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Study of linear buckling analysis on curvilinear fiber composites subjected to uniaxial compressive load

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Abstract- In modern days, composite materials play a major role in many applications specifically in commercial aircraft due to its high specific structural properties and high corrosion resistance as well as being relatively lightweight compared to their metal counterparts. These features that composite materials possess facilitate the designers to come up with various possible designs with high structural efficiency. Subsequently, numerous alternatives in varying fiber orientations have been studied over the years in order to customize in-plane stiffness which leads to enhanced structural performance, especially buckling performance. This paper presents a finite element model of conventional straight fiber and curvilinear fiber composites with respect to compressive load considering different boundary conditions. The variation of orientation variation and buckling load will be investigated for curvilinear fiber composites. And finally the results will compared with the conventional straight fiber composites.

Index Terms- Composite materials, curvilinear fiber, finite element, linear buckling analysis

I. Introduction

Over the years thin plates have been used vastly in aviation and naval applications [1]. And in many cases, these plates usually have to undergo compressive load and this specific condition could potentially lead to buckling phenomenon. Buckling is a phenomenon of change of structural geometry when subjected to load, subsequently a buckled structure becomes ineffective to bear loads. Various studies have been conducted to investigate buckling phenomenon due to its complexity [2-8]. Buckling starts to develop even when the stresses over the structure are below its failure point. Although buckling itself does not necessarily mean as a complete collapse, further loading beyond buckling point might cause startling structural deformation and eventually failure due to loss of load bearing capability. It is also commonly found that elastic instability contributes more to failure of thin plates than the lack of the strength of the plates itself. In aviation industry, many components are slender and thin, naturally it makes buckling analysis necessary in the general analysis of the structure.

II. The Theory of Plate Stability

This section provides brief explanation of the classical theory of stability analysis of thin plates. In addition, the formulation for stability of thin plates has some qualitative similarities with column stability that was introduced by Euler.

In this paper, an ideal thin elastic rectangular composite plate is considered. It is assumed that the plate is perfectly flat in its initial state and subjected to external uniaxial in-plane compressive load. Important to note that the material used is composite materials rather than isotropic materials, thus the formulation would differ from buckling of isotropic plates. Based on the derivation of Von Karman equation, the governing equation of composite plates under uniaxial compressive load could be expressed as:

$$D_{11}\frac{\delta^4 w}{\delta x^4} + 2(D_{12} + 2D_{66}) + D_{22}\frac{\delta^4 w}{\delta y^4} = N_x\frac{\delta^2 w}{\delta x^2}$$

Whereas w being the out-of-plane displacement and N_x is the compressive load applied in x axis. It is worth noting that the governing equation considers an assumption of the variables D_{16} and D_{26} being negligible compared the other variables in the bending-twisting coupling matrix (D_{11}, D_{12}, D_{22} and D_{66}). In order to solve the critical buckling load, solving the nontrivial solution for w that satisfies specific boundary conditions is necessary.

$$w = \sum\sum A_{mn}\, sin\frac{m\pi x}{a}\, sin\frac{\lambda n\pi x}{b}$$

The value of λ is chosen based on the boundary conditions. For instance, λ=1 represents simply supported conditions. Then substitute the equation into the governing equation, rearrange the equation and take aspect ratio of the plate into consideration will give us the critical buckling load of the plate.

Using similar approach as presented equations previously would help figure out the buckling load of rectangular plates with different boundary conditions. In this study itself, the author would consider a simulation study with four boundary conditions; all simply supported edges, simply supported and clamped loaded edges, simply supported and clamped non loaded edges and simply supported with one free edge, which would be presented in the following section.

III. PROBLEM DESCRIPTION

Buckling load of rectangular composite plates

As mentioned previously, four boundary conditions would be considered in this study. The idea is to firstly solve the buckling load of the composite plates using analytical method and afterwards compare the analytical results with the finite element analysis software to ensure accountability of the results. Based on section 6.6 of the reference textbook [9], critical buckling load (N_{cr}) could be solved numerically using these following formulae.

1. Simply supported edges

$$N_{cr} = \frac{\pi^2[D_{11}m^4 + 2(D_{12} + 2D_{66})m^2(AR)^2 + D_{22}(AR)^4]}{a^2m^2}$$

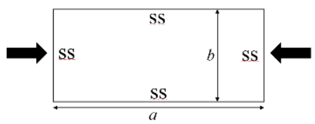

Fig. 1. Plate boundary condition (BC1)

2. Simply supported and clamped on loaded edges

$$N_{cr} = \frac{\pi^2}{b^2}\sqrt{D_{11}D_{22}}(K)$$

$$K = \frac{4}{\lambda^2} + \frac{2(D_{12} + 2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{3}{4}\lambda^2 \qquad (0 < \lambda < 1.662)$$

$$K = \frac{m^4 + 8m^2 + 1}{\lambda^2(m^2 + 1)} + \frac{2(D_{12} + 2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{\lambda^2}{m^2 + 1} \qquad (\lambda > 1.662)$$

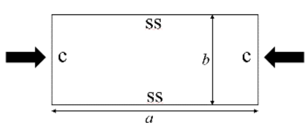

Fig. 2. Plate boundary condition (BC2)

3. Simply supported and clamped on unloaded edges

$$N_{cr} = \frac{\pi^2}{b^2}\sqrt{D_{11}D_{22}}(K)$$

$$K = \frac{m^2}{\lambda^2} + \frac{2(D_{12} + 2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{16}{3}\frac{\lambda^2}{m^2}$$

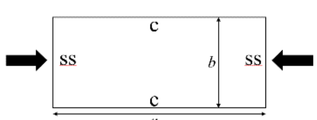

Fig. 3. Plate boundary condition (BC3)

4. Simply supported with one free edge

$$N_{cr} = \frac{\pi^2}{b^2}\sqrt{D_{11}D_{22}}(K)$$

$$K = \frac{12}{\pi^2}\frac{D_{66}}{\sqrt{D_{11}D_{22}}} + \frac{1}{\lambda^2}$$

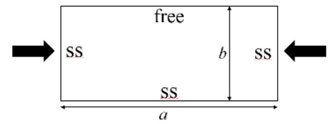

Fig. 4. Plate boundary condition (BC4)

AR refers to the aspect ratio (*a*/*b*) of the plate meanwhile the corresponding value of *K* can be obtained by

$$\lambda = \frac{a}{b}\left(\frac{D_{22}}{D_{11}}\right)^{1/4}$$

In order to simplify the names, simply supported edges will be called BC1; simply supported and clamped on loaded edges will be called BC2; simply supported and clamped on unloaded edges will be called BC3 and simply supported with one free edge will be called BC4.

In this paper, rectangular composite plate of 200 mm length and 100 mm width (AR=2) is considered. For the material, carbon fiber reinforced polyether ether ketone (CF / PEEK) is chosen. The following table shows the material elastic properties.

E1 (MPa)	135000
E2 (MPa)	7540
Nu12	0.3
G12 (MPa)	5000
G13 (MPa)	5000
G23 (MPa)	5000

Table 1. Material elastic properties

The chosen stacking sequence for the laminate is $[45/-45/0/90]_s$ and the ply thickness is 0.125 mm. Total thickness would be 1 mm, relatively thin enough to avoid the significance of transverse shear on the laminate.

Subsequently analytical approach of the buckling load of rectangular composite plate with different boundary conditions were performed and the obtained buckling loads are 20.94 kN (BC1), 22.9682 kN (BC2), 38.703 kN (BC3) and 3.634 kN (BC4).

Finite element model

ABAQUS software was used in order to simulate the linear buckling analysis of the composite plate. S4R reduced integration elements were used to represent the discretized elements and 1600 elements were considered in this simulation. subspace eigensolver was chosen to obtain eigenvalues and buckling shape of the structure. Naturally, the least eigenvalue of each simulation would be the main interest since this would give the “worst” buckling strength as a verification of the structure.

The results of linear buckling load of composite plate with four different boundary conditions using ABAQUS solver are compared with analytical solution and the results are shown in table 2 and buckling shapes obtained for BC1 to BC4 respectively from ABAQUS are depicted in fig.5 – fig.8.

	Analytical (kN)	FEA (kN)	*Diff*
BC1	20.94	21.107	0.79
BC2	22.968	23.356	1.69
BC3	38.703	39.499	2.05
BC4	3.614	3.522	-2.54

Table 2. Buckling load result comparison

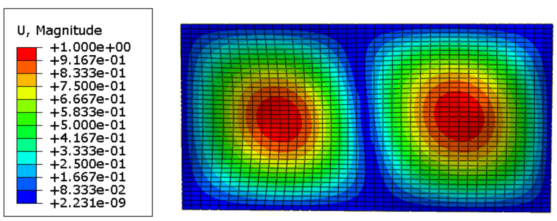

Fig. 5. Buckling shape of BC1

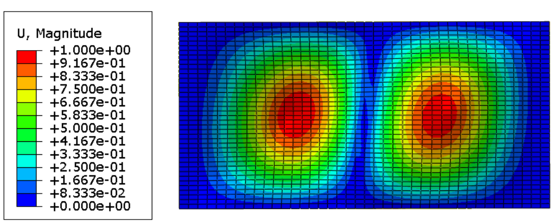

Fig. 6. Buckling shape of BC2

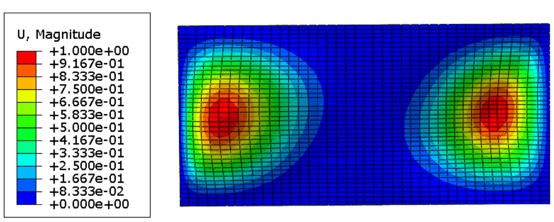

Fig. 7. Buckling shape of BC3

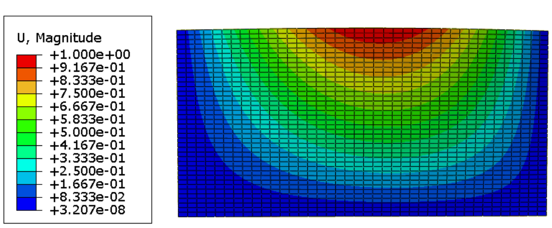

Fig. 8. Buckling shape of BC4

IV. Curvilinear Fiber Composites

Curvilinear fiber composites introduction

For conventional straight fiber composite material manufacturing, ply stacking method has been used over the years in order to achieve desired structural stiffness. By understanding the loadings that the structure might experience would help the designers design the laminates that suits the requirements. Correspondingly, composite materials have been rapidly developed due to its promising features, various alternatives have been studied in order to improve composite materials in structural design. One of the alternatives is curvilinear fiber composites, sometimes also called variable angle tow (VAT) composites. This approach allows the fiber orientation to vary within the plate. Subsequently this would lead to in-plane stiffness variation, which could be beneficial depending on the situation.

Biggers [10,11] studied composite plates using a unique approach. Their approach used a distinctive stiffening patterns of the fibers of the composite plates by manipulating the of fiber distribution. This customized fiber tailoring approach let the fibers act like local stiffeners. These composite plates later were subjected to compressive and shear load. The results were satisfactory, without sacrificing weight efficiency and changing the average of in-plane stiffness, buckling load of the plates could be enhanced. Nagendra et al. [12] carried out research to develop an optimization method of tow paths using NURBS (non-uniform rational B-splines). A single curve through a set of fixed control points based on a single cubic were implemented for the tow paths to pass. They studied optimal frequency and buckling load design of laminated composite plates with a central hole subject to deformations, ply failure and inter-laminar stress constraints using finite element analysis which eventually their experiment demonstrated an increased performance. Meanwhile Honda et al. [13] used linear combined B-spline functions to define the fiber paths. Parnas et al. [14] expressed the trajectories of the fiber in terms of either Bezier curves or cubic polynomials. On the other hand, Olmedo and Gürdal et al. [15,16] proposed a simple definition of linear variation of fiber paths identification by using three independent angles to characterize each VAT ply. Although in the beginning had a limitation due to the fiber variation only in one axis, later on other studies were conducted to expand the method by allowing the fiber orientation to vary in two dimensions. This specific method has been vastly used in curvilinear fiber composite plate studies over the years. In this paper, the presented plate would use their approach of linear fiber variation as well.

Model parameterization

As mentioned before, linear function that was proposed by Gürdal and Olmedo [15,16] would be used to carry out this study and it is expressed in this following equation

$$\theta(x) = \frac{2(T_1 - T_0)}{a}|\tilde{x}| + T_0$$

With a being the representation of the length of the panel whereas T_0 is the angle of fiber trajectory at the center of the panel ($\tilde{x} = 0$) and T_1 is the angle of fiber trajectory at the end of the panel ($\tilde{x} = \pm a/2$). Therefore, a variable angle tow composite lamina could be represented by the notation of $\langle T_0|T_1\rangle$ and the corresponding negative ply would be referred as $-\langle T_0|T_1\rangle$.

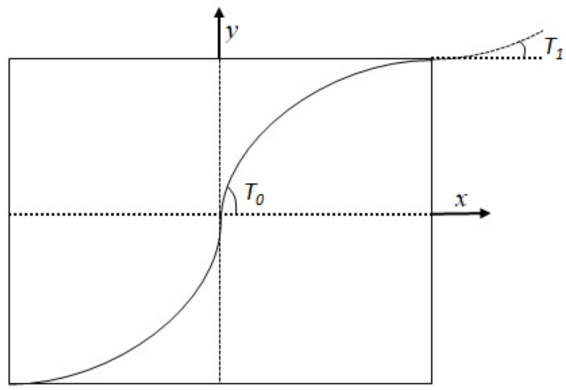

Fig. 9. Reference of fiber trajectory

Furthermore, when T_0 is equal to T_1, it indicates that the fiber orientation is constant within the lamina. In addition the reference of fiber trajectory could be defined using these following equations

$$y = \frac{a}{2(T_1 - T_0)}\left\{-\ln\left[\cos\left(T_0 + \frac{2(T_1 - T_0)\tilde{x}}{a}\right)\right] + \ln[\cos T_0]\right\}$$

$$for\ \ 0 \le \tilde{x} < \frac{a}{2}$$

$$y = \frac{a}{2(T_1 - T_0)}\left\{\ln\left[\cos\left(T_1 + \frac{2(T_1 - T_0)\tilde{x}}{a}\right)\right] - \ln[\cos T_1]\right\}$$

$$for\ -\frac{a}{2} \le \tilde{x} < 0$$

Python script was used in order to create a composite plate then mesh will be generated and ABAQUS S4R shell element is used. Parameter of the model are established within the code, this will also help further parametric studies in the future of related topic. Firstly the center of each element would be calculated by the coordinates of each element nodes and the connectivity between nodes is ensured. Then fiber angle would be mapped according to the fiber trajectory definition in the above equation then the data of varying angles will be stored in discrete field in ABAQUS.

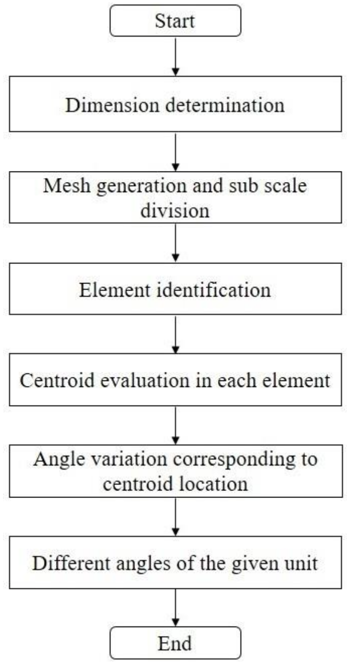

Fig. 10. Modeling steps of curvilinear fiber design

Curvilinear fiber composites utilization

In this section, parametric study will be done to observe the effect of varying the fiber orientation towards the buckling load. Plies with certain angle will be modified in order to see enhancement possibilities in the buckling load under uniaxial compressive load. 45, -45 and 90 degrees plies will be replaced with curvilinear fiber composites. Furthermore, the simulation will be divided into two groups, replacing 45 and -45 plies while maintaining the 90 degree ply and vice versa. T_1 will remain constant (45, -45, 90 and -90) meanwhile T_0 will vary within some certain range. Important to note that -90 degrees will be considered due the variation of the fiber orientation in negative direction in order to balance with the 90 degree counterpart. The following table 3 and 4 show the buckling loads of modified laminates.

T_0 (°)	Buckling load (kN)			
	BC1	BC2	BC3	BC4
60	19.609	23.688	41.004	3.653
58	19.840	23.67	41.046	3.654
56	20.065	23.64	41.011	3.649
54	20.283	23.609	40.9	3.638
52	20.491	23.571	40.714	3.622
50	20.687	23.525	40.375	3.6
40	21.410	23.09	38.155	3.411
38	21.442	22.949	37.523	3.357
36	21.605	22.788	36.844	3.3
34	21.616	22.604	36.123	3.237
32	21.625	22.395	35.366	3.17
30	21.612	22.159	34.58	3.1

Table 3. Buckling load result (first group)

T_0 (°)	Buckling load (kN)			
	BC1	BC2	BC3	BC4
80	21.279	23.552	39.471	3.517
70	21.8	24.163	39.402	3.5
60	22.716	25.299	39.334	3.473
50	23.84	26.807	39.315	3.439
45	24.39	27.569	39.327	3.421
40	24.978	28.472	39.407	3.403
30	25.886	29.9	39.637	3.372
20	26.433	30.784	39.967	3.35
10	26.641	31.098	40.312	3.34
0	26.635	31.043	40.586	3.328

Table 4. Buckling load result (second group)

88 models were observed in this section. As can be seen that the variation of fiber orientation affect the buckling load differently in four different boundary conditions. Variation of 90 degree ply for BC1 and BC2 cases show major improvement while the modification of 45 degree ply seem only improve the buckling performance by a small margin. And for BC3, varying in both 45 and 90 plies also enhanced the buckling load but not as distinct as in BC1 and BC2. Variation for 45 degree ply in BC4 also shows improved buckling performance. In general, varying the fiber angle could improve the buckling load. However, the variation of 90 degree ply in BC4 seems to be ineffective compared to the others. Correspondingly, it is worth noting that the purpose of this paper is not to show optimization method but rather show the potential of customized fiber tailoring approach. In conclusion, via this approach the buckling load could possibly be improved without having to sacrifice the structural weight. Structural strength could be improved by manipulating the fiber orientation in a certain manner while at the same time maintaining the same material efficiency, this definitely expands the design space for composite laminates manufacturing. Subsequently, table 5 shows the best stacking sequence obtained from the simulation result and their buckling load improvement compared to the initial stacking sequence.

BC	Stacking seq.	Buckling load diff. (%)
BC1	$[\pm\langle 32\|45\rangle/0/90]_s$	2.45
	$[\pm 45/0/[\pm\langle 10\|90\rangle/0/\pm 45]$	26.22
BC2	$[\pm\langle 60\|45\rangle/0/90]_s$	1.42
	$[\pm 45/0/[\pm\langle 10\|90\rangle/0/\pm 45]$	33.15
BC3	$[\pm\langle 58\|45\rangle/0/90]_s$	3.92
	$[\pm 45/0/[\pm\langle 10\|90\rangle/0/\pm 45]$	2.75
BC4	$[\pm\langle 60\|45\rangle/0/90]_s$	3.75
	none	*none*

Table 5. New laminates and their buckling loads

V. Conclusion

Finite modeling of composite plate was presented in this study. 4 initial models of conventional straight fiber composites with various boundary conditions were proposed and their buckling loads were obtained by analytical solution and ABAQUS software, both results were compared in order to investigate the accountability of the results. 88 models in total were simulated afterwards to see whether the variation of fiber orientation could improve the buckling performance of the plate. Simulation was divided into 2 groups, variation of 45 degree ply and 90 degree ply. In most cases, enhanced buckling performance could be seen, especially variation of 90 degree ply in BC2 and BC3 cases meanwhile the others were improved in relatively small margin and variation of 90 degree ply in BC4 seems to be not effective for this case study. However, the essence of this study is to show that the varying fiber angle approach could improve the structural performance without having to sacrifice the weight aspect and material efficiency. This also means more freedom in the design of composite materials.

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