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Nonlinear observers for attitude estimation in gyroless spacecraft via Extended Kalman filter algorithm

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***Abstract*:**This paper designs an observer to estimate the spacecraft's angular velocity using only Euler angles attitude without gyroscope measurement. This work contributes as an alternative or backup system during unavailable gyroscope measurement due to faulty sensor or reduction of sensor hardware for cost reduction.In this work, the observer model for satellite attitude is derived in their nonlinear form, and the observability of the nonlinear system is investigated using Lie derivative. The performance of the designed observer is analyzed and verified using real flight data of Malaysian satellite by employing Extended Kalman Filter algorithm.

Index Terms- Nonlinear observer; Satellite attitude estimation; ExtendedKalman Filter; Lie derivative

I. Introduction

Satellite attitude determination is one of the important aspects in Attitude Determination and Control System (ADCS) of a satellite. Satellite attitude is important to be determined in a satellite to be fed back to controller in accomplishing a specific satellite mission such as Earth observation, communication, scientific research and many other missions. However not all states are directly available may be due to faulty sensor or as a way to obtain a substantial reduction of sensors which represents a cost reduction.

In most practical implementations of ADCS, the angular velocity and attitude information of a spacecraft are obtained respectively from measurement of gyroscopes and attitude sensor such as sun sensor, star sensor, or magnetometer. However, gyroscopes are generally expensive and are often prone to degradation or failure [1].Therefore, as an alternative or backup system to circumvent the problem of gyroless measurement, an observer can be designed to provide the information of angular velocity by using only the measurement of Euler angles attitude.

Since decades, a great number of research works have been devoted to the problem of estimating the attitude of a spacecraft based on a sequence of noisy vector observations such as [2][3][4][5].Different algorithms have been designed and implemented in satellite attitude estimation problem. Early applications relied mostly on the Kalman filter for attitude estimation. Kalman filter was the first applied algorithm for attitude estimation for the Apollo space program in 1960s. Due to limitation of Kalman filter which work optimal for linear system only, several famous new approaches have been implemented to deal with the nonlinearity in satellite attitude system including Extended Kalman Filter (EKF) [6][7][4], Unscented Kalman Filter (UKF) [8][9][10], Particle Filter (PF)[11][12][13], and predictive filtering [14][15]. EKF is an extended version of Kalman filter for nonlinear system whereby the nonlinear equation is approximated by linearized equation through Taylor series expansion. UKF, an alternative to the EKF uses a deterministic sampling technique known as the unscented transform to pick a minimal set of sample points called sigma points to propagate the non-linear functions. EKF and UKF approaches is restricted assume the noise in the system is Gaussian white noise process. While, PF is a nonlinear estimation algorithm that approximates the nonlinear function using a set of random samples without restricted to a specific noise distribution as EKF and UKF. However, EKF was found as most widely used algorithm both in theory and in real practice by spacecraft community due to simplicity for implementation and theoretically attractive in the sense that it minimizes the variance of the estimation error.

In the open literature, spacecraft attitude estimation use different attitude representation either Euler angles, Rodrigues parameter, or quarternion parameteras their kinematic model [16]. Each kinematic model of different parameter is governed by different differential equation [17]. The researchers also studied the performance of estimated states by varying different type of sensor measurement such as gyroscope, magnetometer, sun sensor or star sensor. In this paper, an observer to estimate the angular velocity of a satellite using measurement of Euler angles attitude provided by attitude sensor is designed and implemented via EKF algorithm. EKF was used in this work due to its well-known and established algorithm and theoretically attractive in the sense that it minimizes the variance of the estimation error. While Euler angles is used to represent the satellite's attitude instead of quartenion and Rodrigues parameter as its straightforward physical interpretation for analysis. The performance of the nonlinear observer to estimate the attitude is also verified using real flight data of Malaysian satellite, RazakSAT.

The organization of this paper proceeds as follows. Section II presents mathematical model of nonlinear satellite attitude dynamic. Section III describes two important concepts related to nonlinear observer including system observability via Lie derivative and also EKF, an estimation algorithm used in the observer system. Section IV presents and discusses the results of the observer system which was tested and verified using actual flight data and Section V presents the paper's conclusions.

II. MATHEMATICAL MODEL OF NONLINEAR SATELLITE ATTITUDE DYNAMICS

Mathematical model of satellite attitude dynamics is described by both the dynamics equation of motion and kinematics equation of motion [18].

Dynamic equation of motion relates the angular velocity to the exerted torque as defined by Euler's Moment Equation [18][17]

$$I\dot{\omega} + \omega \times I\,\omega = T \tag{1}$$

Or similarly is written in component-wise

$$\dot{\omega}_x = -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + \frac{T_x}{I_x} \tag{2a}$$

$$\dot{\omega}_y = -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + \frac{T_y}{I_y} \tag{2b}$$

$$\dot{\omega}_z = -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + \frac{T_z}{I_z} \tag{2c}$$

with $I = diag[I_x, I_y, I_z]$, $\dot{\omega} = [\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z]$, $\omega = [\omega_x, \omega_y, \omega_z]$, $T = [T_x, T_y, T_z]$ represent satellite's moment of inertia, angular acceleration, angular velocity and space environmental disturbances torque vectors respectively.

While, kinematic equation of motion relates the attitude parameter to the angular velocity. In this work, Euler angles parameter is used to represent the satellite's attitude as its straightforward physical interpretation for analysis. Euler angles are defined as the rotational angles about the body axis as follows: $\emptyset$ is rotational angle about X-axis (roll); θ is rotational angle about Y-axis (pitch); and φ is rotational angle about Z-axis (yaw). The kinematic equation of Euler angles parameter using $\varphi - \theta - \emptyset$ (or some literature use notation 3-2-1) sequence rotation is

$$\begin{bmatrix}\dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}[\omega_x + \omega_0 c\theta s\varphi] + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\end{bmatrix} \tag{3}$$

where *c*,*s* and *t* denote cosine, sine, and tangent functions, respectively. While, ω_0 is the orbital rate of the spacecraft.

A complete formulation of the satellite attitude dynamics is obtained by combining both the dynamics equation of motion and kinematics equation of motion as follow

$$\begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\\ \dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}-\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z\\ -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z\\ -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y\\ [\omega_x + \omega_0 c\theta s\varphi] + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\end{bmatrix} + \begin{bmatrix}\frac{T_x}{I_x}\\ \frac{T_y}{I_y}\\ \frac{T_z}{I_z}\\ 0\\ 0\\ 0\end{bmatrix} \tag{4}$$

For low Earth orbit satellite, gravity gradient torque must be taken into consideration as part of external torque since it is continuously acting on the spacecraft body and influence the satellite's attitude motion. The external torque dominated by gravity gradient torque is written as [17][19]

$$T = \begin{bmatrix}T_x\\ T_y\\ T_z\end{bmatrix} = \begin{bmatrix}3{\omega_0}^2(I_z - I_y)s\emptyset c\emptyset c^2\theta\\ 3{\omega_0}^2(I_z - I_x)s\theta c\theta c\emptyset\\ 3{\omega_0}^2(I_x - I_y)s\emptyset c\theta s\theta\end{bmatrix} \tag{5}$$

Hence, by substituting the gravity gradient torque in Equation (5) into Equation (4), the complete model of satellite attitude under Low Earth Orbit is

$$\begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\\ \dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}-\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + 3{\omega_0}^2\frac{(I_z - I_y)}{I_x}s\emptyset c\emptyset c^2\theta\\ -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + 3{\omega_0}^2\frac{(I_z - I_x)}{I_y}s\theta c\theta c\emptyset\\ -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + 3{\omega_0}^2\frac{(I_x - I_y)}{I_z}s\emptyset c\theta s\theta\\ [\omega_x + \omega_0 c\theta s\varphi] + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\\ \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)]\end{bmatrix} \tag{6}$$

III. Nonlinear observers for spacecraft attitude estimation

A. Nonlinear Observers

A nonlinear observer is a nonlinear dynamic system that is used to estimate the unknown states from one or more measurements. Mathematically, the nonlinear observer design is described as follows. Given the actual nonlinear system dynamics and measurement described by continuous-time model [19]

$$\dot{x} = f(x) + w \quad (7)$$

$$y = h(x) + v \quad (8)$$

Then, the observer is modeled as

$$\dot{\hat{x}} = f(\hat{x}) + L(y - \hat{y}) \quad (9)$$

$$\hat{y} = h(\hat{x}) \quad (10)$$

In Equation (7)-(10),$x \in R^n$ is the state vectorand $y \in R^p$ is the output vector, w and v denote the noise or uncertainty vector in the state and measurement respectively. While $\hat{x}$and $\hat{y}$ denotes the corresponding estimate and L is the gain matrix of the observer depending on the observer design.

In this work, the system is designed toestimate the satellite's angular velocity $(\omega_x, \omega_y, \omega_z)$ by using Euler angles attitude $(\emptyset, \theta, \varphi)$ measurement only. Hence the state vector is $x = [\omega_x, \omega_y, \omega_z, \emptyset, \theta, \varphi]^T$, while the state equation is

$$\dot{x} = f(x) = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{\emptyset} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + 3{\omega_0}^2\frac{(I_z - I_y)}{I_x}s\emptyset c\emptyset c^2\theta \\ -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + 3{\omega_0}^2\frac{(I_z - I_x)}{I_y}s\theta c\theta c\emptyset \\ -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + 3{\omega_0}^2\frac{(I_x - I_y)}{I_z}s\emptyset c\theta s\theta \\ [\omega_x + \omega_0 c\theta s\varphi] + s\emptyset t\theta[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \\ c\emptyset[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \\ \frac{s\emptyset}{c\theta}[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \end{bmatrix} \quad (11)$$

and the measurement equation

$$y = h(x) = \begin{bmatrix} \emptyset \\ \theta \\ \varphi \end{bmatrix} \quad (12)$$

B. Observability Test

Observability is one of important concepts in estimation. Observability provides an indication of the state quantities that can be observed from measurements. If a system is not observable, this means the current values of some of its states cannot be determined through output of sensors measurement. Nonlinear observability is intimately tied to the Lie derivative [20]. The Lie derivative is the derivative of a scalar function along a vector field. The Lie derivative of a scalar function *h*with respect to vector field *f*, denoted$L_f(h)$ is given by

$$L_f(h) = \frac{\partial h}{\partial x} f(x) \quad (13)$$

Given a system as in Equation (7) and (8), the step for observabilty test of the nonlinear system is described as follows: [19]

Step 1: Compute a matrix, *G* of Lie derivative

$$G = \begin{bmatrix} L_f^0(h_1) & \cdots & L_f^0(h_p) \\ \cdots & \cdots & \cdots \\ L_f^{n-1}(h_1) & \cdots & L_f^{n-1}(h_p) \end{bmatrix} \quad (14)$$

with Lie derivative is defined as

$$L_f^i(h) = \begin{cases} h & ; \quad \text{for } i = 0 \\ \frac{\partial}{\partial x}\left[L_f^{i-1}(h)\right]f & ; \quad \text{for } i = 1,2,3,\dots,n \end{cases} \quad (15)$$

Step2: Compute the gradients operator on matrix *G*.

$$dG = \begin{bmatrix} dL_f^0(h_1) & \cdots & dL_f^0(h_p) \\ \cdots & \cdots & \cdots \\ dL_f^{n-1}(h_1) & \cdots & dL_f^{n-1}(h_p) \end{bmatrix} \quad (16)$$

Step3: Check the rank of matrix *dG*. Matrix *dG* must have rank *n* for the system to be observable.

C. Extended Kalman filter

In this work, EKF is used as the estimation algorithm in the nonlinear observer system due to its well-known and established algorithm and theoretically attractive in the sense that it minimizes the variance of the estimation error. EKF is an on-line, recursive algorithm trying to estimate the true state of an observable nonlinear system where only some noisy measurements are available. EKF algorithm is described as below.[18]

Let the continuous model in Equation (7) and (8) is transformed into the discrete-time model such that

$$x_k = f(x_{k-1}) + w_{k-1} \quad (17)$$

$$y_k = h(x_k) + v_k \quad (18)$$

Here the subscriptof the variables denotes the time step, while w_{k-1} and v_k are restricted assumed as Gaussian distributed noises with mean zero and covariance R_w and R_v respectively such that $w_{k-1} \sim N(0, R_w)$and $v_k \sim N(0, R_v)$. Then, the estimated state is obtained through the following step:

Step 1: Set the initial state estimate $\hat{x}_0 = \hat{x}_{0|0}$ and variance $P_0 = P_{0|0}$.

Step 2: Repeat

(i) Prediction step (priori estimate)

Jacobianof $f(x_{k-1})$: $F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}}$ (19)

Predicted state estimate: $\hat{x}_{k|k-1} = f\left(\hat{x}_{k-1|k-1}\right)$ (20)

Predicted covariance estimate: $P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + R_w$ (21)

(ii) Update step (posteriori estimate)

Jacobianof $h(x_k)$: $H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}$ (22)

Kalman gain: $K_k = P_{k|k-1} H_k^T \left[H_k P_{k|k-1} H_k^T + R_v\right]^{-1}$ (23)

Updated state estimate: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left[y_k - h\left(\hat{x}_{k|k-1}\right)\right]$(24)

Updated covariance estimate: $P_{k|k} = [I - K_k H_k] P_{k|k-1}$(25)

IV. RESULT AND DISCUSSION

This section investigates the performance of the designed observer to provide the information of the angular velocity during gyroless condition.

Before implementing the observer system, the nonlinear observability test is carried out to investigate whether the designed system is possible to be executed. The nonlinear observability test is carried out via Lie derivative with the aid of MATLAB using step described in Section III. The test has shown that

Rank of matrix dG =6

implies that the system is full rank. Hence it is said that the system is locally observable everywhere for the system during gyroless condition and only Euler angles attitude information was available. The test proves that the satellite's angular velocity can be estimated using the designed observer.

The performance of the nonlinear observer designed in this work has been analyzed and validated using real sensors data from the RazakSAT satellite. RazakSAT is a Malaysian satellite which was launched into Low Earth Orbit near Equatorial in 2009. In the mission, the attitude was provided directly using sun sensor, one of the attitude sensor, while the angular velocity was provided by gyroscope sensor. The satellite's characteristics of RazakSAT are given in Table 1, which was provided by Astronautic Technology SdnBhd (ATSB), the Malaysian company that responsible for RazakSAT's mission.

Table 1: RazakSAT's characteristics.

Moment of inertia, I_x	25.4 kg.m^2
Moment of inertia, I_y	26.2 kg.m^2
Moment of inertia, I_z	21.0 kg.m^2
Orbital rate, ω_0	0.001063 rad/s or 0.0609 deg/s

Figures 1, 2 and 3 show the real measurement of Euler angles attitude respectively around roll, pitch, and yaw angles as obtained by using sun sensor in RazakSAT mission. The available measurements are for about 6 orbits sequentially, available to the ground system at a sampling rate of about 1 minute.

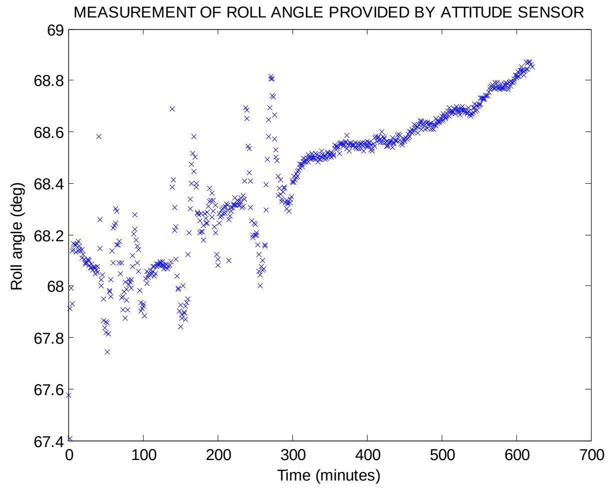

Figure 1: Real measurement of roll angle provided by attitude sensor.

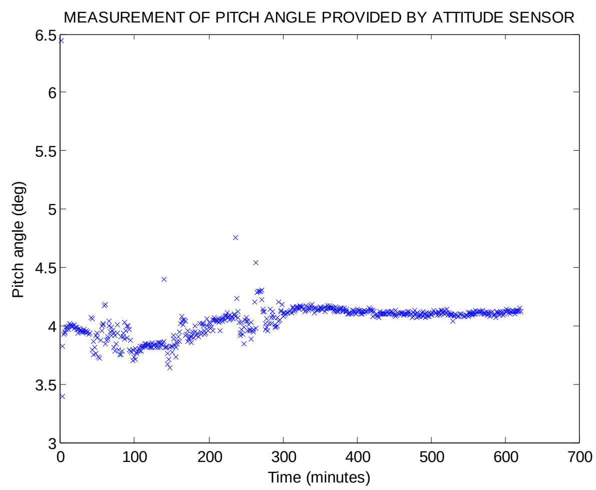

Figure 2: Real measurement of pitch angle provided by attitude sensor.

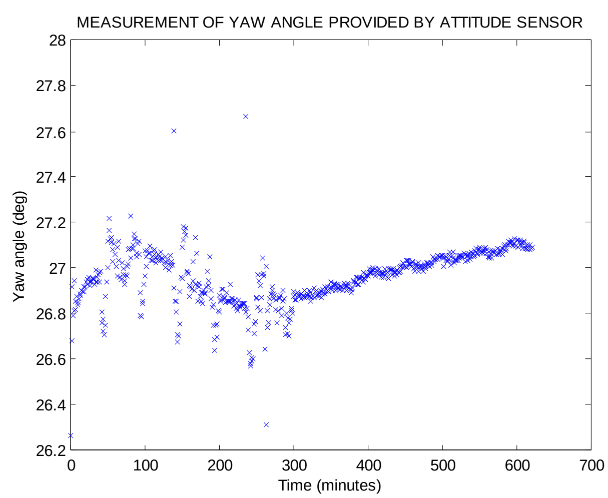

Figure 3: Real measurement of yaw angle provided by attitude sensor.

In order to validate and analyze the performance of the designed system, the estimated result is compared with the real sensor data of RazakSAT. Figures 4, 5, and 6 show comparison between the estimated angular velocities through the designed observer and the real angular velocities measurement provided by gyroscope in RazakSAT mission respectively around X-axis, Y-axis, and Z-axis. From all the three figures, it is observed that behaviour of the estimated states scattered randomly around zero about the first half of the period analysed. While in the second half of the period, the estimated states are not scattered randomly as to the first half period. However, the estimated angular velocity around X-axis and Z-axis seem deviate from zero values of the real states measured by gyroscope. This deviation may be due to mis-modeling or the influence of unmodeled space environmental disturbance torque including aerodynamic, solar radiation pressure and Earth magnetic torques.

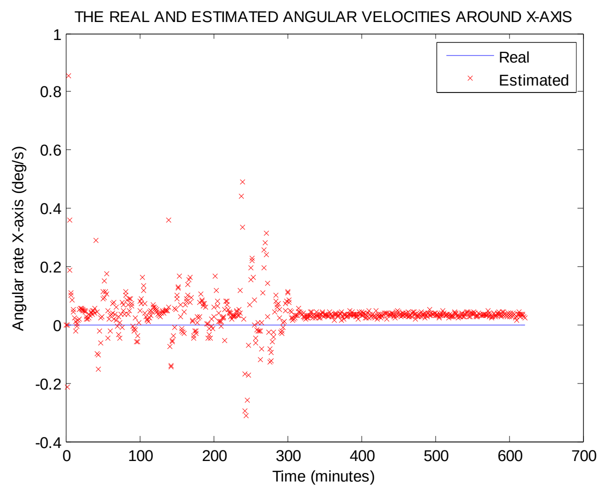

Figure 4: Comparison between the estimated and the real angular velocity around X-axis.

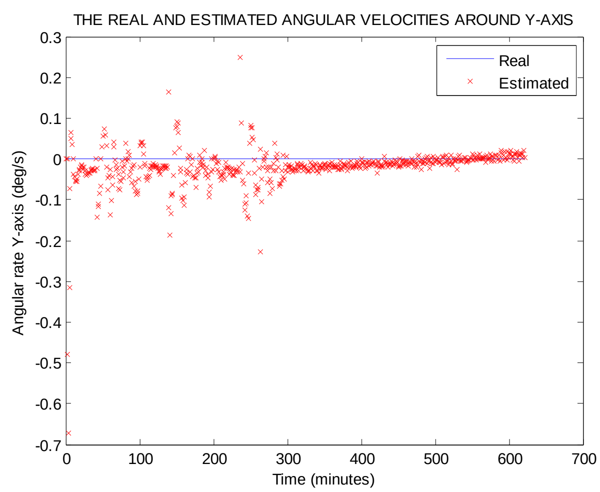

Figure 5: Comparison between the estimated and the real angular velocity around Y-axis.

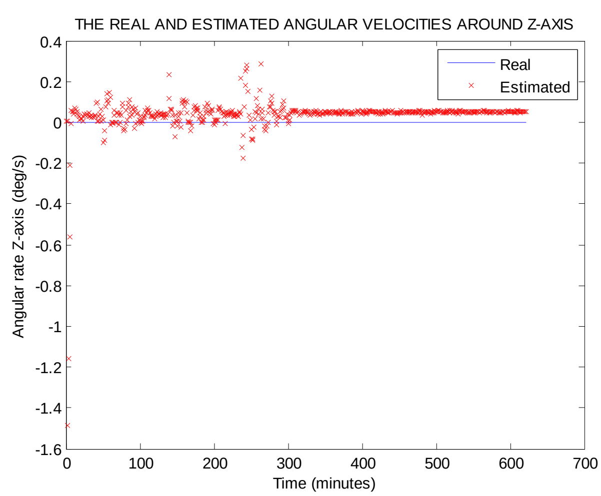

Figure 6: Comparison between the estimated and the real angular velocity around Z-axis.

Figures 7, 8 and 9 show the plot error of the estimated angular velocities when compared to the real angular velocities measured by gyroscope of RazakSAT respectively around X-axis, Y-axis, and Z-axis. From all the three figures, it is also observed that behaviour of the error scattered randomly around zero value for about the first half of the analyzed period, while in the second half of the period, the estimated states are less scattered randomly as to the first half period. However, the error around X-axis and Z-axis is seem to have constant value about -0.05 deg/s for the second half of the analysed period. While the error around the Y-axis for the second half of analysed period decrease with a tendency to stabilize around a zero value. This could be arising from the mis-modeling factor which ignores the effects of the space environmental disturbance torque as described before.

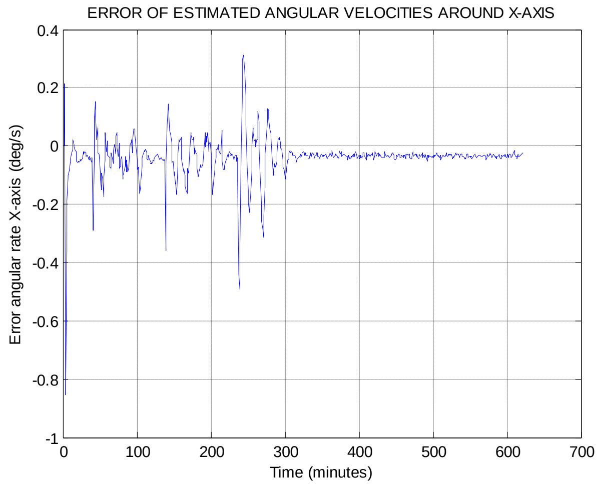

Figure 7: Error of estimated angular velocity around X-axis.

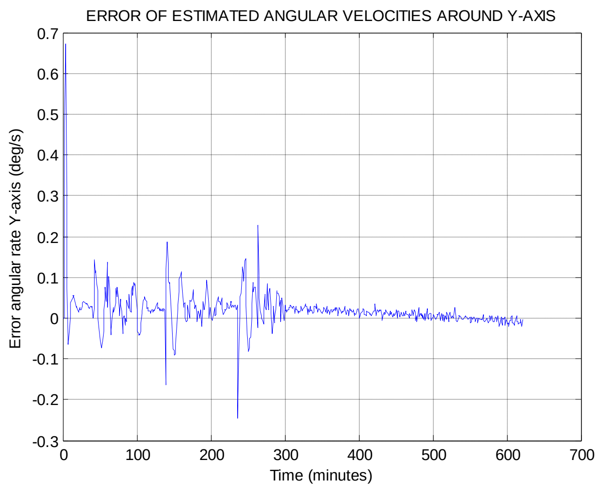

Figure 8: Error of estimated angular velocity around Y-axis.

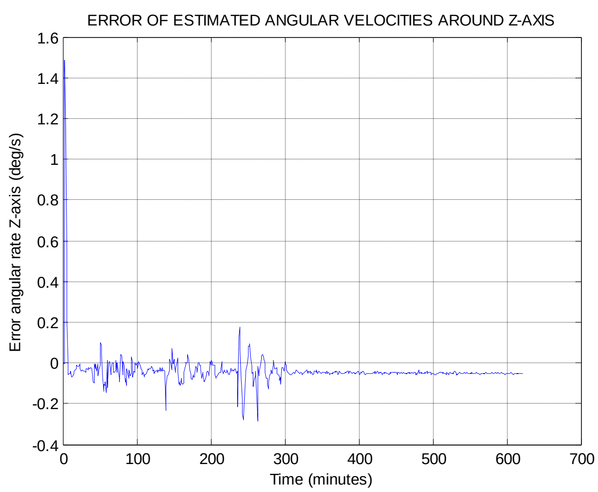

Figure 9: Error of estimated angular velocity around Z-axis.

Table 2 shows the Root Mean Squared Error (RMSE) of the estimated angular velocity compared to real angular velocity obtained by gyroscope measurement in RazakSAT mission. The result shows that the system is able to provide the information of angular velocity within 0.1 deg/s accuracy, which is suitable for moderate accuracy attitude determination mode such as during housekeeping and detumblingtask, but unsuitableforpointingmodethat requiresmorepreciseaccuracy such as during imaging task.

Table 2: RMSE of the estimated angular velocity.

States	**RMSE value**
Angular velocity around X-axis, ω_x	0.0825 deg/s
Angular velocity around Y-axis, ω_y	0.0530 deg/s
Angular velocity around Z-axis, ω_z	0.0982 deg/s

V. CONCLUSION

In this paper, a nonlinear observer to provide the angular velocity information in gyroless condition was designed and developed using Extended Kalman Filter algorithm. The system observability is first investigated to check the possibility of the nonlinear system states estimation by employing test rank using Lie derivatives technique. The rank test shows that the designed observer is observable everywhere, and hence can estimate the angular velocity from the Euler angles attitude information only without the gyroscopes. The performance of the designed observer by employing Extended Kalman Filter algorithm is then investigated and validated using real flight data of RazakSAT, the Malaysian satellite. The result shows that the system is able to provide the information of angular velocity within 0.1 deg/s accuracy, which is suitable for moderate accuracy attitude determination such as during housekeeping and detumbling mode. The designed observer can be used as an alternative or backup attitude determination system of a Low Earth Orbit satellite during unavailable gyroscope measurement due to faulty sensor or reduction of sensor hardware for cost reduction.

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