Orbital (Mechanical, Kinetic, Potential) Energy in Planetary Orbits

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Abstract- The values of mechanical, potential, and kinetic energy were found for each planet. The orbit of the planet was treated as an ellipse, producing two values- an aphelion and perihelion value. The data from "Equations for Planetary Orbits" was used as the "measured" values and the data from NASA was used as the accepted values. The percent error between the two values was between 0.1% and 0.9%. An adjustment to the equations in "Equations for Planetary Orbits" showed a decrease in this error.

Index Terms- Aphelion, Circular Orbits, Elliptical Orbits, Kinetic Energy, Mechanical Energy, Perihelion, Potential Energy

I. INTRODUCTION

The orbital energy for the Planets in our solar system was always an area of interest for me. Therefore with the values from the paper "Equations for Planetary Ellipses" and the values from NASA, the mechanical, potential and kinetic energy was able to be calculated. I will use the terms orbital energy and mechanical energy interchangeably, despite their true definitions. The equations found in most high school physics textbooks only show mechanical, potential and kinetic energy in a circular motion (or circular orbit). It took me a decent about of time to find and learn how to use the equations for elliptical motion. With a quick discussion with a physics professor, a few new terms were clarified for me. These important terms will help me convey the mathematical calculations with more efficiency.

II. IDENTIFY, RESEARCH AND COLLECT IDEA

This idea for calculating orbital energy (mechanical, potential and kinetic energy) came as an extent of other papers I have published. The first time I drew any interest to the difference between circular orbital energy and elliptical orbital energy was during a high school physics lecture. The lecture talked mostly about circular orbital energy and very little about elliptical orbital energy. This lack of information caught my interest and was another contributing factor into conducting this research.

III. WRITE DOWN YOUR STUDIES AND FINDINGS

The equations for circular orbital energy are noted by equations 1.1, 1.2 and 1.3. Circular modeling is important but it is not how the planets move. All of the planets in our solar system have an eccentricity greater than 0 (but less than 1) and are therefore elliptical. A circular orbit means that aphelion and perihelion are equal, and that the radius of the orbit is always the same value. The true model for orbital energy relies on the use of equations 2.1, 2.2, 2.3, 2.4, 2.5, and 2.6 (will be referred to as the elliptical orbital energy equations (EOEEs)). The values produced by the EOEEs will be noted as Energy of Elliptical Orbits Values or EEOVs. The term(s) a_{min} *and* a_{max} are the distances at perihelion and aphelion respectively. Values (distances at perihelion and aphelion) from "Equations for Planetary Orbits" were used and noted as the measured values. The "accepted" values come from NASA's planetary factsheet. The EOEEs were used for finding the orbital energy for both aphelion and perihelion for all eight planets. This data can be found in table 1.1, 1.2, 2 and 3. Table 1.1 and 1.2 are exclusively for Earth values, which include the aphelion and perihelion values, measured and accepted, for the EEOVs and the percent error. Earth has its own table for a very important reason. Initially, the values produced by "Equations for Planetary Ellipses" were written as decimals. With these rounded values, the percent error exceeded 0.2%. A new, modification in the original equations was proposed: leaving the values as fractions to insure less and less percent error. Using the factional values produced different values for the orbital energy set than the decimal values, and thus lowered the percent error. This was only implemented for the values for Earth at this time. Future research will produce values for the other planets. Table 2 is the raw numbers produced by the EOEEs. Table 3.1 and 3.2 also includes these values but also shows the percent error. The new, adjusted Earth data can also be found in table 2, 3.1 and 3.2, however, these tables do *not* compare old values to new values. The M and A found in table 3.1 and 3.2 corresponds to measured and accepted respectively.

Equation Sets:

Equation 1.1: $E_{mec\,h} = -G\frac{Mm}{2r}$

Equation 1.2: $K = -E_{mec\,h} = -\frac{1}{2}U$

Equation 1.3: $U = 2E_{mec\,h}$

Equation 2.1: $E_{mec\,h} = -G\frac{Mm}{2a}$

Equation 2.2 (Perihelion Only): $U = \frac{2E_{mec\,h}}{(1-\epsilon)}$

Equation 2.3 (Perihelion Only): $K = \frac{-E_{mec\,h}(1+\epsilon)}{(1-\epsilon)}$

Equation 2.4 (Aphelion Only): $U = \frac{2E_{mec\,h}}{(1+\epsilon)}$

Equation 2.5 (Aphelion Only): $K = \frac{-E_{mec\,h}(1-\epsilon)}{(1+\epsilon)}$

Tables:

Table 1.1: *Old* EEOVs for Earth and % Error

	Perihelion		*Aphelion*	
	Measured	*Accepted*	*Measured*	*Accepted*
Semi Major Axis	1.4745E+11	1.4710E+11	1.5255E+11	1.5210E+11
Mechanical Energy	-2.6883E+33	-2.6947E+33	-2.5984E+33	-2.6061E+33
Potential Energy	-5.4695E+33	-5.4825E+33	-5.1099E+33	-5.1250E+33
Kinetic Energy	2.7812E+33	2.7879E+33	2.5115E+33	2.5190E+33
% Errors				
Semi Major Axis	0.2379%		0.2959%	
Mechanical Energy	0.2375%		0.2955%	
Potential Energy	0.2371%		0.2946%	
Kinetic Energy	0.2403%		0.2977%	
Adjustment Needed?	Yes, % error needs to be below 0.1		Yes, % error needs to be below 0.1	

Table 1.2: *New* EEOVs for Earth and % Error

	Perihelion		*Aphelion*	
	Measured	*Accepted*	*Measured*	*Accepted*
Semi Major Axis	1.4706E+11	1.4710E+11	1.5214E+11	1.5210E+11
Mechanical Energy	-2.6954E+33	-2.6947E+33	-2.6054E+33	-2.6061E+33
Potential Energy	-5.4840E+33	-5.4825E+33	-5.1237E+33	-5.1250E+33
Kinetic Energy	2.7886E+33	2.7879E+33	2.5183E+33	2.5190E+33
% Errors				
Semi Major Axis	0.0272%		0.0263%	
Mechanical Energy	0.0260%		0.0269%	
Potential Energy	0.0274%		0.0254%	
Kinetic Energy	0.0251%		0.0278%	
Adjustment Needed?	No, % error is below 0.05		No, % error is below 0.05	

Table 2: EEOVs for All Planets						
	Measured Perihelion			*Accepted Perihelion*		
Planet	Emech	U	K	Emech	U	K
Mercury	-4.75850E+35	-1.19710E+36	7.21250E+35	-4.76160E+35	-1.19790E+36	7.21730E+35
Venus	-3.01410E+36	-6.07060E+36	3.05660E+36	-3.00690E+36	-6.05620E+36	3.04930E+36
Earth	-2.68830E+33	-5.46950E+33	2.78120E+33	-2.69470E+33	-5.48250E+33	2.78790E+33
Mars	-2.06290E+35	-4.55380E+35	2.49090E+35	-2.06250E+35	-4.55310E+35	2.49050E+35
Jupiter	-1.70050E+38	-3.57620E+38	1.87570E+38	-1.70130E+38	-3.57780E+38	1.87660E+38
Saturn	-2.79570E+37	-5.92950E+37	3.13370E+37	-2.78730E+37	-5.91150E+37	3.12420E+37
Uranus	-2.10422E+36	-4.41130E+36	2.30710E+36	-2.10170E+36	-4.40600E+36	2.30430E+36
Neptune	-1.52120E+36	-3.07630E+36	1.55500E+36	-1.52330E+36	-3.08040E+36	1.55710E+36
	Measured Aphelion			*Accepted Aphelion*		
Planet	Emech	U	K	Emech	U	K
Mercury	-3.13940E+35	-5.21060E+35	2.07120E+35	-3.13800E+35	-5.20830E+35	2.07030E+35
Venus	-2.97220E+36	-5.90300E+36	2.93080E+36	-2.96820E+36	-5.89520E+36	2.92700E+36
Earth	-2.59840E+33	-5.10990E+33	2.51150E+33	-2.60610E+33	-5.12500E+33	2.51900E+33
Mars	-1.70840E+35	-3.12320E+35	1.41480E+35	-1.71000E+35	-3.12610E+35	1.41610E+35
Jupiter	-1.54160E+38	-2.93920E+38	1.39760E+38	-1.54270E+38	-2.94130E+38	1.39860E+38
Saturn	-2.49420E+37	-4.71940E+37	2.22520E+37	-2.48930E+37	-4.71010E+37	2.22080E+37
Uranus	-1.91910E+36	-3.66950E+36	1.75030E+36	-1.91810E+36	-3.66750E+36	1.74940E+36
Neptune	-1.48810E+36	-2.94380E+36	1.45570E+36	-1.48940E+36	-2.94630E+36	1.45690E+36

(Tables 3.1 and 3.2 on next page)

Table 3.1 - Perihelion EEOVs and Percent Error			
Planet	Emech- M	Emech- A	% Error
Mercury	-4.75850E+35	-4.76160E+35	0.07%
Venus	-3.01410E+36	-3.00690E+36	0.24%
Earth	-2.68830E+33	-2.69470E+33	0.24%
Mars	-2.06290E+35	-2.06250E+35	0.02%
Jupiter	-1.70050E+38	-1.70130E+38	0.05%
Saturn	-2.79570E+37	-2.78730E+37	0.30%
Uranus	-2.10422E+36	-2.10170E+36	0.12%
Neptune	-1.52120E+36	-1.52330E+36	0.14%
Planet	U - M	U - A	% Error
Mercury	-1.19710E+36	-1.19790E+36	0.07%
Venus	-6.07060E+36	-6.05620E+36	0.24%
Earth	-5.46950E+33	-5.48250E+33	0.24%
Mars	-4.55380E+35	-4.55310E+35	0.02%
Jupiter	-3.57620E+38	-3.57780E+38	0.04%
Saturn	-5.92950E+37	-5.91150E+37	0.30%
Uranus	-4.41130E+36	-4.40600E+36	0.12%
Neptune	-3.07630E+36	-3.08040E+36	0.13%
Planet	K - M	K - A	% Error
Mercury	7.21250E+35	7.21730E+35	0.07%
Venus	3.05660E+36	3.04930E+36	0.24%
Earth	2.78120E+33	2.78790E+33	0.24%
Mars	2.49090E+35	2.49050E+35	0.02%
Jupiter	1.87570E+38	1.87660E+38	0.05%
Saturn	3.13370E+37	3.12420E+37	0.30%
Uranus	2.30710E+36	2.30430E+36	0.12%
Neptune	1.55500E+36	1.55710E+36	0.13%

Table 3.2 - Aphelion EEOVs and Percent Error			
Planet	Emech- M	Emech- A	% Error
Mercury	-3.13940E+35	-3.13800E+35	0.04%
Venus	-2.97220E+36	-2.96820E+36	0.13%
Earth	-2.59840E+33	-2.60610E+33	0.30%
Mars	-1.70840E+35	-1.71000E+35	0.09%
Jupiter	-1.54160E+38	-1.54270E+38	0.07%
Saturn	-2.49420E+37	-2.48930E+37	0.20%
Uranus	-1.91910E+36	-1.91810E+36	0.05%
Neptune	-1.48810E+36	-1.48940E+36	0.09%
Planet	U - M	U - A	% Error
Mercury	-5.21060E+35	-5.20830E+35	0.04%
Venus	-5.90300E+36	-5.89520E+36	0.13%
Earth	-5.10990E+33	-5.12500E+33	0.29%
Mars	-3.12320E+35	-3.12610E+35	0.09%
Jupiter	-2.93920E+38	-2.94130E+38	0.07%
Saturn	-4.71940E+37	-4.71010E+37	0.20%
Uranus	-3.66950E+36	-3.66750E+36	0.05%
Neptune	-2.94380E+36	-2.94630E+36	0.08%
Planet	K - M	K - A	% Error
Mecrury	2.07120E+35	2.07030E+35	0.04%
Venus	2.93080E+36	2.92700E+36	0.13%
Earth	2.51150E+33	2.51900E+33	0.30%
Mars	1.41480E+35	1.41610E+35	0.09%
Jupiter	1.39760E+38	1.39860E+38	0.07%
Saturn	2.22520E+37	2.22080E+37	0.20%
Uranus	1.75030E+36	1.74940E+36	0.05%
Neptune	1.45570E+36	1.45690E+36	0.08%

IV. GET PEER REVIEWED

Peer reviewed by my high school physics teacher. Attempted to get this reviewed by another person but they were/are unavailable.

V. IMPROVEMENT AS PER REVIEWER COMMENTS

Rewording and renaming the tables, clarifying confusing concepts.

VI. CONCLUSION

The EEOVs from “Equations for Planetary Ellipses” determined for each planet had an approx. 0.1% to 0.9% error. The adjustment in the values, by leaving terms as fractions, decreased this error to approx. 0.02% to 0.08%. As a follow-up to this, the values in “Equations for Planetary Ellipses” should be adjusted (left as fractions). The values that are actually being left as fractions are the semi major axis, semi minor axis, and the c value.

APPENDIX

- Amin: Perihelion distance.
- Amax: Aphelion distance.
- Elliptical Orbital Energy Equations (EOEEs): The equations used for producing the mechanical, kinetic and potential energy values for ellipses.
- Energy of Elliptical Orbits Values (EEOVs): Values produced from the EOEEs.

ACKNOWLEDGMENT

I would like to acknowledge my high school physics department and my high school math department. Additionally, I would like to especially thank Mr. Bruce Peachy for his support.

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Eric Sullivan - Equations for Planetary Ellipses - published at: "International Journal of Scientific and Research Publications (IJSRP), Volume 6, Issue 5, May 2016 Edition".

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