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Design of Nonlinear Controller and Analysis of Nonlinear Phenomena in Non-Minimum Phase DC-DC Switched Mode Converter

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Abstract- This paper provides design and implementation details of sliding mode controller for buck-boost converter in continuous conduction mode (CCM). This paper also looks in to the nonlinear dynamics of the power converter such as chaos and bifurcation in particular. MATLAB-Simulink based simulation has been carried out to illustrate the simulation results.

Index Terms- *Buck-Boost converter; sliding mode controller; chaos; bifurcation*

I. Introduction

Switched mode power converter (SMPC) plays a vital role in different applications ranging from distributed generation, electric vehicle, portable electronic products and consumer electronics. SMPC is a nonlinear electronic circuit whose dynamics change with the change in the position of the switch. There are numerous topologies of SMPC and boost and buck-boost converter are some widely used converter topology. The output voltage to duty cycle transfer function of boost and buck-boost converter exhibit non-minimum phase behavior which means a zero in the right half of s-plane. There are several ways to mitigate the non-minimum phase behavior of the converter.

The main aim of switched mode power converter is to either step-down or step-up the unregulated DC voltage and provide regulated DC voltage. There are different controllers used for controlling the DC voltage of switched mode power converter. The controllers can be classified as linear controller or nonlinear controller. In linear controller, classical PID controller are good option but the PID controller fails to control the system with large signal disturbance. Therefore, nonlinear controllers are used. Sliding mode control is one of the widely used control scheme used for control of DC-DC converter as the controller is robust to change in parameters.

Mathematical model of buck-boost converter in transient state has been formulated using Laplace transform and z-transform in [1]. Buck-boost converter is a non-minimum phase system because the transfer function of buck-boost converter possess a right half plane zero (RHPZ) in transfer function. Due to such characteristics if the duty cycle of the converter crosses 0.5 in peak current mode control, harmonic oscillations are created which leads to chaos. The complete analysis of bifurcation and chaos of buck-boost converter has been reported in [2,5,12,14]. For wide range of operation of buck-boost converter, double loop control is used where current loop is considered as the inner loop and voltage loop is considered as the output loop [3]. Robust control of buck-boost converter is presented in [4]. Adaptive back-stepping control of buck-boost converter is presented in [6]. Cascaded PID control of buck-boost converter is studied in [7]. PWM based sliding mode control of DC-DC converter operating in discontinuous conduction mode (DCM) has been studied in [8]. There are different ways to control the chaotic behavior of DC-DC converter which has been presented in [10]. Period-doubling bifurcation of DC-DC converter has been reported in [11]. Current controlled Buck-Boost converter with ramp compensation operating in DCM has been studied in [13]. Discrete time map for analysis of bifurcation and chaos in DC-DC converter is reported in [15]. Maity*et al.*, studied the bifurcation of voltage mode controlled Buck converter using exact discrete model [16].

From the above discussion it can be observed that the nonlinear analysis and nonlinear controller design has been an emerging area of research for power electronics professional. This paper attempts to provide a detailed analysis of nonlinearity of SMPC and aspects of nonlinear controller design. This paper provides design details of sliding mode controller for buck-boost converter and provides details of nonlinear dynamics of buck-boost converter.

This paper is organized as follows. Section II provides mathematical model of buck-boost converter (state-space average model and discrete-time model). Section III provides sliding mode control of buck-boost converter under CCM and DCM. Section IV provides preliminary concepts of bifurcation and chaos. Section V provides nonlinear phenomena of buck-boost converter. Section VI presents simulation results. Section VII concludes the paper.

II. Mathematical Model of Buck-Boost Converter

The circuit diagram of buck-boost converter is shown in Figure 1.

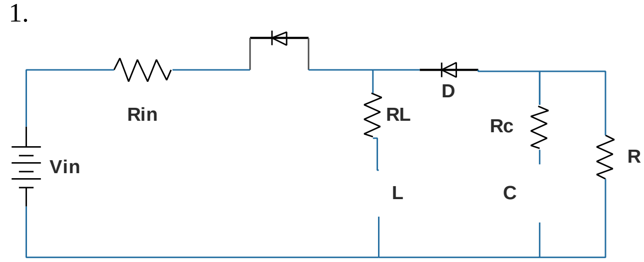


Fig. 1. Circuit diagram of buck-boost converter

The voltage gain of Buck-Boost Converter is $\frac{V_o}{V_{in}} = \frac{-d}{1-d}$

The current ripple of Buck-Boost Converter is $\frac{\Delta I_L}{I_L} = \frac{(1-d)^2 RT_s}{L}$

The voltage ripple of Buck-Boost converter is $\frac{\Delta V_o}{V_o} = \frac{dT_s}{RC}$

The converter has two states i.e ON state and OFF state. The dynamics of converter in these states can be represented as

ON State $\begin{cases} \dot{x} = A_1 x + B_1 u \\ y = C_1 x + D_1 u \end{cases}$

OFF State $\begin{cases} \dot{x} = A_2 x + B_2 u \\ y = C_2 x + D_2 u \end{cases}$

Here the state variables are defined as

$$x = \begin{bmatrix} i_L \\ V_c \end{bmatrix} \; u = V_{in} \; y = V_o$$

$$A_1 = \begin{bmatrix} \frac{R_{in} + R_L}{L} & 0 \\ 0 & -\frac{1}{C(R+R_c)} \end{bmatrix} B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & \frac{R}{R+R_c} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{-R_L + (R \| R_c)}{L} & \frac{-R}{L(R+R_c)} \\ \frac{R}{C(R+R_c)} & -\frac{1}{C(R+R_c)} \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -R \| R_c & \frac{R}{R+R_c} \end{bmatrix}$$

The dynamics of ON state can be represented as

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} i_L \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{-R_{in} + R_L}{L} & 0 \\ 0 & \frac{1}{C(R+R_c)} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in} \\ V_o = \begin{bmatrix} 0 & \frac{R}{R+R_c} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} \end{cases}$$

The dynamics of OFF state can be represented as

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} i_L \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{-R_L + R \| R_c}{L} & \frac{-R}{L(R+R_c)} \\ \frac{R}{C(R+R_c)} & \frac{-1}{C(R+R_c)} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_{in} \mathrm{T} \\ V_o = \begin{bmatrix} -R \| R_c & \frac{R}{R+R_c} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} \end{cases}$$

he state-space average method can be represented as

$$\begin{cases} \dot{x} = \left(A_1 d + (1-d) A_2\right) x + \left(B_1 d + (1-d) B_2\right) u \\ y = \left(C_1 d + (1-d) C_2\right) x + \left(D_1 d + (1-d) D_2\right) u \end{cases}$$

From the small-signal model, the transfer function of buck-boost converter can be derived.

$$\frac{i(s)}{V_{in}(s)} = \frac{1}{R(1-D)^2} \left(\frac{1 + sCR}{1 + \frac{Ls}{R(1-D)^2} + \frac{LCs^2}{(1-D)^2}} \right)$$

Control current gain of buck-boost converter

$$\frac{i(s)}{d(s)} = \frac{V_{in}(1+D)}{R(1-D)^3} \left(\frac{1 + \frac{CR}{1+D} s}{1 + \frac{Ls}{R(1-D)^2} + \frac{LCs^2}{(1-D)^2}} \right)$$

Audio susceptibility of buck-boost converter

$$\frac{V_o(s)}{V_{in}(s)} = \frac{-D}{(1-D)} \left(\frac{1}{1 + \frac{Ls}{R(1-D)^2} + \frac{LCs^2}{(1-D)^2}} \right)$$

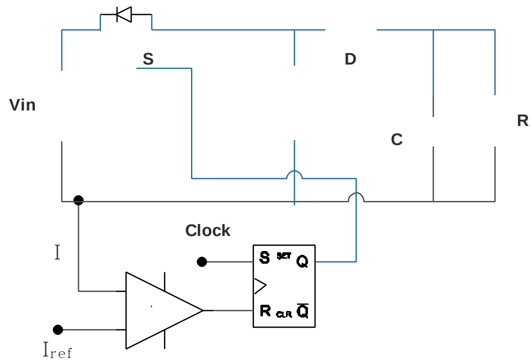
Control voltage gain of buck-boost converter

$$\frac{V_o(s)}{d(s)} = \frac{-V_{in}}{(1-D)^2} \left(\frac{1 - \frac{LD}{R(1-D)^2} s}{1 + \frac{Ls}{R(1-D)^2} + \frac{LCs^2}{(1-D)^2}} \right)$$

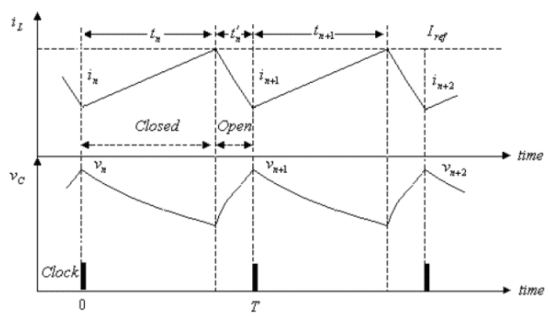
III. Controller Design for DC-DC Buck-Boost Converter

The basic control scheme for DC-DC buck-boost converter is voltage mode controller. But the voltage mode controller has different limitations such as (a) poor line and load regulation, (b) poor current control and (c) slower transient response.

Therefore, current mode control scheme is used. In current mode control scheme, the inductor current is measured and the current is compared with a reference current to derive PWM signal for the switch. Fig. 2(a) shows the current mode control (CMC) of Buck-Boost converter and Fig. 2(b) shows the waveform of CMC.



(a)



(b)

Fig. 2. (a) Buck-Boost converter in current mode control, (b) Output voltage and current of converter in current mode control

Sliding Mode Controller

Sliding mode control is a particular type of the variable structure control system (VSCS), which is characterized by a discontinuous feedback control structure that switches as the system crosses certain manifold in the state space to force the system state to reach, and subsequently to remain on a specified surface within the state space called sliding surface. The switching function (sliding variable) is a function of the states and the sliding surface represents a relationship between the state variables. The system dynamics when confined to the sliding surface is referred as an ideal sliding motion and represents the controlled system behaviour, which results in reduced order dynamics with respect to the original plant.

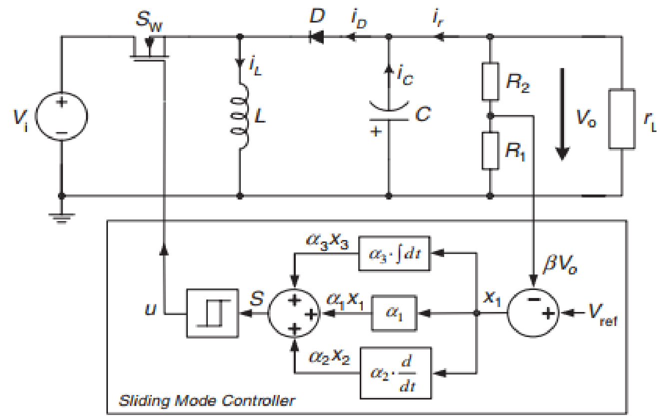


Fig. 3. Fixed-frequency sliding mode control of buck-boost converter under CCM

Fig. 3 shows the fixed-frequency sliding mode control of buck-boost converter under CCM. In the sliding mode control, three state variables are considered such as (a) voltage error, (b) derivative of voltage error and (c) integration of voltage error. The state-variables for the sliding mode control can be defined as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} V_{ref} - \beta v_o \\ \dfrac{d}{dt}\left(V_{ref} - \beta v_o\right) \\ \int \left(V_{ref} - \beta v_o\right) dt \end{bmatrix}$$

The above equation can be further simplified as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} V_{ref} - \beta v_o \\ \dfrac{\beta v_o}{r_L C} + \int \dfrac{\beta v_o u_B (1-u)}{LC} dt \\ \int \left(V_{ref} - \beta v_o\right) dt \end{bmatrix}$$

The time differentiation of above equation can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\dfrac{1}{r_L C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\beta v}{LC} u_B - \dfrac{\beta v}{LC} u_B \\ 0 \end{bmatrix} \bar{u}$$

The switching function of generalized sliding mode control can be represented as $u = \begin{cases} 1 & S > 0 \\ 0 & S < 0 \end{cases}$

Where $S = \sum_{i=1}^{3} \alpha_i x_i = J^T x$

Here $J^T = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}$

To ensure existence condition, following reachability condition must be satisfied $\lim\limits_{S \to 0} S\dot{S} < 0$

$$\frac{dS}{dt} = J^T Ax + J^T B\bar{u}_{eq}$$

Taking $\frac{dS}{dt} = 0$, we get

$$\bar{u}_{eq} = -\left[J^T B\right]^{-1} J^T Ax$$

Substituting the values, we get

$$\bar{u}_{eq} = \frac{\beta L}{\beta V_o u_{B_{eq}}}\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_c - \frac{\alpha_3 LC}{\alpha_2 \beta V_o u_{B_{eq}}}\left(V_{ref} - \beta V_o\right)$$

IV. Bifurcation and Chaos

A sudden change in the qualitative behavior of a system is termed a bifurcation. Successive bifurcations lead to instability in power electronic converters. The type of bifurcation is classed by the qualitative change that takes place when a parameter is varied. How far a system is from instability is termed the stability margin. There are two metrics to consider; the phase margin and the gain margin.

Bifurcations that occur in power electronic converters are typically classed as standard (smooth) bifurcations or non-standard (non-smooth) bifurcations. Smooth bifurcations do not involve any structural change associated with the loss in stability. In continuous time systems, they occur when the real part of one, or more, of the eigenvalues of the system is greater than zero. In discrete-time systems, they occur when the magnitude of the eigenvalue is greater than 1.

Typically, for power electronic converters, smooth bifurcations can be classified into two categories; slow-scale bifurcations and fast-scale bifurcations which lead to slow scale instability and fast-scale instability. Typical types of bifurcations that occur in power electronic converters are Hopf bifurcations and period-doubling bifurcations. Non-smooth bifurcations do cause a structural change and are characterized by a sudden jump of the operating point. They occur due to interactions between system trajectories and state-space boundaries where the system switches from one configuration to another. Border collision bifurcations and grazing bifurcations are types exhibited by power electronic converters. Fig. 4 illustrate the classification of different bifurcation in nonlinear dynamics whereas Fig. 5 shows the different bifurcation diagram.

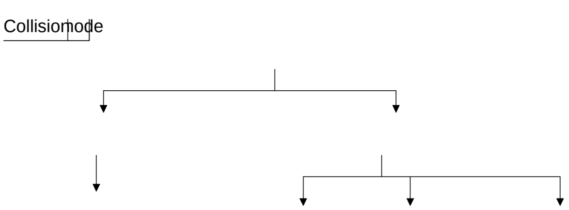


Fig. 4. Classification of Bifurcation

The equivalent control function is derived as

$$0 < u_{eq}^{*} = -\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_c - LC\frac{\alpha_3}{\alpha_2}\left(V_{ref} - \beta V_o\right) + \beta V_o u_{Beq} < \beta V_o u_{Beq}$$

The control signal and ramp signal are represented as

$$\begin{cases} V_c = -\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_L C}\right) i_c + LC\frac{\alpha_3}{\alpha_2}\left(V_{ref} - \beta V_o\right) + \beta V_o u_{beq} \\ V_{ramp} = \beta V_o u_{beq} \end{cases}$$

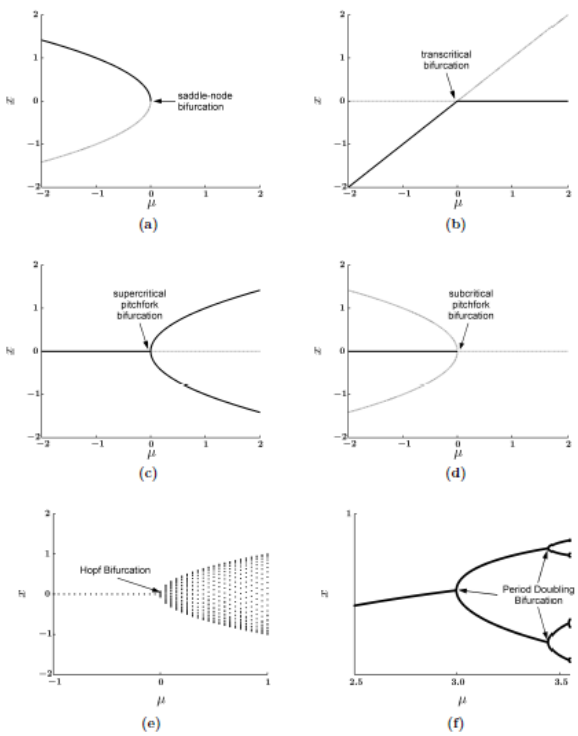


Fig. 5. (a) Saddle-node bifurcation, (b) transcritical bifurcation, (c) supercritical bifurcation (d) Super-critical pitch fork bifurcation (e) Hopf bifurcation (f) Period doubling bifurcation

Chaos occurs in nonlinear systems and results in the seemingly random movement of trajectories within a bounded state-space. A chaotic trajectory is unpredictable in the long term; knowing the trajectory now does not guarantee knowing where the trajectory will end up. This contradicts the definition of a deterministic system. However, deterministic systems can exhibit chaotic behavior. The key property of chaos is the sensitivity of nonlinear systems to initial conditions. Even a small error in specifying the initial conditions to a system can result in large differences in the output as time evolves. Hence, the long term predictability of a chaotic system is unpredictable in a practical sense. Consider a logistic map $x_{k+1} = rx_k\left(1 - x_k\right)$

Fig. 6 shows the bifurcation diagram of above logistic map.

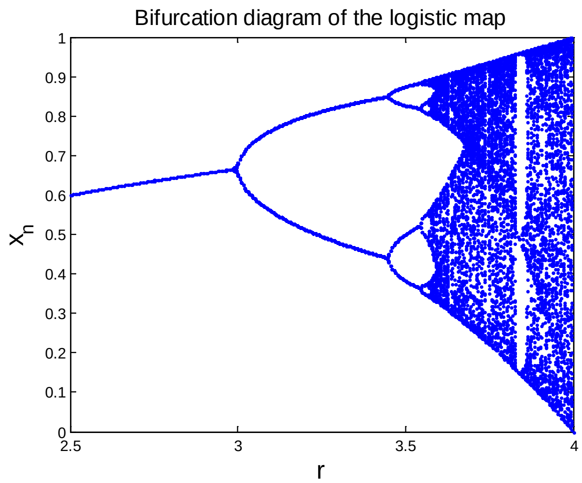


Fig. 6. Bifurcation diagram of logistic map function

V. Nonlinear Phenomena of Buck-Boost Converter

The difference equation of the converter can be represented by

$$x_{n+1} = f\left(x_n, \alpha\right), x \in R^N, \alpha \in R$$

Where $x = \left[i_L \quad V_c\right]^T$

When S is closed, $$\begin{cases} L\dfrac{di_L}{dt} = V_{in} & S = closed \\ C\dfrac{dv_c}{dt} = \dfrac{-v_c}{R} & S = closed \end{cases}$$

The solution of the above equation is $t_n = \dfrac{L\left(I_{ref} - i_n\right)}{V_{in}}$

The capacitor voltage at the said time can be represented as

$$V_c\left(t_n\right) = V_n \exp\left(\frac{-t_n}{RC}\right)$$

When S is open, $$\begin{cases} L\dfrac{di_L}{dt} + v_c = V_{in} & S = open \\ C\dfrac{dv_c}{dt} + \dfrac{v_c}{R} = i_L & S = open \end{cases}$$

From the above condition, the second order differential equation can be obtained as

$$\frac{d^2}{dt^2}\left(i_L\right) + \left(\frac{1}{RC}\right)\frac{d}{dt}\left(i_L\right) + \left(\frac{1}{LC}\right)\left(i_L\right) = \frac{V_{in}}{LCR}$$

The solution for above homogeneous equation can be represented as

$$\lambda_{1,2} = \frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

The final homogeneous solution can be represented as

$$i_L\left(t\right) = \exp\left(\frac{-t}{2\tau_{RC}}\right)\left(a_1 \sin \omega t + a_2 \cos \omega t\right) + \frac{V_{in}}{R}$$

$$\bar{t}_n = T\left(1 - \left(\frac{t_n}{T}\right)\right)$$

where

$$a_1 = \frac{\dfrac{L}{2\tau_{RC}}\left(I_{ref} - \dfrac{V_{in}}{R}\right) + V_{in} - v_n \exp\left(\dfrac{-t_n}{\tau_{RC}}\right)}{\omega L}$$

$$a_2 = I_{ref} - \frac{V_{in}}{R}$$

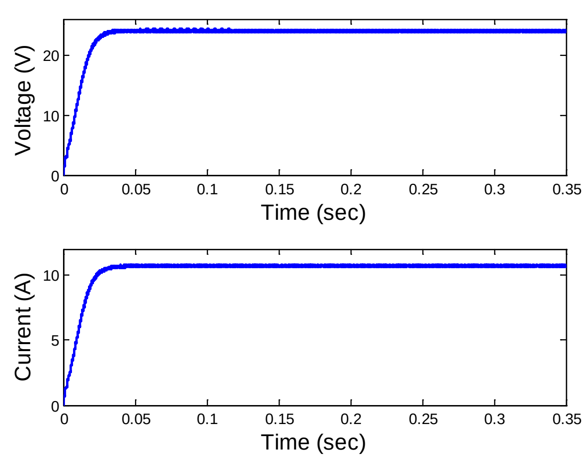
VI. Simulation Results

This section provides simulation results of the said controller and the converter. MATLAB-Simulink has been used to carry out the necessary simulations.

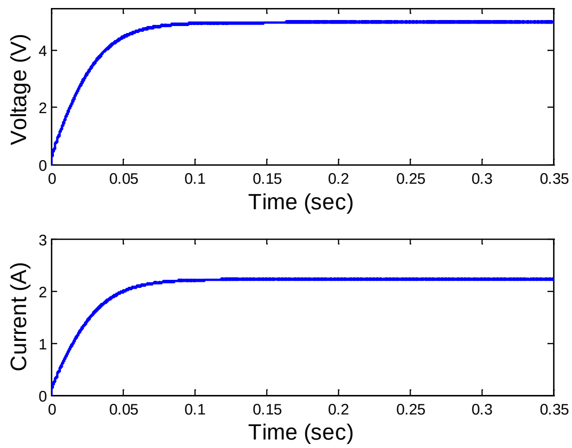
TABLE I. System Parameters

Parameters	Symbol	Value
Input Voltage	V_{in}	12V
Capacitance	C	700µF
Capacitor ESR	R_c	0.01Ω
Inductance	L	100µH
Inductor Resistance	R_L	0.12Ω
Switching Frequency	f_{sw}	200kHz
Load	R	2.25Ω to 5Ω
Desired Output Voltage	V_{ref}	24V (Boost), 5V (Buck)

Figure 7(a) and Figure 7(b) shows the voltage and current of buck-boost converter operating in CCM in boost mode and controlled using fixed-frequency sliding mode controller. Figure 8(a) and Figure 8(b) shows the voltage and current of buck-boost converter operating in CCM in buck mode and controlled using fixed-frequency sliding mode controller. The line regulation and the load regulation property of the sliding mode controller is shown in Table II and Table III.



(a)



(b)

Fig. 7. Output voltage and current of converter of buck-boost converter in (a) Boost mode (b) Buck mode

TABLE II. LINE REGULATION

INPUT VOLTAGE	VOLTAGE DEVIATION	% CHANGE
V_{in} = 20V	1.2V	2.5%
V_{in} = 26V	1.5V	1.8%
V_{in} = 28V	1.8V	1.5%

TABLE III. LOAD REGULATION

LOAD	VOLTAGE DEVIATION	% CHANGE
Minimum load	1.2V	1.8%
Half load	1.35V	1.6%
Full load	1.8V	1.2%

In nonlinear analysis, the effect of change of inductor current and the effect of change of input voltage has been shown in Figure 8 and Figure 9 respectively. The period doubling bifurcation behavior can be seen due to the change in inductor current and input voltage.

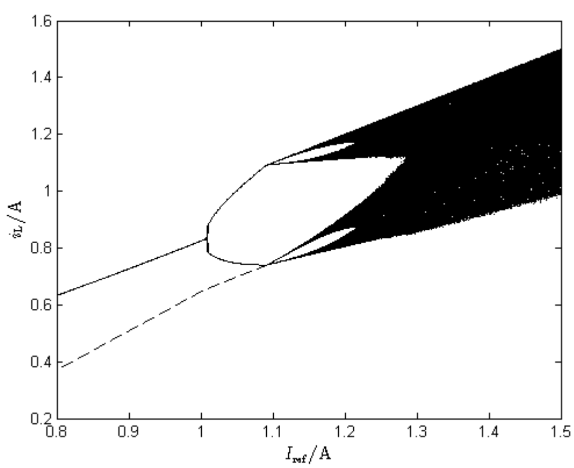


Fig. 8. Bifurcation behavior of buck-boost converter with variation of inductor current and reference current

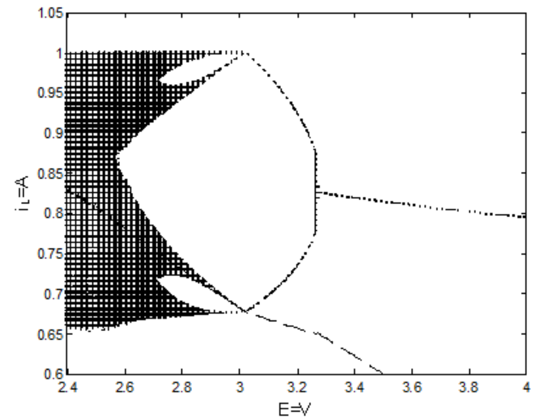


Fig. 9. Bifurcation behavior of buck-boost converter with variation of inductor current and voltage

VII. CONCLUSIONS

This paper provides sliding mode controller design for buck-boost converter. The sliding mode converter is used because it is faster than conventional PI type controller and easier to design and implement. As converter is a nonlinear entity, sliding mode controller is one of the best suitable choice for its control. Nonlinear dynamics such as chaos and bifurcation in switched mode power converter has also been analyzed in this paper.

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