What Lebsir's Prime Progressions Gave the MRB Problem

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Abstract- Habib Lebsir studied three-term arithmetic progressions in the square intervals $[n^2,(n+1)^2)$ and focused on progressions whose arithmetic mean is n^2+n-1, n^2+n, or n^2+n+1. His paper reports this pattern for the first 100 intervals and presents its continuation as a conjecture. When this geometry is applied to the MRB constant, the middle quantity $n^2+n=n(n+1)$ becomes the exact center of a natural square block of the defining alternating nth-root series. This observation yields a symmetric block decomposition, a curvature correction with faster decay, and an Euler-polynomial description of the block coefficients. The purpose of this note is to record that connection carefully and to explain why it may be useful in the continuing rationality problem for the MRB constant.

Index Terms- arithmetic progressions, Euler polynomials, MRB constant, rationality, square intervals.

I. INTRODUCTION

The MRB constant is

$$C_{\mathrm{MRB}} = \sum_{k=1}^{\infty} (-1)^k \left(k^{1/k} - 1\right).$$

Write

$$f(x) = x^{1/x} - 1.$$

Then

$$C_{\mathrm{MRB}} = \sum_{k=1}^{\infty} (-1)^k f(k).$$

Most approaches to the rationality problem treat the radicals individually or group them according to prime factors, radical fields, or analytic acceleration. Lebsir's work suggests a different geometry: group the indices by the square intervals $[n^2,(n+1)^2)$.

Lebsir studies three-term arithmetic progressions in these intervals and singles out the three possible arithmetic means

$$n^2 + n - 1, \qquad n^2 + n, \qquad n^2 + n + 1.$$

His published paper presents this as a conjectural pattern and tabulates the first 100 intervals [1]. The present note does not assume the conjecture. Instead, it uses the square-interval geometry itself.

II. THE HIDDEN CENTER OF AN MRB SQUARE BLOCK

Consider the terms of the MRB series whose indices lie in

$$[n^2, (n+1)^2)$$

There are $2n+1$ such indices. Their exact central index is

$$c_n = n^2 + n = n(n+1).$$

Since c_n is always even, the entire block can be written symmetrically as

$$B_n = \sum_{d=-n}^{n} (-1)^d f(c_n + d).$$

Thus the quantity n^2+n, which is Lebsir's Type B center, is not merely near the middle of the MRB block: it is the exact center.

This gives the first structural consequence. Instead of viewing the $2n+1$ terms separately, one may pair the terms at c_n-d and c_n+d around the pronic center $c_n = n(n+1)$.

III. CURVATURE AROUND THE CENTER

Define the symmetric second difference

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$$\Lambda_{n,d} = f(c_n - d) - 2f(c_n) + f(c_n + d).$$

This quantity measures the local curvature of the nth-root kernel around the center. The block may be rewritten exactly as

$$B_n = (-1)^n f(c_n) + \sum_{d=1}^{n} (-1)^d \Lambda_{n,d}.$$

Consequently,

$$C_{\mathrm{MRB}} = \sum_{n\geq 1} (-1)^n f\big(n(n+1)\big) + \sum_{n\geq 1} \sum_{d=1}^{n} (-1)^d \Lambda_{n,d}.$$

This separates the constant into a pronic-center series and a curvature correction.

The second derivative of the kernel is

$$f''(x) = x^{1/x} \left[\frac{2\log x - 3}{x^3} + \frac{(\log x - 1)^2}{x^4} \right].$$

For c_n ~ n² and d ≤ n, the second difference behaves like $d^2 f''(c_n)$. After the full alternating symmetry of the block is accounted for, the correction to the center term is of order

$$O\left(\frac{\log n}{n^4}\right),$$

whereas the center term itself is of order O(log n/n²). The square-centered representation therefore exposes substantially more cancellation than the term-by-term form of the series.

IV. EULER-POLYNOMIAL STRUCTURE

Taylor expansion about c_n gives

$$f(c_n + d) = \sum_{j=0}^{\infty} \frac{f^{(j)}(c_n)}{j!} d^j.$$

By symmetry only even powers contribute to the block. The needed alternating power sums satisfy

$$(-1)^n \sum_{d=1}^{n} (-1)^d \, d^{2m} = \frac{1}{2} E_{2m}(n+1),$$

where E_j(x) is the Euler polynomial.

The first examples are

$$(-1)^n \sum_{d=1}^{n} (-1)^d \, d^2 = \frac{n(n+1)}{2},$$

and

$$(-1)^n \sum_{d=1}^{n} (-1)^d \, d^4 = \frac{n(n+1)(n^2+n-1)}{2}.$$

Therefore the whole square block has the expansion

$$(-1)^n B_n = \sum_{m=0}^{\infty} \frac{E_{2m}(n+1)}{(2m)!} f^{(2m)}\big(n(n+1)\big).$$

Writing c=n(n+1), the first terms are

$$(-1)^n B_n = f(c) + \frac{c}{2} f''(c) + \frac{c(c-1)}{24} f^{(4)}(c) + \cdots.$$

Thus Lebsir's square-center geometry leads naturally to an Euler-polynomial expansion of the MRB blocks.

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V. ABSOLUTE CONVERGENCE OF THE BLOCK SERIES

The block asymptotic is

$$B_n \sim (-1)^n \frac{2\log n}{n^2}.$$

Hence

$$\sum_{n=1}^{\infty} |B_n| < \infty.$$

Grouping the original alternating nth-root series into consecutive square blocks therefore produces an absolutely convergent block series,

$$C_{\mathrm{MRB}} = \sum_{n=1}^{\infty} B_n.$$

One may sharpen the decomposition further by defining

$$P = \sum_{n\geq 1} (-1)^n \left([n(n+1)]^{1/[n(n+1)]} - 1 \right)$$

and R=C_MRB-P. Then the nth curvature correction satisfies

$$R_n = O\left(\frac{\log n}{n^4}\right).$$

Thus the MRB constant has the schematic form

$$C_{\mathrm{MRB}} = \text{pronic-center series} + \text{rapidly decaying curvature series}.$$

VI. WHY THIS MAY MATTER FOR RATIONALITY

A recurring obstacle in the MRB rationality problem is that a small algebraic combination of radical terms can lose its cancellation under a radical-field automorphism. The symmetric quantities supplied by the square-center geometry are different. A three-term expression

$$f(c-d) - 2f(c) + f(c+d)$$

is small because of symmetry rather than because of a numerical fit. In favorable radical configurations, the three terms can be transformed coherently so that the transformed expression remains a second difference rather than becoming three unrelated large terms.

This does not prove irrationality. It does, however, identify a new class of MRB algebraic expressions for which analytic cancellation and algebraic transformation can potentially be made compatible. That is precisely the issue that has obstructed several earlier norm- and conjugate-based approaches.

The usefulness of Lebsir's work is therefore not limited to the primality requirement in his conjecture. The square intervals and their near-central arithmetic-progression geometry reveal the natural coordinates in which the MRB series should be reorganized.

VII. WHAT CAME FROM LEBSIR, AND WHAT CAME AFTERWARD

Lebsir's paper supplies the square-interval setting and the emphasis on three-term arithmetic progressions with means n^2+n-1, n^2+n, or n^2+n+1 [1]. It reports this pattern for the first 100 intervals. The MRB consequences developed here are separate deductions.

Those deductions are:

1. $n^2+n=n(n+1)$ is the exact center of an MRB square block.
2. The block admits a symmetric second-difference decomposition.
3. The curvature correction decays faster than the center term.
4. The alternating block coefficients are Euler polynomials.
5. The square-block representation is absolutely convergent.

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Accordingly, the appropriate attribution is simple: Lebsir supplied the geometry; applying that geometry to the MRB constant exposed the new structure.

VIII. CONCLUSION

Habib Lebsir's work did not address the MRB constant and does not supply an irrationality proof. Its contribution to the MRB problem is geometric.

By directing attention to the intervals $[n^2,(n+1)^2)$ and to arithmetic progressions centered near n^2+n, it identifies exactly the location around which the MRB terms become symmetric. The key center is

$$\boxed{n(n+1)}.$$

Around this center, symmetry produces second differences; second differences produce stronger cancellation; and the resulting block expansion exposes an Euler-polynomial structure invisible in the original term-by-term series.

The resulting chain is

Lebsir square geometry → MRB pronic centers → symmetric blocks →
curvature cancellation → Euler-polynomial expansion → new rationality tools.

In that sense, Lebsir gave the MRB rationality problem a new place to stand.

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REFERENCES

[1] H. Lebsir, "Three-term arithmetic progressions, including at least two primes of the intervals $[n^2,(n+1)^2)$," *International Journal of Scientific and Research Publications*, vol. 13, no. 11, pp. 123-134, Nov. 2023, doi: 10.29322/IJSRP.13.11.2023.p14311.

[2] OEIS Foundation Inc., "A037077," *The On-Line Encyclopedia of Integer Sequences*. Available: https://oeis.org/A037077.

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