P-Fuzzy Algebra with Congruence Class

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Abstract- The concept of P-fuzzy algebra is elaborated in this paper with conjugate fuzzy sets, congruence and pseudo fuzzy cosets. Well known existence theorem in algebra is proved under P-fuzzy algebra in new manner. Some properties in P-fuzzy algebra are introduced.

Index Terms- P-fuzzy algebra, Conjugate fuzzy subsets, Pseudo fuzzy cosets, t-level set, Fuzzy normal subgroup

I. Introduction

As it well known, in the "classical" fuzzy theory established by L.A.Zadeh in 1965, a fuzzy set A is defined as a map from A to the real unit I=[0.1]. the set of all fuzzy set on A is usually denoted by I_A. In its trajectory of stupendous growth, it have also come to include the theory of fuzzy algebra. Several researchers have been working on concepts like fuzzy groups, fuzzy rings etc.

Fuzzy algebra is developed to the concept of P-fuzzy algebra with conjugate fuzzy subsets and pseudo fuzzy cosets. We unify and generalize these in this paper.

II. Preliminaries

Definition2.1 A P – fuzzy set $\mu \in P^A$ is called a P – fuzzy algebra or fuzzy subalgebra on the algebra A, if

- For any n – ary ($n \geq 1$) operation f Є F
  $\mu(f(x_1 \ldots\ldots\ldots x_n)) \geq \mu(x_1) * \ldots\ldots\ldots\ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots\ldots x_n$ Є A
- For any constant (nullary operation) C
  $\mu(c) \geq \mu(x)$ for all x Є A.

Definition2.2 Let μ be a fuzzy subgroup of a group G and aЄG, then the pseudo fuzzy coset $(a\mu)^p$ is defined by $((a\mu)^p)(x)=p(a)\mu(x)$ for every xЄG and for some pЄP.

Definition2.3 Let λ and μ be two fuzzy subsets of a group G. We say that λ and μ are conjugate fuzzy subsets of the group G if for some gЄG, We have $\lambda(x)=\mu(g^{-1}xg)$ for every xЄG.

Definition2.4 A fuzzy subgroup μ of a group G to be fuzzy normal subgroup of a group G if $\mu(xy)=\mu(yx)$ for every x,yЄG.

Definition2.5 Let μ be a fuzzy subset of a set X. For tЄ[0,1], the set $X^t_\mu=\{x \in X/\mu(x)\geq t\}$ is called a t-level subset of the fuzzy subset μ.

Result2.6 P-fuzzy algebra is a fuzzy subgroup.

III. Results

Theorem3.1 Let λ and μ be two fuzzy subsets if an abelian group G from P-fuzy algebra, then λ and μ are conjugate fuzzy subsets if the group G if λ=μ.

Proof:Let G be a fuzzy group from P-fuzzy algebra A.

Here λ and μ are two fuzzy subset of the group G then λ and μ are conjugate if

$\lambda(x_1) * \ldots\ldots\ldots\ldots\ldots * \lambda(x_n) = \mu(x_1) * \ldots\ldots\ldots\ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots\ldots x_n$ Є A

using conjugacy

$\lambda(f(x_1 \ldots\ldots\ldots x_n)) = \mu(g(x_1 \ldots\ldots\ldots x_n))$ for all $x_1 \ldots\ldots\ldots x_n$ Є A

$\lambda(x_1) * \ldots\ldots\ldots\ldots\ldots * \lambda(x_n) = \mu(gx_1g^{-1}) * \ldots\ldots\ldots\ldots\ldots * \mu(gx_ng^{-1})$

relating the coefficient of x_1

$$\lambda(x_1)= \mu(gx_1g^{-1}) \text{ for every } x \in G$$

$$= \mu(g^{-1}gx_1) \text{ for every } x \in G$$

$$\lambda(x_1)= \mu(x_1) \text{ for every } x \in G$$

Similarly for n values,

We get, $\lambda(x_2)= \mu(x_2)$

. .

$\lambda(x_n)= \mu(x_n)$

In general, $\lambda(x)= \mu(x)$

$\lambda= \mu$

Conversely, Here $\lambda= \mu$

Then, $\lambda(x)= \mu(x)$ extending x as a function of n variables,

We get,

$\lambda(f(x_1 \ldots\ldots\ldots x_n)) = \mu(g(x_1 \ldots\ldots\ldots x_n))$

$\lambda(x_1) * \ldots\ldots\ldots\ldots\ldots * \lambda(x_n) = \mu(x_1) * \ldots\ldots\ldots\ldots\ldots * \mu(x_n)$ indicates the condition for conjugate.

Theorem3.2 Let μ be a fuzzy subgroup of P-fuzzy algebra A then the pseudo fuzzy coset $(a\mu)^p$ is a fuzzy subgroup of the P-fuzzy algebra A for every a∈A.

Proof: Let μ be a fuzzy subgroup of A,Using result 1,
Consider a P – fuzzy subalgebra on the algebra A then there exists a P –fuzzy set $\mu \in P^A$ then for

(i) n – ary operation

$\mu(f(x_1 \ldots\ldots x_n)) \geq \mu(x_1) * \ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots x_n \in A$-----(1)

from (1) let $x_2 = y$

$\mu(f(xy)) \geq \mu(x)*\mu(y)$---------------------(2)

since $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$-------(3)

from (2) and (3)

$\mu(x)*\mu(y) = \mu(x*y) \geq \min\{\mu(x), \mu(y)\}$-----(4)

Let μ: X→[0,1]

Here μ is a function defined that X is a set which map it each and every element between 0 and 1.

Also the set of all elements of X^{-1} maps the values between 0 and 1.

Therefore $\mu: X^{-1} \to [0,1]$.

This implies $\mu(x^{-1}) = \mu(x)$----------------------------(5)

(ii) nullary operation (for any constant)

If there exist any constant μ(c) then

$\mu(c) * \mu(x) = \mu(c * x) \geq \min\{\mu(c), \mu(x)\} = \mu(x)$------(6)

$\mu(c) \geq \mu(x)$, for all x ∈ A.

Since all the elements of μ(x) lies between 0 and 1. Also μ(c) is any constant and it is greater than or equal to 1. Using the results (4), (5) and (6), it is clear that P – fuzzy subalgebra is a subgroup.

Let $x_1, x_2 \in A$

$A\mu(f(x_1x_2))^p = a\mu(f(x,y^{-1})^p)$ using pseudo fuzzy coset

$=p(a)[\mu(x)*\mu(y^{-1})]$

$\geq p(a)\mu(x,y^{-1})$

$\geq p(a)\min\{\mu(x),\mu(y)\}$

$=\min\{p(a)\mu(x),p(a)\mu(y)\}$

$\geq \min\{(a\mu)^p(x),(a\mu)^p(y)\}$ for every x,y∈G

Theorem3.3 Let λ and μ be any two fuzzy subset of a set X from P-fuzzy algebra A, then for a∈X, $(a\mu)^p C (a\lambda)^p$ iff $\mu \underline{C} \lambda$

Proof: If λ and μ be two fuzzy set.

Let $(a\mu)^p C (a\lambda)^p$

$[a\mu(f(x_1 \ldots\ldots x_n))]^p C [a\lambda(g(x_1 \ldots\ldots x_n)]^p$

Using pseudo fuzzy coset

$p(a)[\mu(x_1)* \ldots\ldots *\mu(x_n)] \underline{C} p(a)[\lambda(x_1)* \ldots\ldots *\lambda(x_n)]$

using cancellation law,

$\mu(x_1)* \ldots\ldots *\mu(x_n) \underline{C} \lambda(x_1)* \ldots\ldots *\lambda(x_n)$

using theorem 1

$\mu(x) \underline{C} \lambda(x)$

$\mu \underline{C} \lambda$

Conversly, Let $\mu \underline{C} \lambda$, implies $\mu(x) \underline{C} \lambda(x)$

$\mu(x_1)* \ldots\ldots *\mu(x_n) \underline{C} \lambda(x_1)* \ldots\ldots *\lambda(x_n)$

Premultiply by p(a) on both sides

$p(a)[\mu(x_1)* \ldots\ldots *\mu(x_n)] \underline{C} p(a)[\lambda(x_1)* \ldots\ldots *\lambda(x_n)]$

$p(a)[\mu(f(x_1 \ldots\ldots x_n))] \underline{C} p(a)[\lambda(g(x_1 \ldots\ldots x_n))]$

$(a\mu)^p C (a\lambda)^p$.

Theorem3.4(Existence Theorem)

Let μ be a fuzzy subgroup of a group G from P-fuzzy algebra A. The congruence class $[x]_\mu$ of μ_t containing the element x of the group G exist only when μ is a fuzzy normal subgroup of the group G.

Proof: Let μ be a fuzzy subgroup of a group G from P-fuzzy algebra A.

If μ is a fuzzy normal subgroup of a group G then the t-level relation μ_t of μ is a congruence relation on the group G and hence the congruence class $[x]_\mu$ of μ_t the containing the element X of the group G exist.

Now, We prove that for the existence of the congruence class $[x]_\mu$ we must have the fuzzy subgroup μ of the group G to be fuzzy normal subgroup of group G.

i.e., if μ is not a fuzzy normal subgroup of the group G, then the congruence class $[x]_\mu$ of μ_t containing the element X of the group G does not exist.

To prove this conside the alternating group A_4

Choose $P_1, P_2, P_3 \in [0,1]$

Such that $1 > P_1 > P_2 > P_3 \geq 0$

Define $\mu: A_4 \to [0,1]$ by

$$\mu(x) = \begin{cases} 1 & \text{if } x=e \\ P_1 & \text{if } x=(12)(34) \\ P_2 & \text{if } x=(14)(23),(13)(24) \\ P_3 & \text{otherwise} \end{cases}$$

The t- level subset of μ are given by

{e},{e,(12),(34)},{e,(12)(34),(13)(24),(14)(23)} and A_4.

μ is a fuzzy subgroup of A_4

For x=(123) y=(143)

$\mu(xy)=\mu((123)(143))$
$=\mu((12)(34))$
$=P_1$

$\mu(yx)=\mu((143)(123))$
$=\mu((14)(23))$
$=P_2$

Therefore $\mu(xy)\neq\mu(yx)$

For x=(123) and y=(143)

M is not a fuzzy normal subgroup of P_4

Let x=(14)(23)

Y=(13)(24)

t-level set

$\mu(xy)=(12)(34)=P_1$

$\mu(yx)=(12)(34)=P_1$

$\mu(xy)=\mu(yx)=P_1=t$

Thus by definition of t-level relation of μ we have (x,y)Єμ_t

For a=(123)

$\mu((ax),(ay^{-1}))=P_2<P_1$

So by the definition of t-level relation of μ we have (ax,ay) not belongs to μ_t for $t=P_1$ and α=(123).

Hence it follows that μ_t is not a congruence relation on the alternating group A_4. So by the definition of congruence class, $[x]_\mu$ does not exist. That is if μ is not a fuzzy normal subgroup of the group G then the congruence class $[x]_\mu$ of μ_t containing the element X of the group G does not exist.

IV. CONCLUSION

In this paper we studied some special properties in P-fuzzy algebra using fuzzy group. Also we proved existence theorem under P-fuzzy algebra.

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