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A Review on Stress Analysis of an Infinite Plate with Cut-outs

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Abstract- In this paper an effort is made to review the investigations that have been made on the "stress analysis of an infinite plate with cut-outs". A number of analytical and experimental techniques are available for stress analysis around the different types of cut-outs for different condition in an infinite plate, made up of different materials under different loading condition has been reported in this article. The methods compared are tabulated with their findings. Singularities of circular hole in rectangular plate and elliptical hole in rectangular plate are considered in present study.

Index Terms- Stress concentration factor, Discontinuity, Complex variable, Stress function, Conformal mapping

I. Introduction

Openings/cut-outs are made into structures in order to satisfy some service requirements, results in strength degradation. In practice different shape of holes are used for different applications for example manhole of any pressure vessel is either circular or elliptical while the window or door of an airplane is rectangular hole having chamfer of some radius at corners.

The stress analysis of infinite plate with openings is interesting field of investigation. Many researchers have contributed in this area by taking different hole geometry and loading pattern. Here, an attempt is made to solve elastostatic problem of infinite plate subjected to different type of loading, with different shape of cutouts.

This hole/opening works as stress raisers and may lead to the failure of the structure/machine component. Hence it is an important aspect of stress analysis to predict stress concentration for regular or irregular holes. The irregularity in the hole shape may be because of chemical degradation. Under the effect of external loading and chemical process some irregular shapes may evolved. It is necessary to know stress distribution around such irregular shaped hole which may be useful to know hole shape evolution.

A. Stress Concentration

Stress concentration is localization of high stresses mainly due to discontinuities in continuum, abrupt changes in cross section and due to contact stresses. To study the effect of stress concentration and magnitude of localized stresses, a dimensionless factor called Stress Concentration Factor (SCF), K_t as defined by Eq. (1) is used.

$$K_t = \sigma_{max} / \sigma_{nom} \quad (1)$$

Where, σ_{max} is maximum stress at the discontinuity and σ_{nom} is nominal or background stress.

The stress concentration factor can be determined analytically by applying elasticity theory. For a large thin plate with a small circular hole at the center, that is subjected to uni-axial far-field tension, σ, acting along the x-axis, the stresses (radial, circumferential and tangential) around the vicinity of the hole are given in polar coordinates (r,θ) which shown in figure 1:

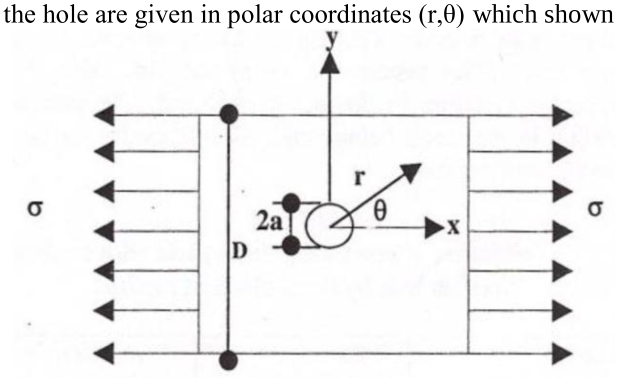

Figure 1: Plate with single circular hole subjected to uni-axial stress

B. Measurement of stress

Stress analysis of the critical elements under various loading conditions is carried out by the researchers for safe design of the element. Stress is measured by experimental methods or analytical/numerical method.

1. Experimental Methods

From the various experimental methods available for stress analysis, the following methods are long established.

1.1 Photo Elasticity

Photo elastic stress analysis is a full field technique for measuring the magnitude and direction of principal stresses. When polarized light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principals of stress optic law, Eq. (5) [2].

$$\sigma_1 - \sigma_2 = N\, f_\sigma / t \qquad (2)$$

where, σ_1 and σ_2 are the values of the maximum and minimum principal stresses at the point under consideration, N is the fringe number or fringe order at the point, f_σ is the material fringe value and t is the model thickness.

1.2 Brittle Coating

The brittle-lacquer technique of experimental stress analysis relies on the failure by cracking of a layer of a brittle coating which has been applied to the surface under investigation. Specially prepared lacquers are usually applied by spraying on the actual part. Pattern of small cracks appear on the surface of this coating where the strain is high indicating the presence of stress concentration. The cracks also indicate the directions of maximum strain at these points since they are always aligned at right angles to the direction of the maximum principal tensile strain. These crack data could be used to locate strain gauges for precise measurement of the stress. The method is however, sensitive to temperature and humidity [2].

1.3 Electrical Strain Gauges

The method is one of the most popular and widely accepted for strain measurements and stress analysis. The strain gauge consists of a grid of strain-sensitive metal foil bonded to a plastic backing material. Any change in length will result in a change of resistance. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained [3].

Change of resistance and strain may be expressed as follows:

$$\Delta R / R = K \times \Delta L / L$$
$$K = (\Delta R / R) / (\Delta L / L)$$
$$\varepsilon = (\Delta R / R) / K \qquad (3)$$

Where, ΔR and ΔL are the changes in resistance and length respectively, K is termed as the gauge factor and ε is the strain. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained.

2. Analytical/Numerical Methods

Distribution of stresses in a structure with boundary conditions, i.e. displacements and/or forces on the boundary can be determined by using either the closed form analytical methods or by approximate numerical methods. Boundary value problems can be solved analytically by using constitutive equations based on the elastic or plastic behavior of the material under load. Analytical or close-form solutions can be obtained for simple geometries, constitutive relations and boundary conditions. Approximate solutions for boundary-value problems can be obtained through the use of numerical methods such as finite element method, finite difference method, boundary element method and finite volume method.

2.1 Finite Element Method

The structural model to be analyzed is divided into many small pieces of simple shapes called elements. Finite Element Analysis (FEA) program writes the equations governing the behavior of each element taking into consideration its connectivity to other elements through nodes. These equations relate the unknowns, for example displacements in stress analysis, to known material properties, restraints and loads. The program assembles the equations into a large set of simultaneous algebraic equations - thousands or even millions. These equations are then solved by the program to obtain the stress distribution for the entire model.

In recent years, with the advent of advanced software's, the FEA based software ANSYS, COSMOL, DIANA, ABACUS and NASTRAN have been very useful for stress analysis. These software's are preferred by users according to the type of stress analysis, the type of elements to be analyzed and the depth of accuracy required.

2.2 Boundary Element Method

In this method the governing differential equation is converted into an integral form, often involving only integrals over the boundary of the domain. Consequently, only the boundary has to be discretized in order to carry out the integrations. The dimensionality of the problem is thereby effectively reduced by one. A three dimensional volume problem becomes a two dimensional surface one, while a two-dimensional plane problem involves only one-dimensional line integrations. Also, because the interior of a solution domain is not discretized, there is much less approximation involved in representing the solution variables and rapid variations of, for example, stresses and displacements can be resolved very accurately. Stresses are accurate as there are no approximations imposed on the solution in interior domain points. The method is suitable for modeling problems of rapidly changing stresses. Boundary Element Method (BEM) uses less number of nodes and elements for the same level of accuracy as other methods. The Boundary Element Method is unsuitable if information is required at a large number of internal points [3].

2.3 Complex Variable Approach

Complex analysis, traditionally known as the theory of functions of a complex variable, is the branch of mathematics investigating functions of complex numbers. The difference between real functions and complex function is that complex functions can handle areas while real functions can handle only direction. So it can use easily for two dimensional problems of physics. Complex analysis is one of the classical branches in mathematics with its roots in the 19th century and some even before. Important names are Euler, Gauss, Riemann, Cauchy, Weierstrass, and many more in the 20th century. In 1909 G V Kolossoff wrote his doctoral degree dissertation of Dorpat University, USSR on the subject on "One application of the theory of functions of a complex variable to a plane problem in the mathematical theory of elasticity." In which systematic use of the complex variable theory was first proposed, making use of Gousat's, a French Mathematician's and two complex stress functions to represent the biharmonic equations. Then in next 40 years theory come to successful conclusion. In 1933, N I Muskhelishvili wrote a book on the basis of this theory titled "Some basic problems of the mathematical theory of elasticity." in Russian language which was translated in English by J M Radok in 1953.

In case of complex variable approach complex functions are used in which independent as well as dependent variables all are complex numbers. Let us consider one complex function f (z) where, z = x + iy and the relation w = f (z) will give representation of complex variable function. w = u(x, y) + iv(x, y) represents the function in form of two real valued functions u and v [5].

II. LITERATURE REVIEW

In order to satisfy certain service requirements holes/openings are made in structures and machine components. These holes are of different shape and size. Also, the chemical erosion may lead to irregularity in the hole shape. These holes are the stress raisers and it is essential to know the stress pattern around such holes. There are different methods to calculate stress concentration factors and to know the stress field around holes of different geometry under different loading conditions.

1) Close form solutions: Complex function theory (conformal mapping, boundary collocation method, Laurent series expansion, integral transforms (Fourier, Mellin, Hanckel transforms, Eigen function expansion) limited to very simple cases.
2) Computational solutions (Finite element method, Boundary element method, Finite difference method.)
3) Experimental solutions (photo elasticity, moiré interferometry….)

An attempt is made to review few of the important contribution for stress analysis in the infinite plate with the different types of cut-out in the present work.

A. Stress analysis of plate with single hole

Stress concentration around irregular holes using complex variable methods are reported by K. R. Y. Simha and S. S. Mahapatra [6].Conformal mapping method has been used for evaluation of stresses. The method is an operation in complex mathematics which maps a set of points in one coordinate system to a corresponding set in another, keeping the angle of intersection between two curves constant, and is widely used in solving elasticity problem. Nine hole shapes with same area and different perimeter are studied. Irregular holes may change their shape if not their size by exchanging surface energy with strain energy. A linear elastic analysis followed in this paper may not support the physics of change in shapes but can be extended to a linear visco-elastic material.

The analytical solution of infinite elastic plate with an circular hole and elliptic hole subjected to arbitrary biaxial loading is obtained by Xin-Lin Gao [7]. The complex potential method is used to formulate the boundary-value problem, and the elliptic-hyperbolic coordinate system is adopted to simplify the formulation. Two adjustable parameters-the biaxial loading factor λ and the orientation angle β-contained in the expressions for stress and displacement field are derived. He should be pointed out that in his analysis the hole boundary is assumed to be traction-free.

Formulation which given by the Savin [8] for stresses around holes in anisotropic plates under inplane loading a general solution is obtained to consider an arbitrary shape of hole and arbitrarily oriented uniaxial, biaxial, and shear stresses at infinity as well as uniform tangential force, and uniform pressure around the hole. This is achieved by V.G. Ukadgaonker and Rao [9] introducing a general form of mapping function and an arbitrary biaxial loading condition into the boundary conditions. They extended the basic formulation for multilayered plates. They introducing the computer program, the constants of the mapping function, the arbitrary biaxial loading factor, the orientation angle and the complex parameters for the laminate, the stresses around the hole can be easily obtained.

Zirka et al. [10] have analyzed stress concentration around circular hole in a rectangular plate for orthotropic and isotropic plates under dynamic and static loading. They have used photo elastic method for analysis.

Ukadgaonkar and Vyasraj [11] have analyzed the stresses in an orthotropic plate with an irregular shaped hole for different in-plane loading. They find the solutions for in-plane loading is applicable for isotropic as well as for orthotropic cases. They are found that angle ply laminates are inducing more stress concentration for shear loads and are not suitable. They are produce many results with different shape of hole for uni-axial, (loading in x- or y-direction), as well as for biaxial loading conditions and also a pure shear loading case is considered. These results are supported with finite element solutions.

Jike Liu and Chengwu Cai [12] determine the stress concentration in structures with a circular or elliptic hole and it investigated by analytical methods. They studied the problem with a rectangular hole and approximate results are derived. They deduce the analytical solutions to the stress concentration problem in plates with a rectangular hole under biaxial tensions. By using the U-transformation technique and the finite element method, the analytical displacement solutions of the finite element equations are derived in the series form. They derive the stress concentration factors for various ratios of height to width of the hole are obtained.

D S Sharma , Khushbu Panchal and Patel Nirav [13,14] obtain the general solution for determining the stress field around circular hole and triangular hole in infinite orthotropic plate subjected to internal pressure by use of the Muskhelisvili's complex variable method and numerical results are obtain using MATLAB 7.6. They studied the effect of fiber orientation and material parameter on stress pattern around pressurized circular hole. They use the ANSYS for preparation of model and results are compared with the Muskhelisvilli's complex variable method. The formulation which they find is a good tool for the designer to predict stress pattern around internally pressurized hole and to predict failure pattern of mechanical component and structures.

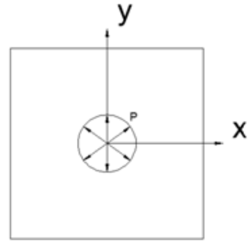

Figure 2: Plate with circular hole

They find the stress components for plane stress conditions in terms of Muskhelisvilli's complex function ϕ (z_1) and Ψ (z_2) as follows:

$$\sigma_x = 2\,\mathrm{Re}\left[s_1^{\,2}\phi'(z_1) + s_2^{\,2}\psi'(z_2)\right]$$
$$\sigma_y = 2\,\mathrm{Re}\left[\phi'(z_1) + \psi'(z_2)\right]$$
$$\tau_{xy} = -2\,\mathrm{Re}\left[s_1\phi'(z_1) + s_2\psi'(z_2)\right] \quad (4)$$

Table 1: Comparison of results of present method with ANSYS results for circular hole [13]

Material	Stress	Present Method Result	ANSYS Result
Isotropic	Max σ_X	1.00	1.001
	Max σ_Y	1.00	1.001
	Max τ_{XY}	1.00	0.9955
Glass/Epoxy Fiber orientation $=0^o$	Max σ_X	1.6521	1.555
	Max σ_Y	1.3085	1.209
	Max τ_{XY}	0.8537	0.8554
Glass/Epoxy Fiber orientation $=0^o/90^o$	Max σ_X	1.7916	1.611
	Max σ_Y	1.7916	1.599
	Max τ_{XY}	0.7164	0.7942

D K N Rao, Ramesh Babu and K R Reddy [15] find the solution is useful for finding the stress distribution around holes in symmetric laminates as well as in isotropic plates and also to determine the failure strength of the laminate on first ply failure basis by Tsai-Hill, Hashin-Rotem and Tsai-Wu criteria. This is a one stop solution for all kinds of in-plane loading on symmetric laminates as well as isotropic plates with any shape of cutout. They using the Savin's basic solution for anisotropic plates, the stress functions are derived for generalized mapping function for the hole and arbitrarily oriented in-plane loading. Square and rectangular holes in symmetric laminates of Graphite/epoxy and Glass/epoxy are studied. The stress results are also obtained by ANSYS for comparison.

In Table 2, contributions of work from many researchers have been summarized. Different techniques over the years have been used for stress analysis by the researchers. These have been compared on the basis of type of discontinuity in the element like notches or holes, circular/elliptical hole etc., the types of material used for the element, different types of loading applied to the element like axial transverse, biaxial loading etc.

Table 2: Review of stress analysis in plate with singularities

Sr. No	Author	Year	Discontinuity	Material	Loading	Analysis Techniques
1	R. E. Peterson [4]	1966	Holes, Notches	Isotropic plate	Axial, bending moment, twisting moment	Complied work of many researchers

2	Xin-Lin Gao [7]	1995	Circular hole and elliptic hole	Infinite elastic plate	Arbitrary biaxial loading	Complex potential method
3	K. R. Y. Simha and S. S. Mahapatra [6]	1998	Ir-regular holes	Infinite elastic plate	Hydrostatic tension, pure shear loading, pure shear state	Kolosov – Muskhelisvilli's complex variable approach
4	V.G. Ukadgao-nker and D.K.N. Rao [9]	1999	Triangular hole	Isotropic and orthotropic plate	Uniaxial, biaxial, shear stress, tangential shear, uniform pressure	Savin's conformal mapping and Cauchy integrals
5	A. I. Zirka [10]	2004	Circular Hole	Orthotropic plate	Static and dynamic	Photo elastic method
6	Ukadgao-nkar and Vyasraj [11]	2005	Different shape of hole (circular, elliptical, etc.)	Orthotropic laminate	Uniaxial, biaxial, pure shear stress	Savin's complex variable method and Finite element method
7	Jike Liu and Chengwu Cai [12]	2008	Rectangular hole		Biaxial loading	U-transformation technique and Finite element method
8	D S Sharma , Khushbu Panchal and Patel Nirav [13]	2010	Circular hole	Orthotropic plate	Internal pressure	Muskhelisvilli's complex variable method and Finite element method
9	D S Sharma , Khushbu Panchal and Patel Nirav [14]	2010	Triangular hole	Orthotropic plate	Uniaxial, Biaxial and Shear loading	Muskhelisvilli's complex variable method and Finite element method
10	D K N Rao, Ramesh Babu and K R Reddy [15]	2010	Square and Rectangular hole	Symmetric laminates	Arbitrary biaxial loading	Savin's complex variable method and Finite element method

B. Stress analysis of plate with multiple hole

In Table 3, contributions from many researchers on Stress analysis of plate with multiple holes have been summarized. Different techniques over the years have been used for stress analysis of plate with multiple holes by the researchers.

Table 3: Review of stress analysis in plate with multiple holes

Sr. No	Author	Year	Discontinuity	Material	Loading	Analysis Techniques
1	V. G. Ukadgaonker and Patil [16]	1993	Two elliptical holes	Isotropic	Uniform pressure and tangential stresses	Complex variable approach
2	T. K. Paul and K. M. Rao [17]	1995	Two circular holes	Thick FRP-finite laminated plate	Transverse loading	Finite element displacement method along with high order bending theory
3	J.M. Henshaw, J. R. Sorem Jr, Glaessgen [18]	1996	Multiple holes	Laminated composite plate	Tensile and shear loading	Finite element method
4	K. Ting, K. T. Chen, W. S. Yang [19]	1999	Multiple elliptical holes	Infinite domain	Remote uniform stresses	Boundary alternating method

5	E. Pan, B. Yang, G. Cai , F.G. Yuan [20]	2000	Multiple circular hole	Infinite composite laminate	Uniform loads on the hole boundaries & at infinity	Boundary element method
6	Jianlin Wang, Steven L. Crouch, Sofia G.Mogilevskaya [21]	2003	Multiple circular hole	Infinite isotropic plate	Uniform loads on the hole boundaries & at infinity	Complex integral equation method
7	L Q Zhang, A Z Lu, Z Q Yue,Z F Yang [22]	2009	Two elliptical holes	Infinite elastic plate	Uniform loads on the hole boundaries	Schwarz's alternating method and Muskhelishvili's approach

L Q Zhang, A Z Lu, Z Q Yue and Z F Yang [22] using the Schwarz's alternating method and the Muskhelishvili's complex variable function techniques to determine the stress solution for an infinite elastic plate around two elliptic holes which shows in figure 3 subjected to uniform loads on the hole boundaries and at infinity. This algorithm can be used to compute the stress concentration factors.

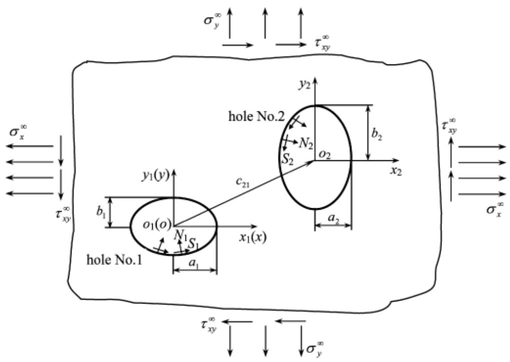

Figure 3: An infinite elastic plate containing two elliptical holes subjected to any uniform stress at the infinity, normal tractions N_1 and N_2 and tangential shears S_1 and S_2 on the boundaries

III. CONCLUSION

In this paper an effort is made to review the investigations that have been made on the stress analysis of infinite plate with cut-out. An attempt has been made in the article to present an overview of various techniques developed for stress analysis of infinite plate. Singularities of circular hole, elliptical hole, triangular hole and rectangular hole and multiple hole like circular hole and elliptical hole in infinite plate are considered.

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