On the comparative Study of Compactness and some of its relative notion in Metric and topological spaces.

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Abstract- In this paper, we compared compactness and its related properties in metric and topological spaces and determine what topological spaces can do that metric spaces cannot.

Index Terms- Metric Spaces, Topological Spaces, Compact Spaces, Lindelöf space , Countably compact Spaces, Separable Space, Sequentially Compact Space, Second Countable Space, Complete Space.

I. Introduction and Definitions

The general topology has become foundation of knowledge for all branches of mathematics. Its methods have enriched other fields of mathematics and also give enough clues to their new development.

In this work, we compared compactness property and its related notions within metrics and topological spaces, we further study the implication of the notions. As we know, sequences are not sufficient for the study of general topological spaces, that is why sequential compactness is not a best notion. Compactness is the most important property of topological space, it has very wild application to both analysis and functional analysis [1]. [5] The following definition below leads us to the main work as we can see in [5], [7].

Definition 1.1: Suppose X is a topological space. A cover of X is a family $O \subseteq p(X)$ of subset of X such that $\cup O = \underset{A \in O}{\cup O} = X$. This cover is open if every $A \in O$ is closed.

Definition 1.2: Suppose X is a topological space. A subfamily $B \subseteq O$ of a cover O is said to be a subcover iff $\cup B =$.

Definition 1.3: A topological space X is said to be compact if every open cover of X has a finite subcover. i.e for each open cover $\{O\}_{\propto_i}$ of X ∃ a finite subset of index set $\{\propto_1, \propto_2, \dots, \propto_n\}$ such that $\cup O_{\propto_i} = X$.

Definition 1.4: [6] A topological space X is Lindelöf if every open cover has a countable subcover.

Example: (i) Every finite space is compact

(ii) Closed and bounded interval $[a, b]$ in $\mathbb{R}$ is compact while bounded interval (a, b) in R is not compact. To show this, let $L = b - a$ and consider the collection $\cup = \{U_n : n \in N\}$ of open subsets of (a, b) given by $U_n = \left(n + \frac{l}{n}, b\right)$. Then, U is a cover of (a, b) infact $a < x < b \Rightarrow x - a > \frac{l}{n}$ for some integer $n \geq 1$ and then $x \in U_n$. However, if F is any finite subcollection of U, then $a + \frac{L}{M+1} \notin U_n, U_n \in F$, where $M = max\{n : U_n \in F\}$, so F is not a cover of (a, b).

(iii) No any half-open interval $(a, b]$ or $[a, b)$ is compact

(iv) The real line $\mathbb{R}$ is not compact for $(-n, n)$ is an open cover of $\mathbb{R}$ that has no finite subcover of $\mathbb{R}$.

(v) The discrete topology on a contably infinite set gives an example of a space which is Lindelöf but not compact.

(vi) The sorgenfrey line is Lindelöf

(vii) Countable and cocountable spaces are Lindelöf

Definition 1.5: [6] A collection F of sets is said to have the finite intersection property if F is nonempty and each nonempty finite subcollection of F has nonempty intersection. [2]

Lemma: [3] A topological space X is compact iff each collection of closed subsets of X having the finite intersection property itself has nonempty intersection.

Two generalization of compactness will be derive by weakening the requirement that subcovers must be finite. A topological space is σ - compact if it can be express as the union of countably many compact sets, by comparing this definition of σ - compact to definition 1.4 above, we see that every compact space is σ - compact and every σ - compact space is Lindelöf. A topological space X is countably compact if every countable open cover has a fininte subcover and two other compactness notions are closely related, but not equivalent to countable compactness a topological space X is said to be sequencially compact if every sequence in X has a convergent subsequence and X is weakly countably compact if every infinity set has a limit point [4]. These are the view among other compactness properties we would start comparing between metric and topological spaces to determine what topological spaces can do that metric space cannot.

II. Compactness properties and its related notions.

The following charts (Fig. A and B) below demonstrate the comparative study of compactness properties and its related notions in topological spaces and metrics spaces respectively. We would study notions like σ - compactness, Lindelöf, countably compactness and determine that they are all equivalent to compactness on metrics spaces and show where they failed to do so in topological spaces, that is , what topological notions can do that metric notions cannot. We start our comparative study from **theorem 2.1** below.

Theorem 2.1: Every σ - compact space is Lindelöf
σ – compactness $\Longrightarrow$ Lindelöf

Proof: suppose X is a σ - compact space. Then there exist a countable family $\{F_n: n = 1,2, \ldots\}$ of compact subsets of X such that $X = \bigcup\{F_n: n = 1,2, \ldots\}$. Now, consider any open cover U of X so that for each $n = 1,2, \ldots$ $\exists$ a finite subfamily U_n of cover U which cover a countable family F_n is compact, $\bigcup\{U_n: n = 1,2, \ldots\}$ is a countable subfamily of cover U which cover X. Therefore X is Lindelöf.

Proposition 2.1: Sequentially compactness properties implies countably compactness in a topological space. i.e. if a topological space X is sequentially compact, then is countably compact.

Proof: First, we shall show that if $\{x_n\}$ is a sequence which has a subsequence $\{x_{n_k}\}$ converging to t in a topological space X, the t is an accumulation point of the sequence $\{x_n\}$. By let O be an open neighbourhood of t, provided that t is an accumulation point of subsequence $\{x_{n_k}\}$, $\exists$ N $\in \mathbb{N}$ such that $k \geq$ N implies that $x_{n_k} \in O$, that means there are infinitely many n such that $x_n \in O$. So, t is an accumulation point. Therefore, the sequentially compactness implies that every sequence in a topological space X has an accumulation point.

From the figure B below, we have the **Lemma 2.1** below.

Lemma 2.1: A metric space is totally bounded iff it is sequentially compact.

Totally boundedness $\Longrightarrow$ Sequentially boundedness.

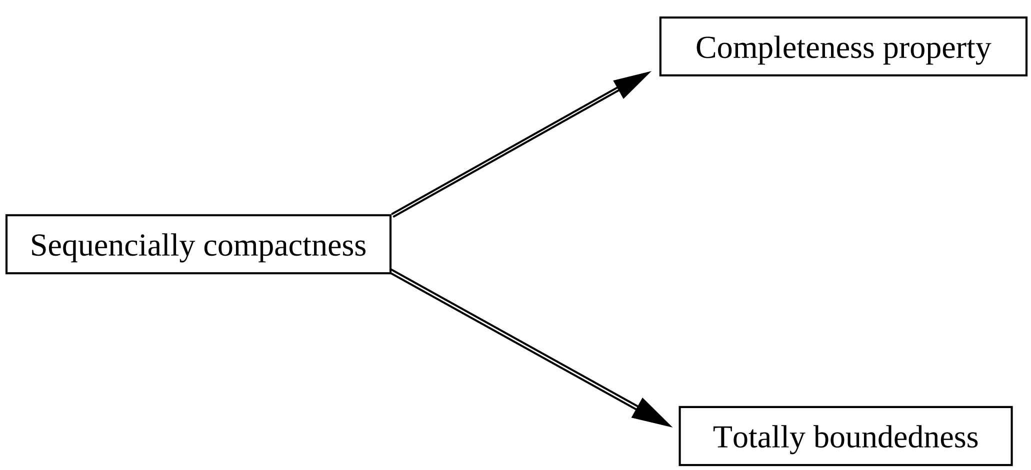
Proof: We shall provide proof of this Lemma from contradictory point of view. Suppose a metric space X is not totally bounded, that means there does not exist finite many points for $\in > 0 x_1, x_2, \ldots x_n$ such that $\bigcup_{C=1}^{n-1} B(x_i, \mathcal{E}) = X$, that is X cannot be covered by fintely many $\mathcal{E}$-balls. Let $\{x_n\}$ be a sequence such that x_n is a member of $X \setminus \bigcup_{C=1}^{n-1} B(x_i, \mathcal{E}), \forall\, n \Longrightarrow d(x_i, x_j) \geq \mathcal{E}\ \forall\, i > j$

$X \setminus \bigcup_{C=1}^{n-1} B(x_i, \mathcal{E}), \forall\, n \Longrightarrow d(x_i, x_j) \geq \mathcal{E}\ \forall\, i > j$, this provides that $i \neq j$ gives that $d(x_i, x_j) \geq \mathcal{E}$, so that the sequence $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ which is not Cauchy.

Conversely, suppose $\{x_n\}$ is a sequence in a metric space X. Since X is totally bounded, that means it can be covered by finitely many ε - balls of radius 1. Let B_1 be one of the ball and $x_n \in B_1$ for $n > 0$. Then B_1 can be covered by finitely many balls of radius $\frac{1}{2}$. Also, let B_2 be another ball which satisfy that $B_1 \cap B_2$ holds and $x_n \in B_1 \cap B_2$ for $n > 0$. If we continue like this, we have a sequence B_i of an open balls of radius $1/_i$ such that $B_1 \cap B_2 \cap \ldots \cap B_m$ holds and contains x_n for $n, m > 0$. Thus, if we take a subsequence $\{x_{n_k}\}$ such that for each $k, B_1 \cap B_2 \cap \ldots \cap B$ contains x_{n_k}. Now, if $j \geq k$, both x_{n_j} and x_{n_k} are also contained in B_k, then $d\left(x_{n_k}, x_{n_j}\right) < \frac{2}{i}$. Therefore, the subsequence x_{n_k} contains in $i = 1$ $B_1 \cap B_2 \cap \ldots \cap B_i$ which converges to k is a Cauchy sequence and since the subsequence x_{n_k} converge to k, therefore, the metric space X is sequentially compact.

Proposition 2.3: suppose (X, d) is a metric space. Then, the following must satisfy: (i) sequentially compactness $\Longrightarrow$ completness and sequentially compactness $\Longrightarrow$ totally boundedness.

Proof:



Suppose $\{x_n\}$ is a Cauchy sequence, by sequential compactness, $\exists$ a subsequence x_{n_k} of x_n such that $x_{n_k} \longrightarrow K$. due to the fact that a Cauchy sequence which has a convergent subsequence also converge. This shows that the metric space is complete. And for the second statement. Let X be a metric space and for every radius $r > 0$, there exist the family of open ball $\{B(x, r)\}$ which serves as an open cover of X, for each $x \in X$, there is an open cover of X, the compactness of X provides a finite subcover. Therefore X is totally bounded. The proof is complete.

Suppose X is a metric space, we have from the **Fig. B** below Compactness property $\Longrightarrow$ Lebsesque property $\Longrightarrow$ completeness.

Theorem 2.2: If X is compact metric space then X is complete. i.e. If X is compact metric space, Compactness $\Longrightarrow$ completeness.

Proof: Suppose $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in the subspace $A \subset X$ and $S \subseteq A$ be the following subset $S = \{x_1, x_2, \dots x_n, \dots\} \subseteq A$. If we take S to be infinite, then, we follow the proof as follows:

Step A: $\exists$ an accumulation point $z \in A$, for each $\varepsilon > 0$, $B_{z,\varepsilon} \cap S \neq 0$ and also $B_{z,\varepsilon} \cap S = \{z\}$. Suppose there does not exist such $a \in A$, then for each $x \in A$ $\exists$ some $\mathcal{E}(x) > 0$ such that $B_{z,\mathcal{E}(x)} \cap S = \emptyset$ and also $\{x\}$. As shown in **Lemma 2.1** above, the collection of open balls $\{B_{x,\mathcal{E}(x)}\}_{x\in A}$ covers set A and is open, so, by compactness, it will contain a finite subcover; for this, $\exists$ $x_1, x_2, \dots x_k$ such that $S \subseteq A \subseteq \bigcup_{i=1}^{k} B_{x_i,\mathcal{E}(x_i)}$------------------------------------- (*).

But since S is infinite and $B_{z,\mathcal{E}(x)} \cap S \neq \emptyset$ and also $\neq \{x\}$, $\forall x \in A$, then (*) above is not possible.

Step B: Suppose $z \in A$ is an accumulation point as we shown in step A above. Then the sequence has a subsequence $\{x_{n_k}\}$ where $k \in \mathbb{N}$ and has z as its accumulation point. If we let $z \notin S$, this part is done by using an induction method. Now, let $n(1) \in \mathbb{N}$ such that $x_{n(1)} \in B_{z,1} \cap S$. Assume that we have $n(1) < n(2) < \cdots < n(k)$ such that $x_{n(1)} \in B_{z,\frac{1}{t}} \cap S$, $t = 1, \dots, k$ and if $\mathcal{E} = min\left\{\frac{1}{KM}, d_X(z, x_1), d_X(z, x_2), \dots, d_X\big(z, x_n(k)\big)\right\}$. Provided that $x_n \neq z$ $\forall n$, we have that $\mathcal{E} > 0$ and by part A above, $\exists$ some $n(k+1)\, in\, \mathbb{N}$ such that $x_n(k+1)$ contains in $B_{z,\varepsilon} \cap S$ and $B_{z,\varepsilon} \cap S \subseteq B_{z,\frac{1}{k+1}} \cap S$. Then, we see that choice of $\mathcal{E}$ provides that $n(k) < n(k+1)$ and this ends the induction method of Step B.

Step C: We have z as an accumulation point of the sequence $\{x_n\}_{n\in\mathbb{N}}$ as shown above due to the fact that it is a Cauchy sequence. For this let $\mathcal{E} > 0$, then there are two reasons to study here

(i) $N \in \mathbb{N}$ such that, if $p, q \geq N$, then $d_X\big(x_p, x_q\big) < E$ and (ii) $R \in \mathbb{N}$ such that, if $r \geq R$, then $d_X\big(z, x_n(r)\big) < E$. If we let $r \in N$ such that $n(r) \leq N, R$. Therefore if $n \geq N$, we have $d_X(z, x_n) \leq d_X\big(z, x_{n(r)}\big) + d_X\big(x_{n(r)}, x_n < 2\mathcal{E}\big)$ (by triangle inequality). Therefore, the sequence $\{x_n\}_{n\in\mathbb{N}}$ has z has its accumulation point.

Definition 2.1: [6] If A is an open cover of a metric space X, a real number $\lambda > 0$ is called a Lebesgue number for the cover A, if for every $B \subseteq X$ with $diam(B) < \lambda \ \exists\ a\ K \in A$ such that $B \subseteq K$.

Theorem 2.3: [2] Let X be metric space and $\{U_\propto\}$ be an open cover of X. Then there is a positive number $\lambda(\{U_\propto\})$ called Lebesgue number of the cover of X which satisfy the following property: Each ball $B(X\ \lambda)$ is contained in at least one $U_\propto$.

From implication ϕ above, we shall develop this result below by follow **theorem 2.3** above.

Theorem 2.4: Compactness property $\Longrightarrow$ Lebesgue property for compact metric space X.

Proof: Let us assume $O_x \in F$ such that $x \in O_x$ for every $x \in O_x$ and consider O_x open. Since O_x is open $\exists\ \lambda_x > 0$ such that the open ball $B(x, 2\lambda_x) \subseteq U_x$. Then, for every $x \in X$,

X has $\{B(x, \lambda_x)\}$ as its open cover so that for $x_1, x_2, \dots, x_n \in X$ $U_{m=1}^{n} B\big(x_n, \lambda_{x_m}\big) = X$ by compactness property.

If Lebesgue number $\lambda = min\big(\lambda_{x_1}, \lambda_{x_2}, \dots, \lambda_{x_m}\big) > 0$ and if $A \subseteq X$ such that $D(Y) < \lambda$, then, we choose $a \in A$ and $m \in \{1, \dots, n\}$ such that $a \in B\big(x_m, \lambda_{x_m}\big)$. Now, we need to consider the inclusion relation below.

$A \subseteq B(a, \lambda) \subseteq \big(x_m, 2\lambda_{x_m}\big) \subseteq O_{x_m}$---------------------------------- (K)

In the inclusion relation (K) above, $A \subseteq B(a, \lambda)$ follow from $a \in A$ and $D(A) < \lambda$ and by choice of λ_x, the third inclusion relation holds. Lastly, for the inclusion $B(a, \lambda) \subseteq \big(x_m, 2\lambda_{x_m}\big)$ if $t \in B(a, \lambda)$, then, we have $d(t, x_m) \leq d(t, a) + d(a, x_m) < \lambda + \lambda_{x_m} \leq \lambda_{x_m} + \lambda_{x_m} = 2\lambda_{x_m}$ (by triangle inequality).

Proposition 2.2: In a metric space X, if X, is countably compact, then is sequentially compact.

Proof: If X is countably compact, then every sequence in X has an accumulation point, this shows that the any sequence $\{x_n\}_{n\in\mathbb{N}} \in X$ has an accumulation point. By the fact that if X is first countable, then for every accumulation point m of a sequence $\{x_n\}_{n\in\mathbb{N}} \in X$ $\exists$ a subsequence $\{x_{n_k}\}$ which converges to m, which proof for sequentially compactness.

Theorem 2.5: Suppose X is a topological space. If X is second countable, then is separable space.

Second countability $\Longrightarrow$ Separability

Proof: Note that a topological space X is said to be second countable if it has a countable base. Then, if L is a base for topology, then, $\exists$ a dense subset $A \subseteq X$. Such that card (A) $\leq$ card (L). Which shows that X is separable.

Theorem 2.6: If topological space X is second countable then is Lindelöf.

Second countability $\Longrightarrow$ Lindelöf

Proof: If $\{O_i\}_{i\in J}$ is an open cover of topological space X.

We shall show that there must be a countable subcover by follow the existence of countable basis L for the topology τ of a topological space X. Suppose one of the cover $O_i = \emptyset$. We shall adopt element of basis L to direct to certain $U_i's$. Specifically, for each set $Y \in L$, we select one set O_Y from $U_{i's}$ by taking the following steps:

Step A: We verify if there is at least one of the open sets O_i such that $Y \in O_i$. If not, we let $O_Y = \emptyset$. If the set $Y \in O_i$, then, we choose one such O_i and consider it as O_Y. We can also choose the same O_i corresponding to many $Y's$ as the given O_i contains many basis, but we have at most one O_i chosen for each Y, so the set $\{O_Y : Y \in L\}$ is countable.

Step B: Here, we shall show that the set $\{O_Y : Y \in L\}$ in step A above is a cover of topological space X. for every $x \in X$, we need to show that one of the set O_Y contains x in X. Since $O_i's$ is a cover of X, then $\exists\ O_i$ such that $x \in O_i$. Provided that we have L as a basis as defined in step A above, there is $Y \in L$ such that $x \in Y \in O_i$. As $Y \in O_i's$, Y will be one of the basis sets for which a set $Y \subseteq O_Y$ such that $x \in Y \in O_Y \Longrightarrow x \in O_Y$, which ends the proof.

Theorem 2.7: In a metric space X, if X is separable, then is a second countable space.

Proof: If the sequence $\{x_n\}_{n\in\mathbb{N}}$ is a countable dense in X. Then for each x_n, let O_n be the set of all open balls centred at x_n in which its radius is rational. Then the O_n is countable for each n, so, we have that $O = U\{A_n : n \in \mathbb{N}\}$ is a basis. Now, choose an open set $K \subseteq X$, we shall show that K is a union of our basis O defined above. If $x \in K$, we shall show that there is one of basis set O such that $x \in A \subseteq K$. Then, the point x contained in some $\mathcal{E}$ - balls in K. If we consider an $\frac{\varepsilon}{10^2}$. Ball around x, this ball must contain some point x_n from dense subset of countable dense set $\{x_n\}$ so that $d(x_n, y) < \frac{\varepsilon}{10^2}$. Then, if we take a rational radius which is strictly more than $\frac{\varepsilon}{10^2}$, then, we have a rational radius ball with centre x_n containing $x \in K$.

Theorem 2.8: Every compact topological space is σ - compact.
Compactness $\Longrightarrow$ σ - compactness

Proof: Suppose X is a compact topological space. Let $K \subset X$ be a compact subset of X. Let $\{A_n : n \in \mathbb{N}\}$ be the family of compact subset of X that covers X, $K \cap A_n$ is a subset of the subspace $A_n \subset X$, for every n in $\mathbb{N}$. For this, $K \cap A_n$ is compact in K for every n in $\mathbb{N}$. Then K will be union of countable family $\{K \cap A_n : n \in \mathbb{N}\}$ of its compact subsets. Therefore, K is σ – compact. This completes the proof.

Theorem 2.9: Every totally bounded metric space is separable.
Total boundedness $\Longrightarrow$ separability

Proof: Let X be a metric space and is totally bounded. Then, there exist $x_{p,1}, x_{p,2}, \ldots x_{p,n_p}$ such that $X = U_{k=1}^{n_p} B\left(x_{p,q}, \frac{1}{p}\right)$. Then there exist $A \in X$ such that $A = \{x_{p,q} : p \in \mathbb{N}, q = 1,2, \ldots, n_p\}$ is countable choose $x \in X$ and $\mathcal{E} > 0$, also ε^{-1} such that $p > \varepsilon^{-1}$, so that there exist a such that $d(x, x_{p,q}) < \frac{1}{p} < \varepsilon$. Then, A is dense and the metric space X is separable.

Theorem 2.10: Every totally bounded space is bounded.
Totally bounded $\Longrightarrow$ Boundedness

Proof: Let X be a metric space and is totally bounded. From definition of totally boundedness, there exist finitely many points $x_1, x_2, \ldots, x_n$ such that $U_n B(x_n, 1) = X \Longrightarrow$ the open ball of radius 1 covers X. By $M = max_{n,m} d(x_n, x_m)$, by triangle inequality, we have $diam(X) \le M + 2 < \infty$. Therefore, X is bounded.

Theorem 2.11: Every compact tmetric space X is totally bounded.
Compactness property $\Longrightarrow$ Totally boundedness property

Proof: For the radius $r > 0$ of the ball $B(x, r)$, $\exists$ family of open ball $\{B(x, r)\}_{x\in X}$ such that $X = U\{B(x, r)\}_{x\in X}$, by compactness, $\{B(x, r)\}$ cover X and $\exists$ a finite subcover for the cover $\{B(x, r)\}$. Therefore, X is totally bounded.

Theorem 2.12: If a metric space X is sequentially compact, then X is Lebesgue.
Sequencially compactness $\Longrightarrow$ Lebesgue property.

Proof: Let $\lambda > 0$ be a Lebesgue number. Now if O is an open cover that not accept λ, there exist open sets of diameter D which is arbitrarily small such that $D \not\subset A \in O$. Particularly, for each $n \in \mathbb{N}$ $\exists$ x_n such that $B\left(x_n, \frac{1}{n}\right) \not\subseteq A, \forall A \subset 0$. Due to sequentially compactness, the sequence $\{x_n\}$ has x as its accumulation point. Provided that O cover λ, we have $x \in A$ for some $A \in O$. Since A is open, $\exists$ rarius $r > 0$ such that the metric $d(x, x_n) < \frac{r}{2}$ and $\frac{1}{n} < \frac{r}{4}$. Assume $q \in B\left(x_n, \frac{1}{n}\right)$, then by triangle inequality, $d(q, x) \le d(q, x_n) + d(x_n, x) < \frac{1}{n} + \frac{r}{2} < \frac{r}{4} + \frac{r}{2} = \frac{3}{4} r < r$. Therefore, the open ball $B\left(x_n, \frac{1}{n}\right) \subseteq A$, which is contrary to the choice of the sequence $\{x_n\}$. This end the proof.

Definition 2.2: [7] A topological space X is pseudocompact, if every continuous real – valued function on X is bounded.

Theorem 2.13: Every pseudocompact metric space X is countably compact.
Pseudocompactness $\Longrightarrow$ Countably compactness

Proof: Suppose the implication above is not true, then there exist a sequence $A = \{a_n\}$ which is not converge. Hence A is closed and also discrete. Therefore, the function $f: A \longrightarrow \mathbb{N}$ defined by $f(a_n) = n$ is continuous, so by Tietze extension theorem, this function can be extended to the space X. But due to the fact that $f(a_n) = n$ is continuous, then, is not bounded which is contrary to pseudocompactness notion, which proved the theorem. The converse of this is also true in topological spaces.

Topological Spaces

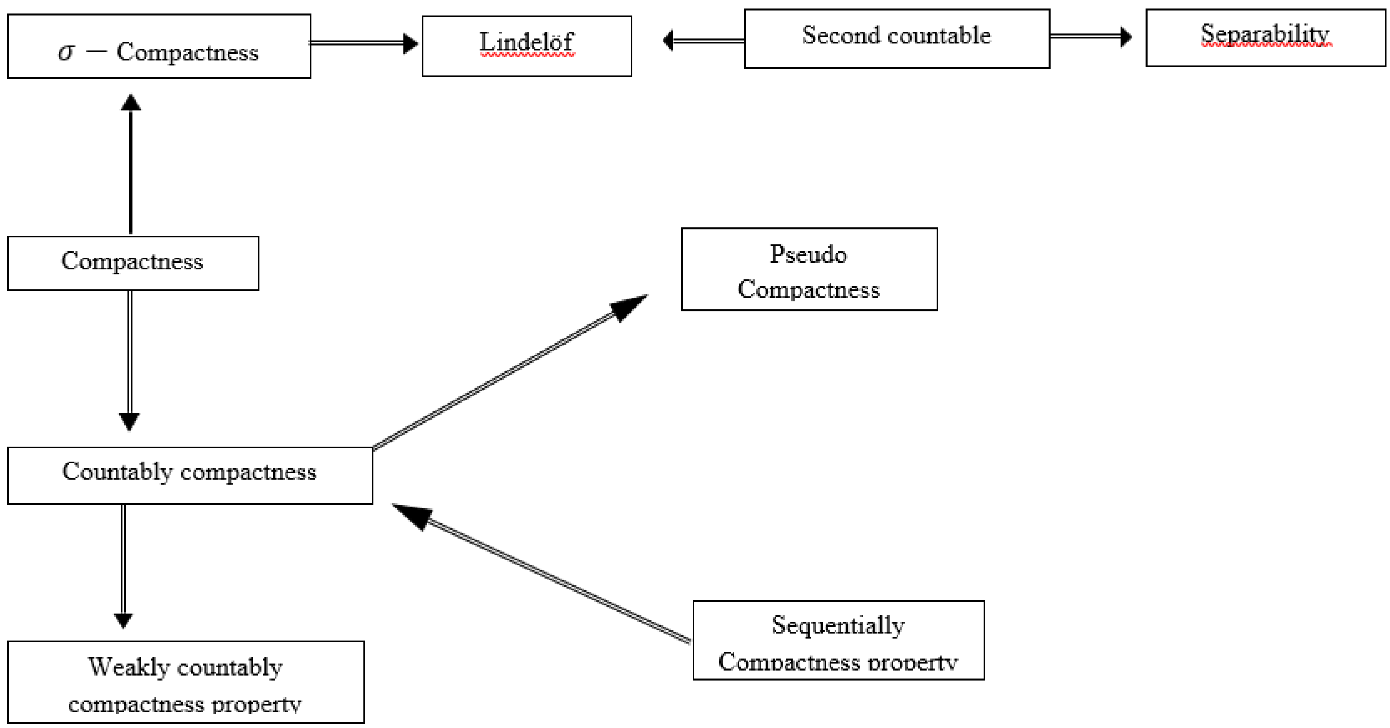


Fig. A

Metric Spaces

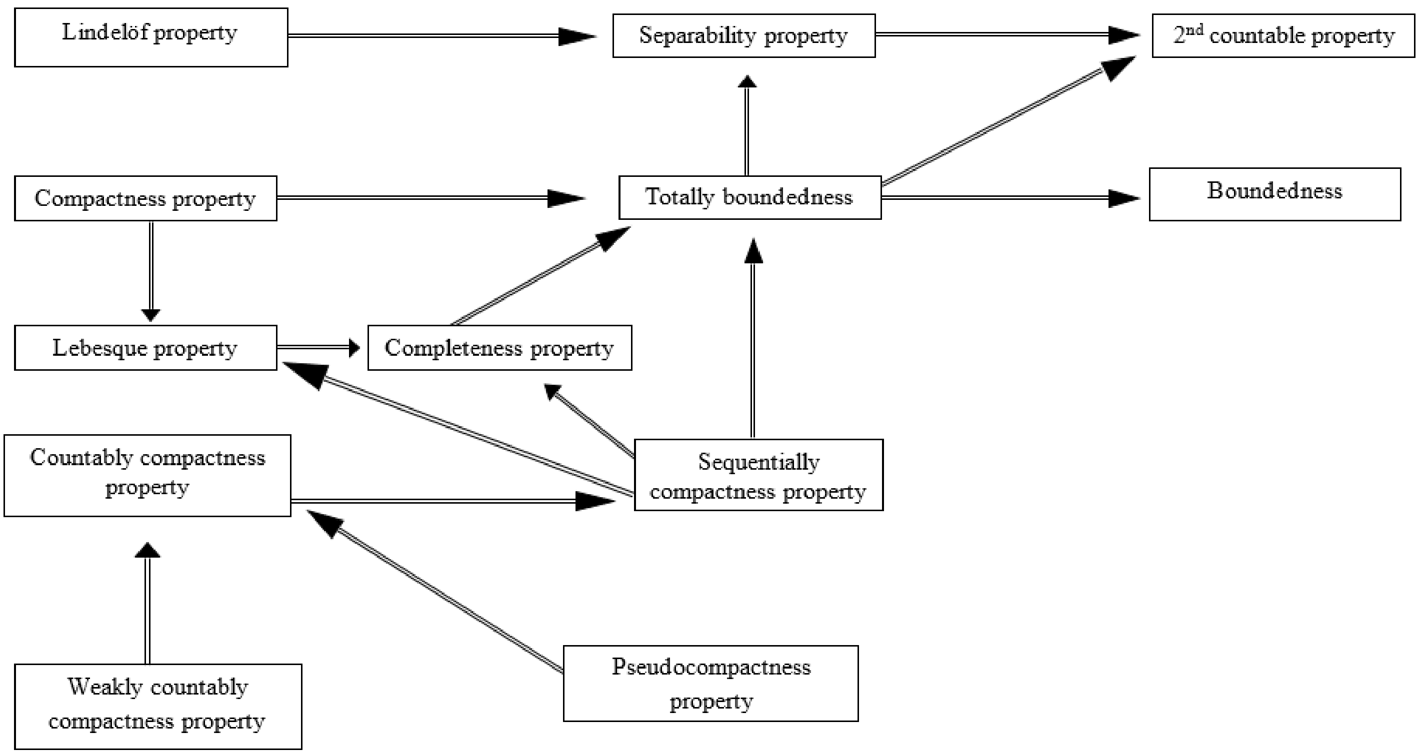


Fig. B

III. Conclusion

In this paper, we noticed that there are some compactness properties and some of its relative notions that satisfied in both topological spaces and metric spaces. But out of these properties, there are some that satisfied in arbitrary topological spaces and metric spaces only and some on metric topological spaces.

The figure A and B above provides us those properties that satisfied in topological spaces and in metric spaces respectively. By comparing these two figures, we conclude that:

(i) Completeness, Lebesgue, Boundedness, Total boundedness are not consider in arbitrary topologicsl spaces, this shows that they are properties of metric spaces

(ii) While some other properties mentioned above like σ – compactness, compactness, second countability, separability, Lindelöf, etc. Plays roles in both metric and topological spaces.

(iii) Converse of **Theorem 2.6** does not hold in topological spaces. Also It does not holds in metric spaces as shown in Fig A and B above.

(iv) Converse of **theorem 2.7** does not holds in metric space but holds in topological spaces as in **Fig A and B** above.

(v) Converse of **theorem 2.9, 2.10, 2.11 and 2.12** does not holds.

(vi) Converse of **theorem 2.13** holds in topological spaces.

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