m x), A(my)]\}$
$= T[A(m x), A(my)]$
$\leq A(m(x y))$
ii)$A((m x)^{-1}) \leq A(m x)$
iii)$\max\{A(m x), A(my)\} =$
$T[\max\{A(m x), A(my)\}, \max\{A(m x), A(my)\}]$
$= \max\{T[A(m x), A(my)], T[A(m x), A(my)]\}$
$= T[A(m x), A(my)]$
$\geq A(m x \vee m y))$
iv)$\max\{A(m x), A(my)\} =$
$T[\max\{A(m x), A(my)\}, \max\{A(m x), A(my)\}]$
$= \max\{T[A(m x), A(my)], T[A(m x), A(my)]\}$
$= T[A(m x), A(my)]$
$\geq A(m x \wedge m y))$
Therefore A is an anti fuzzy lattice ordered m-group of G.

Proposition 3.9: An onto m-homomorphic image of an anti fuzzy lattice ordered m-group with sup property is an anti fuzzy lattice ordered m-group.

Proof: Let f : G → G' be a onto m-homomorphism of G and let A be an anti fuzzy lattice ordered m-group of G with sup property.

Let m x', my' ϵ G'

Let $mx_0 \in f^{-1}(m\,x')$, $my_0 \in f^{-1}(m\,y')$ be such that

$A(m\,x_0) = \sup\{A(m\,x) / m\,x \in f^{-1}(m\,x')\}$ &

$A(m\,y_0) = \sup\{A(m\,y) / m\,y \in f^{-1}(m\,y')\}$

i) $A^f(m(x'y')) = \sup\{A(z) / z \in f^{-1}(m(x'y'))\}$

$= \sup\{A(z) / z \in f^{-1}(m\,x'\,my'))\}$

$\le \sup\{A(mx_0my_0) / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{A(m\,x_0\,y_0) / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{\max\{A(mx_0), A(my_0)\}/mx_0 \in f^{-1}(mx'), my_0 \in f^{-1}(m\,y')\}$

$\le \max\{\sup\{A(m\,x_0) / mx_0 \in f^{-1}(m\,x')\}, \sup\{A(my_0) / my_0 \in f^{-1}(m\,y')$

$\le \max\{A^f(mx'), A^f(mx')\}$

ii) $A^f((m\,x')^{-1}) = \sup\{A(mx_0)^{-1}/(m\,x_0)^{-1} \in f^{-1}(m\,x')^{-1}\}$

$= \sup\{A(m\,x_0)^{-1} / (m\,x_0) \in f^{-1}(m\,x')\}$

$\le \sup\{A(m\,x_0) / (m\,x_0) \in f^{-1}(m\,x')\}$

$\le A^f((m\,x'))$

iii) $A^f(m\,x' \vee my') = \sup\{A(z) / z \in f^{-1}(m\,x' \vee m\,y'))\}$

$\le \sup\{A(z) / z \in f^{-1}(m\,x') \vee f^{-1}\,m\,y'))\}$

$\le \sup\{A(mx_0 \vee m\,y_0) / mx_0 \in f^{-1}(mx'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{A(mx_0 \vee my_0) / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{\max\{A(m\,x_0), A(m\,y_0)\} / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \max\{\sup\{A(m\,x_0) / mx_0 \in f^{-1}(m\,x')\}, \sup\{A(my_0) / my_0 \in f^{-1}(m\,y')$

$\le \max\{A^f(m\,x'), A^f(m\,x')\}$

iv) $A^f(m\,x' \wedge my') = \sup\{A(z) / z \in f^{-1}(m\,x' \wedge m\,y'))\}$

$\le \sup\{A(z) / z \in f^{-1}(m\,x') \wedge f^{-1}\,m\,y'))\}$

$\le \sup\{A(mx_0 \vee my_0) / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{A(mx_0 \vee my_0) / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \sup\{\max\{A(m\,x_0), A(m\,y_0)\} / mx_0 \in f^{-1}(m\,x'), my_0 \in f^{-1}(m\,y')\}$

$\le \max\{\sup\{A(m\,x_0) / mx_0 \in f^{-1}(m\,x')\}, \sup\{A(my_0) / my_0 \in f^{-1}(m\,y')$

$\le \max\{A^f(m\,x'), A^f(m\,x')\}$

Proposition 3.10: Let f: G→G' be a lattice group m-homomorphism and A be an anti fuzzy lattice ordered m-group of G' then $f^{-1}(A)$ is an anti fuzzy lattice ordered m-group of G.

Proof -Let m x, m y ϵ G and A be an anti fuzzy lattice ordered m-group of G'.

i) $f^{-1}(A)(m(x\,y)) = A\,f(m(x\,y))$

$= A(f(m\,x)\,f(m\,y))$

$= A(m\,f(x)\,mf(y))$

$= A(m\,f(x)\,f(y))$

$\le \max\{A(mf(x)), A(mf(y))\}$

$\le \max\{A(f(m\,x)), A(f(my))\}$

$\le \max\{f^{-1}(A)(m\,x), f^{-1}(A)(m\,x)\}$

ii) $f^{-1}(A)((m\,x)^{-1}) = A\,f((m\,x)^{-1})$

$= A(f(m\,x))^{-1}$

$= A(m\,f(x))^{-1}$

$\le A(m\,f(x))$

$\le A(f(m\,x))$

$\le f^{-1}(A)(m\,x)$

iii) $f^{-1}(A)(m\,x \vee m\,y)) = A\,f(m\,x \vee m\,y))$

$= A(f(m\,x) \vee f(m\,y))$

$= A(m\,f(x) \vee m\,f(y))$

$\le \max\{A(mf(x)), A(mf(y))\}$

$\le \max\{A(f(m\,x)), A(f(my))\}$

$\le \max\{f^{-1}(A)(m\,x), f^{-1}(A)(m\,x)\}$

iv) $f^{-1}(A)(m\,x \wedge m\,y)) = A\,f(m\,x \wedge m\,y))$

$= A(f(m\,x) \wedge f(m\,y))$

$= A(m\,f(x) \wedge m\,f(y))$

$\le \max\{A(mf(x)), A(mf(y))\}$

$\le \max\{A(f(m\,x)), A(f(my))\}$

$\le \max\{f^{-1}(A)(m\,x), f^{-1}(A)(m\,x)\}$

Therefore $f^{-1}(A)$ is an anti fuzzy lattice ordered m-group of G.

IV. SECTION-4 DIRECT PRODUCT OF ANTI FUZZY LATTICE ORDERED M-GROUPS

Definition: 4.1 Let A_i be an anti fuzzy lattice ordered m-group of G_i, for i = 1,2,…,n. Then the product A_i (i = 1,2,….n) is the function

$A_1 x\, A_2 x \ldots x\, A_n : G_1 x\, G_2 x \ldots x\, G_n \to L$ defined by

$(A_1 x\, A_2 x \ldots\ldots x\, A_n)m(x_1, x_2, \ldots, x_n)$

$= \max\{A_1(m\,x_1), A_2(mx_2), \ldots, A_n(mx_n)\}$

Proposition 4.2: The direct product of anti fuzzy lattice ordered m groups is an anti fuzzy lattice ordered m-group.

Proof- Let $x = (x_1, x_2, \ldots, x_n)$,

$y = (y_1, y_2, \ldots, y_n) \in G_1 x\, G_2 x \ldots\ldots x\, G_n$

Let $A_1 x\, A_2 x \ldots\ldots x\, A_n = A$

i) $A(m(x\,y)) = A(m(x_1 y_1, x_2 y_2, \ldots, x_n y_n))$

$= \max\{A_1(m\,x_1 y_1), A_2(m\,x_2\,y_2), \ldots, A_n(m\,x_n\,y_n)\}$

$\le \max\{\max[A_1(m\,x_1), A_1(m\,y_1)], \max[A_2(m\,x_2), A_2(m\,y_2)], \ldots \max[A_n(m\,x_n), A_n(m\,y_n)]\}$

$\le \max\{\max[A_1(m\,x_1), A_2(m\,x_2), \ldots A_n(m\,x_n)], \max[A_1(m\,y_1), A_2(m\,y_2), \ldots A_n(m\,y_n)]\}$

$\le \max\{(A_1 x\, A_2 x \ldots\ldots x\, A_n)\,m(x_1, x_2, \ldots, x_n), (A_1 x\, A_2 x \ldots\ldots x\, A_n)\,m(y_1, y_2, \ldots, y_n)$

$\le \max\{A(m\,x), A(m\,y)\}$

ii) $A(m\,x)^{-1} = A\,m(x_1^{-1}, x_2^{-1}, \ldots\ldots, x_n^{-1})$

$= \max\{A_1(m\,x_1^{-1}), A_2(m\,x_2^{-1}), \ldots\ldots, A_n(m\,x_n^{-1})\}$

$\le \max\{A_1(m\,x_1), A_2(m\,x_2), \ldots\ldots, A_n(m\,x_n)\}$

$\le A\,m(x_1, x_2, \ldots\ldots, x_n)$

$\le A(mx)$

iii) $A(m\,x \vee m\,y)) = A(m\,x_1 \vee my_1, m\,x_2 \vee my_2, \ldots, mx_n \vee m\,y_n))$

$= \max\{A_1(m\,x_1 \vee my_1), A_2(m\,x_2 \vee my_2), \ldots, A_n(mx_n \vee m\,y_n)\}$

$\le \max\{\max[A_1(m\,x_1), A_1(m\,y_1)], \max[A_2(m\,x_2), A_2(m\,y_2)], \ldots \max[A_n(m\,x_n), A_n(m\,y_n)]\}$

$\le \max\{\max[A_1(m\,x_1), A_2(m\,x_2), \ldots A_n(m\,x_n)], \max[A_1(m\,y_1), A_2(m\,y_2), \ldots A_n(m\,y_n)]\}$

$\le \max\{(A_1 x\, A_2 x \ldots\ldots x\, A_n)\,m(x_1, x_2, \ldots, x_n), (A_1 x\, A_2 x \ldots\ldots x\, A_n)\,m(y_1, y_2, \ldots, y_n)$

$\le \max\{A(m\,x), A(m\,y)\}$

iv) $A(m\,x \wedge m\,y)) = A(m\,x_1 \wedge my_1, m\,x_2 \wedge my_2, \ldots, mx_n \wedge m\,y_n))$

$= \max\{A_1(m\,x_1 \wedge my_1), A_2(m\,x_2 \wedge my_2), \ldots, A_n(mx_n \wedge m\,y_n)\}$

$\le \max\{\max[A_1(m\,x_1), A_1(m\,y_1)], \max[A_2(m\,x_2), A_2(m\,y_2)], \ldots \max[A_n(m\,x_n), A_n(m\,y_n)]\}$

$\le \max\{\max[A_1(m\,x_1), A_2(m\,x_2), \ldots A_n(m\,x_n)], \max[A_1(m\,y_1), A_2(m\,y_2), \ldots A_n(m\,y_n)]\}$

$\leq \max \{(A_1 \times A_2 \times \ldots \times A_n)\, m(x_1, x_2, \ldots, x_n), (A_1 \times A_2 \times \ldots \times A_n)\, m(y_1, y_2, \ldots, y_n)$
$\leq \max \{A(m x), A(m y)\}$

V. CONCLUSION

In this paper we studied the notion of an anti fuzzy lattice ordered m-groups and investigated some of its basic properties. We also studied the homomorphic image, pre-image of an anti fuzzy lattice ordered m-groups, arbitrary family of anti fuzzy lattice ordered m-groups and anti fuzzy lattice ordered m-groups using T-norms.

Applications: Lattice structure has been found to be extremely important in the areas of quantum logic, Erogodic theory, Reynold's operations, Soft Computing, Communication system, Information analysis system, artificial intelligences and physical sciences.

ACKNOWLEDGEMENT

The author is highly grateful to the referees for their valuable comments and suggestions for improving the paper.

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