# Bayesian Inference for Mean of the Lognormal Distribution

**Juliet Gratia D'Cunha[*], K. Aruna Rao[**]**

[*] Research Scholar, Department of Statistics, Mangalore University, Mangalagangothri, Karnataka, India.
[**]Professor of Statistics (Retd.), Department of Statistics, Mangalore University, Mangalagangothri, Karnataka, India.

***Abstract-*** Lognormal distribution has wide applications in the analysis of failure time data, stock prices and rainfall. In this paper we derive Bayes estimator and credible regions for the mean of the lognormal distribution. We compare the coverage probability and length of the Bayes credible interval with the confidence interval obtained from the maximum likelihood estimator of the log location and scale parameters. The procedure is illustrated using the failure time data of locomotive control and stock price.

***Keywords-*** Lognormal distribution, stock prices, Bayes estimator and credible regions, confidence interval, maximum likelihood estimator.

## I. Introduction

Lognormal distribution has support on the positive part of the real line. It is right skewed and is widely applicable when normal distribution does not fit well to the data. It is used in the analysis of failure time data, stock market data and in the analysis of rainfall data. The commonly used procedure in this area is to take log transformation and use the technique developed for normal models. The mean and variance of the lognormal distribution are not invariant under distributional transformation. Therefore it is necessary to develop estimators and confidence intervals for the mean and variance of the lognormal distribution.

In applied research coefficient of variation (C.V) is widely used as a measure of variability than the standard deviation. The reason for this is that C.V is unitless and facilitates easy interpretation. Zellner (1971) initiated Bayesian inference for lognormal distribution. He considered Bayes estimator for the mean and median of the lognormal distribution. He observed that Bayes estimator for mean of the lognormal distribution does not exist and he obtained improved estimators of mean and median of the distribution. He used diffuse prior $\pi(\mu,\sigma)=\frac{1}{\sigma}$ which is also the right invariant prior for the location scale family (Ghosh *et al.* (2006)). Padgett and Wei (1977) developed Bayes estimator of reliability function for the lognormal distribution. They used two types of priors namely normal prior for mean and gamma prior for the inverse of the scale parameter; the other prior is the vague prior of Jeffrey (See Ghosh *et al.* (2006) for discussion on this prior). This research was extended by Padgett and Johnson (1983) where they obtained lower bounds on reliability function of the two parameter lognormal distribution. Sarabia *et al.* (2005) proposed a class of Bivariate conjugate priors for $\mu$ and $\sigma$ of the lognormal distribution using the conditional specification. Several procedures were also suggested for the estimation of the hyperparameters. Harvey and Merwe (2010), Harvey *et al.* (2011), and Harvey and Merwe (2012) compared the Bayesian credible interval for the means and variances of lognormal distribution. The performance of the credible interval is compared with credible/confidence interval suggested by Zhou and Tu (2000) and Krishnamoorthy and Mathew (2003). In the last section the authors discuss about bivariate lognormal distribution and obtain Bayesian confidence intervals for the difference between two correlated lognormal means and for the ratio of lognormal variances. The overall conclusion is that Bayes credible interval has shorter width compared to the width of the other intervals. D'Cunha and Rao (2014) developed Bayesian credible interval for the C.V of the lognormal distribution and compared it with the confidence interval obtained by the maximum likelihood estimator. In this paper we show that, under mild regularity conditions, Bayes estimator for the mean of the lognormal distribution exists. The priors used are 1) Left invariant Jeffrey's prior 2) Right invariant prior. For a discussion of these priors see Ghosh et al. (2006), Berger (2005). The 100(1-α)% credible interval is compared with the corresponding level confidence interval obtained through the use of maximum likelihood estimator (M.L.E). The reason for this is that confidence interval is easy to compute and is preferred by the applied researchers. This type of comparison has not been done in the past. The simulation results indicate that Bayes credible interval have smaller length compared to the confidence interval obtained through the M.L.E. The procedure is illustrated to obtain the Bayes estimate of the mean lifetime and the associated credible intervals of the data on locomotive controls given in Schmee and Nelson (1977).

The Organization of the paper is such that section 2 presents the derivation of the Bayes estimator and associated credible interval for the mean of the lognormal distribution, while section 3 presents details of the simulation experiment. The results are presented in section 4. The procedure is used to derive the mean lifetime of locomotive controls and the associated credible interval and it also used to predict the future stock price which is given in section 5, while the concluding remarks are provided in section 6.

## II. Bayes Estimator for the Mean of the Lognormal Distribution

Let$\mu$ dna $\sigma$denote the log location and scale parameter of the lognormal distribution. Given a random sample of size $n$, $x_1,\ldots,x_n$ from this distribution, let $Z_i = \log X_i$, $i=1, ..n$, where Z follows normal distribution with parameter $\mu$ and $\sigma^2$ and maximum likelihood estimator of $\mu$ and $\sigma^2$ are $\bar{Z}$ and $S_z{}^2$respectively, where $\bar{Z} = \frac{\sum_i^n z_i}{n}$ and $S_z{}^2 = \frac{\sum_i^n (z_i-\bar{z})^2}{n}$. Using invariance property of maximum likelihood estimator (Kale (1999)), the maximum likelihood estimate (M.L.E) of the mean of the lognormal distribution namely $\theta = e^{\mu+\frac{1}{2}\sigma^2}$ is given by$\hat{\theta} = e^{\hat{\mu}+\frac{1}{2}\hat{\sigma}^2} = e^{\bar{z}+\frac{1}{2}s_z{}^2}$. The asymptotic variance of $\hat{\theta}$ can be obtained using delta method and is given by

$var(\hat{\theta}) = e^{2\mu+\sigma^2}\frac{\sigma^2}{n}\left(1+\frac{\sigma^2}{2}\right) + o(n^{-1})$(1)

In the above expression we have used $\frac{n-1}{n} \approx 1$. The 100(1-α)% asymptotic confidence interval for $\hat{\theta}$ is given by $\hat{\theta} \pm Z\alpha_{/2}\, S.E(\hat{\theta})$, where $Z\alpha_{/2}$ refers to upper $^{\alpha}/_{2}\, th$ percentile value of the standard normal distribution and $S.E(\hat{\theta})$ refers to estimated $standard\ error(\hat{\theta})$. The estimate of $\mu$ and $\sigma^2$ is obtained by substituting the value of $\hat{\mu}$ and $\hat{\sigma}^2$ in the expression for variance of$\hat{\theta}$.

We have used two objective priors namely the right invariant prior $\pi(\mu,\sigma) = \frac{1}{\sigma}$ and left invariant Jeffrey's prior given by $\pi(\mu,\sigma) = \frac{1}{\sigma^2}$ (Ghosh *et al*. (2006), Berger (2005)). The choice of the right invariant prior stems from the fact that Z follows normal distribution and the right invariant prior used in this paper is the one that is suggested for location scale family (Ghosh *et al*.(2006)). The advantage of objective Bayesian analysis is that the prediction remains the same irrespective of the decision maker. Thus the procedure can be applied universally given the past data. The posterior density $\pi(\mu,\sigma|z_1,\ldots,z_n)$ for the right invariant and the left invariant Jeffry's priors are given by the following expression,

$$\pi(\mu,\sigma|z_1,\ldots,z_n) = \frac{1}{\sqrt{2\pi}\frac{\sigma}{\sqrt{n}}}e^{-\frac{1}{2}\frac{(\bar{z}-\mu)^2}{\sigma^2/n}}\frac{\left(\frac{(n-1)s_z{}^2}{2}\right)^{\frac{n+2}{2}}}{\Gamma\left(\frac{n+2}{2}\right)}\left(\frac{1}{\sigma^2}\right)^{\left(\frac{n+2}{2}\right)-1}e^{-\frac{1}{2}\left(\frac{n-1}{\sigma^2}\right)s_z{}^2} \text{ (using right invariant prior)} \quad (2)$$

$$(\pi(\mu,\sigma|z_1,\ldots,z_n) = \frac{1}{\sqrt{2\pi}\frac{\sigma}{\sqrt{n}}}e^{-\frac{1}{2}\frac{(\bar{z}-\mu)^2}{\sigma^2/n}}\frac{\left(\frac{(n-1)s_z{}^2}{2}\right)^{\frac{n+3}{2}}}{\Gamma\left(\frac{n+3}{2}\right)}\left(\frac{1}{\sigma^2}\right)^{\left(\frac{n+3}{2}\right)-1}e^{-\frac{1}{2}\left(\frac{n-1}{\sigma^2}\right)s_z{}^2} \text{ (using left invariant prior)} \quad (3)$$

It may be noted that although we have used independent prior for $\mu$ and $\sigma$, the posterior density has a bivariate correlated distribution. The Bayes estimator of $\theta$ is $E(\theta|z_1,\ldots,z_n)$, where expectation is taken with respect to the posterior density of $\mu$ and $\sigma$ . For the right invariant prior it is given by

$$\iint e^{\mu+\frac{1}{2}\sigma^2}\frac{1}{\sqrt{2\pi}\frac{\sigma}{\sqrt{n}}}e^{-\frac{1}{2}\frac{(\bar{z}-\mu)^2}{\sigma^2/n}}\frac{\left(\frac{(n-1)s_z{}^2}{2}\right)^{\frac{n+2}{2}}}{\Gamma\left(\frac{n+2}{2}\right)}\left(\frac{1}{\sigma^2}\right)^{\left(\frac{n+2}{2}\right)-1}e^{-\frac{1}{2}\left(\frac{n-1}{\sigma^2}\right)s_z{}^2}d\mu d\sigma = \int e^{\bar{z}+\frac{\sigma^2}{2n}+\sigma^2}\frac{\left(\frac{(n-1)s_z{}^2}{2}\right)^{\frac{n+2}{2}}}{\Gamma\left(\frac{n+2}{2}\right)}\left(\frac{1}{\sigma^2}\right)^{\left(\frac{n+2}{2}\right)-1}e^{-\frac{1}{2}\left(\frac{n-1}{\sigma^2}\right)s_z{}^2}d\sigma \quad (4)$$

In the above integral $\left(\frac{1}{\sigma^2}\right)^{\left(\frac{n+2}{2}\right)-1}e^{-\frac{1}{2}\left(\frac{n-1}{\sigma^2}\right)s_z{}^2}$ tends to zero as $\sigma^2$ tends to $\infty$ and $e^{\frac{\sigma^2}{2}\left(\frac{1}{n}+1\right)}$ tends to $\infty$ as $\sigma^2$ tends to $\infty$. The latter tends to $\infty$ at a faster rate than the former tending to zero. This is the reason for the non existence of the mean of the posterior density. Table 1 shows that the location parameter of the lognormal distribution is bounded.

Therefore under the mild assumptions, $\sigma^2$ is bounded in probability, the expected value of $e^{\mu+\frac{1}{2}\sigma^2}$ exists, where the expectation is taken with respect to the posterior density of $\mu$ and$\sigma^2$. Closed form solution does not exist for this expectation and has to be evaluated using numerical integration or the Monte Carlo integration. In this paper we have used Monte Carlo integration to compute Bayes estimate. In our simulation we did not encounter very large value of this expectation and thus 100(1-α)% credible interval for the mean of the lognormal distribution is justifiable. The samples of $\mu,\sigma^2$ are generated using importance sampling approach. Since the marginal distribution of $\eta = \frac{1}{\sigma^2}$ follows gamma distribution for the right as well as left invariant priors, the observation for $\eta$ is generated from the gamma distribution. The conditional posterior distribution of $\mu$ given $\sigma^2$ follows normal distribution with mean $\bar{Z}$ and variance $\frac{\sigma^2}{n}$. Using the previously generated value of $\sigma^2$, we generate an observation from the normal distribution. This constitutes a pair of observation $\mu,\sigma^2$ from the posterior density. Then $E\left(e^{\mu+\frac{1}{2}\sigma^2}|z_1,\ldots,z_n\right) = \frac{1}{M}\sum_{i=1}^{M}\left(e^{\mu_i+\frac{1}{2}\sigma_i{}^2}\right)$, where M denotes the number of paired samples generated from the posterior distribution. In this paper we have used M=10,000. The generated samples were also used for obtaining equitailed credible intervals. An alternate approach for finding the Bayes estimator and the associated Highest Posterior Density (HPD) credible interval is given in the appendix.

## III. SIMULATION EXPERIMENT

In order to find the finite sample (small sample) performance of the Bayes credible interval and the confidence interval, a simulation experiment was conducted. A sample of size *n* was generated from the normal distribution with mean $\mu$ and variance $\sigma^2$. The value of $\mu$ and $\sigma^2$ are so adjusted so that the C.V of the lognormal distribution ranges from 0.1, 0.3, 0.5, 0.7, 1, 1.5, 2 and 2.5, the mean of the lognormal distribution is 1000. The value of $\mu$ equal to 1000 corresponds to the stock market data analyzed in this paper. Observations from the joint posterior distribution were generated as explained in section 2. The Bayes estimator and the associated equitailed credible interval were also constructed along with the maximum likelihood estimator of the mean and the associated confidence interval. For each simulation, the length of the credible and confidence interval was also noted. Using 1000 simulations, confidence and equitailed credible intervals are constructed. Using the 1000 simulations, the average length and standard deviation of the length of confidence and credible intervals were also computed. Confidence/credible level is fixed at $1 - \alpha = 0.95$. The sample sizes considered are *n* = 10, 20, 40, 60, 80, 100, 150 and 200. The total number of simulation configuration = Number of samples $\times$ Number of C.V = 8 $\times$ 8 = 64; these 64 configuration were common for credible interval based on Jeffrey's prior , right invariant prior and confidence interval.

## IV. RESULTS AND DISCUSSION

Table 2 presents the coverage probabilities across different sample sizes for eight values of C.V taken together. We say that confidence/credible interval maintains the level if the estimated coverage probability is in between the range of 0.940 to 0.960. That is$(1-\alpha) \pm 0.01 \text{ where } \alpha = 0.05$, which is approximately equal to error rate of 10%. From the table it becomes clear that the confidence interval failed to maintain coverage probability even for large samples of size *n* = 100 to 200. On the other hand equitailed credible intervals obtained from left and right invariant priors by and large maintain the credible level. For example, when *n* =100, the credible level is maintained in 7 out of 8 times for right invariant as well as left invariant priors. From the table it also follows that the minimum sample size required for the construction of credible interval is 100 for which one can expect that the confidence level would be maintained. On the contrary the confidence interval based on maximum likelihood estimator requires a sample size more than 200 to maintain the confidence level. This is the advantage of the Bayes credible interval over confidence interval.

Table 3 presents the average length of the confidence/credible interval for various sample sizes whenever the confidence/credible level is maintained. Although it appears that the average length of the confidence interval is shorter compared to average length of credible interval using both right and left invariant priors, caution is exercised to interpret the results of this table. For example, in table 2 when *n* =100, the average length for the confidence interval is 116.71 which is the average length of 3 confidence intervals, while for the left and right invariant priors it is 340.84 which is the average length based on 7 credible intervals. To explore the ideas further, the average length of the confidence/credible interval is cross tabulated across sample sizes and C.V of the distribution and is presented in table 3. A careful examination of the table reveals that when *n*=100, the confidence interval maintains coverage probability for 3 values of C.V and if we compare the average length of the confidence interval it is almost the same as the average length of the credible interval using left and right invariant prior. For n=150, for 5 values of C.V the confidence interval maintains coverage probability and the average length of the confidence/credible intervals are almost equal. When *n*=200, for 5 values of C.V the confidence interval maintains coverage probability. Out of these 5 values of C.V, for 3 values of C.V namely 0.7, 1.5, and 2 the average length of confidence interval is marginally shorter than the average length of the credible interval using left and right invariant priors. And for 2 values of C.V namely 0.3, 0.5 the average length is marginally shorter for equitailed credible interval compared to confidence interval. Further for smaller values of C.V from 0.1 to 0.7, the average length of the credible interval is marginally shorter than the average length of the confidence interval. Thus if we take the average length of the confidence/credible interval over the different values of C.V (for which confidence/credible level is maintained), the picture that emerges is the one given in table 2. Thus for larger values of C.V in several cases the confidence interval does not maintain the coverage probability while the credible interval maintains coverage probability at the expense of larger width.

## V. EXAMPLES

### 5.1 Locomotive Data

To illustrate the procedure we consider the data on the number of thousand miles at which different locomotive controls failed in a life test involving 96 controls. The test was terminated after 135,000 miles by which time 37 failures had occurred. The data were discussed by Schmee and Nelson (1977). This is a well known data set and is analyzed by several researchers in the past. The failure times for the 37 failed units are 22.5, 37.5, 46.0, 48.5, 51.5, 53.0, 54.5, 57.5, 66.5, 68.0, 69.5, 76.5, 77.0, 78.5, 80.0, 81.5, 82.0, 83.0, 84.0, 91.5, 93.5, 102.5, 107.0, 108.5, 112.5, 113.5, 116.0, 117.0, 118.5, 119.0, 120.0, 122.5, 123.0, 127.5, 131.0 132., 134.0. In addition, there are 59 censoring times, all equal to 135.0. Initially to check whether the underlined distribution is lognormal, Q-Q plot of the log of the failure time is plotted and is given in figure 1. From this figure it follows that log of the observations is normally distributed. The box plot given in figure 2 summarizes the characteristic features in the data. From the figure we observe that the first quartile $Q_1$=4.25, second quartile $Q_2$= 4.45 and third quartile $Q_3$=4.75. The distance between $Q_3$ to $Q_2$ is 0.3 and distance between $Q_2$ to $Q_1$ is 0.2. And thus the failure time data of locomotive is right skewed.

Table 4 reports the mean failure time using maximum likelihood estimator and Bayes estimator and also the associated 95% confidence/credible intervals. From the table it follows that the mean failure time is 90.36 years using the maximum likelihood estimator while it is 90.33 years and 97.4033 years using right and left invariant priors respectively. The length of the confidence interval is 19.68 and it is 24.39 for both left and right invariant priors. In the simulation experiment for $n$=40 the coverage probability is maintained only for one value of C.V=0.5 and in this case the average length of the confidence interval is slightly greater than the average length of credible intervals. If we ignore the coverage probability, the length of the confidence interval is greater for the values of C.V = 0.1 and 0.3 and for other values of C.V it is shorter than the length of the credible interval. This example confirms the finding of the simulation study.

### 5.2 Stock price data

This example illustrates the use of Bayes estimator for predicting the future observation. Given a sequence of observation $Y_1, Y_2, \ldots, Y_n$, the predicted value for the observation at time point n+1 is the mean of the posterior distribution (Berger (2005), Ghosh *et al.* (2006)). The data consists of the average daily stock prices of ICICI bank of National Stock Exchange (NSE) limited of India, for the period October, 2013. The date for which the stock price to be predicted was arbitrarily decided as October 31st, 2013. Using 5, 10, 15 and 20 days of the stock price prior to this date, the value of the stock price is predicted for October 31st, 2013. Table 5, reports the predicted value and the associated 95% equitailed credible interval along with the reported average price for October 31st. The results indicate that previous 5 days average price can accurately predict the stock price for the 6th day.

## VI. CONCLUSION

Bayesian analysis is nowadays very common in scientific investigations. The uncertainty in the parameters of the probability model can very well be captured by two prior distributions. The use of objective priors facilitates the comparison with the Frequentist approach. In this paper Bayes estimator and the associated credible intervals are derived for the mean of lognormal distribution. The results of the simulation study indicates that the credible level is maintained for a sample size $n$=100, while the confidence interval requires a larger sample size $n$>200. The average length of the equitailed credible interval is at par with the length of confidence interval whenever the latter maintains the coverage probability. Since the confidence interval using maximum likelihood estimator is approximate it is quite natural that in more number of cases the Bayesian credible interval maintains coverage probability compared to the confidence interval obtained through the maximum likelihood estimator. Right invariant prior has not been used in the previous investigation to compare the coverage probability of the credible and confidence intervals. The Bayesian robustness of the credible interval is also established by using the two objective priors. Therefore we recommend the use of Bayes estimator of the mean and associated credible intervals using the non informative right invariant prior for the analysis of real time data as well as stock market data. The conclusion complements the findings of Harvey and Merwe (2010) and Harvey *et al.* (2011). For the Bayesian computation a program was written in the public domain software R and Matlab and interested persons can obtain it from the first author.

## APPENDIX

**Alternate approach to find Bayes estimator of the mean of the Lognormal distribution.**

The Bayes estimator of the mean of the lognormal distribution is given by

$$\int_{-\infty}^{\infty}\int_{0}^{\infty} e^{\mu+\frac{1}{2\eta n}} \frac{\sqrt{n}}{\sqrt{2\pi}}\eta\, e^{-\frac{\eta}{2}(\bar{z}-\mu)^2 n} \frac{\left(\left(\frac{n-1}{2}\right) S_z{}^2\right)^{n+2}}{\Gamma\left(\frac{n+2}{2}\right)} \eta^{\left(\frac{n+2}{2}\right)} e^{-\frac{\eta}{2}(n-1)s_z{}^2}\, d\mu\, d\eta$$

Interchanging the order of integration this is equal to

$$\int_0^{\infty} e^{\bar{z}+\frac{1}{2\eta n}+\frac{1}{2\eta}} \frac{\left(\left(\frac{n-1}{2}\right) S_z{}^2\right)^{n+2}}{\Gamma\left(\frac{n+2}{2}\right)} \eta^{\left(\frac{n+2}{2}\right)} e^{-\frac{\eta}{2}(n-1)s_z{}^2}\, d\eta$$

This integral cannot be evaluated in a closed form. To evaluate the integral, importance sampling approach can be used. For this, generate M observations from the gamma density with parameters $\left(\frac{n+2}{2}\right)$ and $\left(\frac{n-1}{2}\right) S_z{}^2$. Then the value of above integral is $\frac{1}{M}\sum_{i=1}^{M} e^{\bar{z}} e^{\frac{1}{2n\eta_i}+\frac{1}{2\eta_i}}$. Because of the one to one relation between $\eta$ and $e^{\frac{1}{2n\eta}+\frac{1}{2\eta}}$, the HPD credible interval for the mean of the lognormal distribution can be constructed using above gamma density. The procedure is similar as described by D'Cunha and Rao (2014).

## ACKNOWLEDGMENT

The first author would like to take the opportunity to thank Government of India, Ministry of Science and Technology, Department of Science and Technology, New Delhi, for sponsoring her with the award of INSPIRE fellowship, which enables her to carry out the research programme which she has undertaken. She is much honored to be the recipient of this award.

## References

[1] Zellner, A., "Bayesian and non-Bayesian analysis of the log-normal distribution and log-normal regression," *Journal of the American Statistical Association*, vol.66, 1971, pp.327-330.

[2] Ghosh, J. K., Delampady, M., &Samanta, T.,*An introduction to Bayesian Analysis: Theory and Methods.* Springer texts in Statistics, USA, 2006.

[3] Padgett, W. J.,& Wei, L. J.,"Bayes estimation of reliability for the two-parameter lognormal distribution,"*Communications in Statistics—Theory and Methods*, vol 6, 1977, pp. 443-457.

[4] Padgett, W. J., &Johnson, M. P., "Some Bayesian lower bounds on reliability in the lognormal distribution," *Canadian Journal of Statistics*, vol. 11(2), 1983, pp. 137-147.

[5] Sarabia, J. M., Castillo, E., Gomez, E., & Vazquez-Polo, F. J., " A class of conjugate priors for lognormal claims based on conditional specification,"*The Journal of Risk and Insurance*, vol. 72(3), 2005, pp. 479-495.

[6] Harvey, J., & Merwe A. J. V., " Bayesian confidence intervals for the ratio of means of lognormal data with zeros," 2010,Microsoft word-Article.

[7] Zhou, X.H., Tu, W., " Interval estimation for the ratio in means of log-normally distributed medical costs with zero values,"*Computational Statistics and Data Analysis*, vol. 35, 2000,pp. 201 – 210.

[8] Krishnamoorthy, K., & Mathew, T., "Inferences on the means of lognormal distributions using generalized P-values and generalized confidence intervals,"*Journal of Statistical Planning and Inference*, vol. 115, 2003, pp.103 - 121.

[9] Harvey, J., Groenewald, P. C. N., & Merwe A. J. V., " Bayesian confidence intervals for the mean of a lognormal distribution: A comparison with the MOVER and generalized confidence interval procedures," 2011, Technical Report.

[10] Harvey, J., & Merwe A. J. V.,"Bayesian confidence intervals for means and variances of lognormal and bivariate lognormal distributions," *Journal of Statistical Planning and Inference*, vol. 142(6), 2012, pp.1294-1309.

[11] D'Cunha, J. G.&Rao, K. A., " Bayesian inference for volatility of stock prices (Accepted for publication.)," *JMASM-Journal of Modern Applied Statistical Methods*, 2014, to be published.

[12] Berger, J. O.,*Statistical decision theory and Bayesian analysis (Second Edition).*New York: Springer-Verlag, 2005.

[13] Schmee, J., and Nelson, W. B., "Estimates and Approximate Confidence Limits for (Log) Normal Life Distributions from Singly Censored Samples by Maximum Likelihood," General Electric Company Corporate Research and Development TIS Report 76CRD250.*, 1977

[14] Kale, B. K.,*A First Course on Parametric Inference (Second Edition)*. Narosa , 1999.

## Authors

**First Author** – Juliet Gratia D'Cunha, M.Sc, Mangalore University, Mangalagangothri, Karnataka, India.
Email: gratiajuliet@gmail.com
**Second Author** – Aruna Rao K, Ph.D,Mangalore University, Mangalagangothri, Karnataka, India.

**Correspondence Author** – Juliet Gratia D'Cunha, gratiajuliet@gmail.com.

**Table 1: Range of values of $\sigma^2$ for various values of C.V of lognormal distribution**

| C.V | $\sigma^2$ |
|---|---|
| 0.1 | 0.01 |
| 0.3 | 0.0862 |
| 0.5 | 0.2231 |
| 0.7 | 0.3988 |
| 1 | 0.6931 |
| 1.5 | 1.1787 |
| 2 | 1.6094 |
| 2.5 | 1.9810 |
| 3 | 2.3026 |
| 3.5 | 2.5840 |
| 4 | 2.8332 |
| 4.5 | 3.0564 |
| 5 | 3.2581 |
| 10 | 4.6151 |

**Table 2: Coverage probability of the credible and confidence interval for the C.V across sample sizes for 8 combinations of specified values of C.V**

| Sample size | Bayes Procedure (Equitailed) | | | | Maximum Likelihood (Equitailed) | |
|---|---|---|---|---|---|---|
| | No. of times the Coverage probability is maintained | | Average length | | No. of times the Coverage probability is maintained | Average length |
| | Right invariant prior | Left Invariant prior | Right invariant prior | Left Invariant Prior | | |
| 10 | 0 | 0 | * | * | 0 | * |
| 20 | 0 | 0 | * | * | 1 | 262.38 |
| 40 | 1 | 1 | 306.14 | 306.14 | 1 | 308.37 |
| 60 | 5 | 5 | 597.81 | 597.81 | 4 | 236.13 |
| 80 | 6 | 6 | 344.84 | 344.84 | 5 | 223.35 |
| 100 | 7 | 7 | 340.84 | 340.84 | 3 | 116.71 |
| 150 | 6 | 6 | 253.33 | 253.33 | 5 | 145.73 |
| 200 | 7 | 7 | 306.75 | 306.75 | 5 | 254.76 |
| Overall | 32 | 32 | 358.28 | 358.28 | 24 | 221.06 |

*Whenever coverage probability is not maintained average length has not been calculated.

**Table 3: Average length of the confidence/credible interval for various values of C.V and sample sizes.**

| Sample size | Confidence/ credible interval based on | Average length (Coverage probability) when C.V equal to | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.1 | 0.3 | 0.5 | 0.7 | 1 | 1.5 | 2 | 2.5 |
| 100 | M.L.E | 39.13 (0.952) | 116.77 (0.947) | 194.23 (0.950) | 269.57 (0.936) | 377.31 (0.935) | 541.30 (0.938) | 667.99 (0.910) | 798.70 (0.934) |
| | right invariant prior | 38.80 (0.956) | 116.11 (0.943) | 194.25 (0.950) | 269.25 (0.944) | 381.00 (0.948) | 555.69 (0.951) | 691.28 (0.939) | 830.75 (0.950) |
| | Left invariant prior | 38.81 (0.956) | 116.11 (0.943) | 194.25 (0.950) | 269.25 (0.944) | 381.00 (0.948) | 555.70 (0.951) | 691.28 (0.939) | 830.75 (0.950) |
| 150 | M.L.E | 31.92 (0.942) | 95.89 (0.947) | 159.28 (0.952) | 219.26 (0.943) | 308.41 (0.932) | 441.57 (0.945) | 548.62 (0.939) | 652.95 (0.927) |
| | right invariant prior | 31.75 (0.944) | 95.48 (0.951) | 158.88 (0.953) | 220.21 (0.945) | 310.44 (0.939) | 449.33 (0.942) | 564.31 (0.948) | 670.95 (0.932) |
| | Left invariant prior | 31.75 (0.944) | 95.48 (0.951) | 158.88 (0.953) | 220.21 (0.945) | 310.44 (0.939) | 449.33 (0.942) | 564.31 (0.948) | 670.95 (0.932) |
| 200 | M.L.E | 27.64 (0.938) | 83.18 (0.952) | 137.61 (0.948) | 191.69 (0.954) | 266.41 (0.939) | 385.74 (0.948) | 475.56 (0.942) | 553.06 (0.918) |
| | right invariant prior | 27.52 (0.940) | 82.86 (0.949) | 137.41 (0.948) | 192.22 (0.955) | 267.82 (0.952) | 389.90 (0.948) | 482.29 (0.953) | 567.23 (0.939) |
| | Left invariant prior | 27.52 (0.940) | 82.86 (0.949) | 137.41 (0.948) | 192.22 (0.955) | 267.82 (0.952) | 389.90 (0.948) | 482.29 (0.953) | 567.23 (0.939) |

**Table 4: Mean life time of locomotive using Maximum Likelihood Estimator and Bayes Estimator and associated 95% confidence/credible interval**

| Maximum Likelihood Estimation | | Bayes Estimation | | |
|---|---|---|---|---|
| | | | Right invariant | Left invariant |
| MLE | 90.36 | Bayes Estimator | 90.33 | 97.40 |
| Confidence Interval | (80.51,100.21) | Credible Interval | (79.42,103.81) | (79.42,103.81) |

**Table 5: Prediction of stock price for October 31st, 2013 (the 6th day) using Bayes estimation for the script ICICI BANK**

| Average price reported by NSE | Bayes Estimator | | Credible interval | |
|---|---|---|---|---|
| | Right invariant | Left Invariant | Right invariant | Left invariant |
| 1107.95 | 1052.10 | 1047.20 | (1008.2, 1101.3) | (1028.5, 1066.1) |

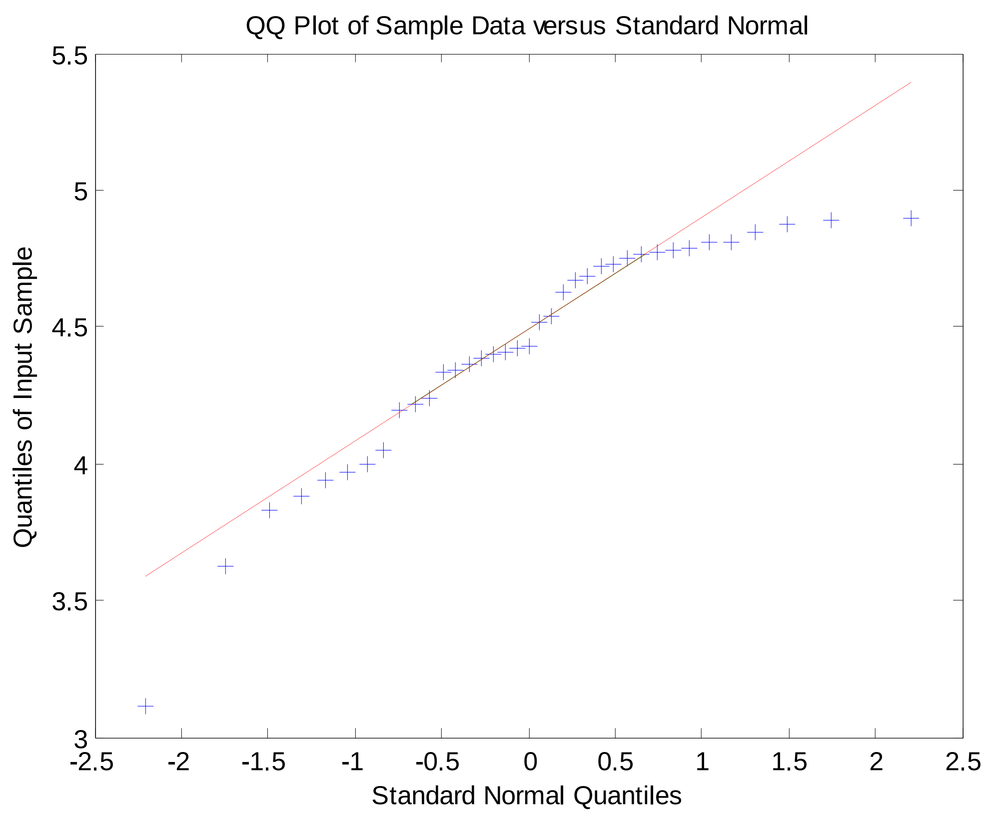


**Figure 1: Q-Q plot of log of failure times**

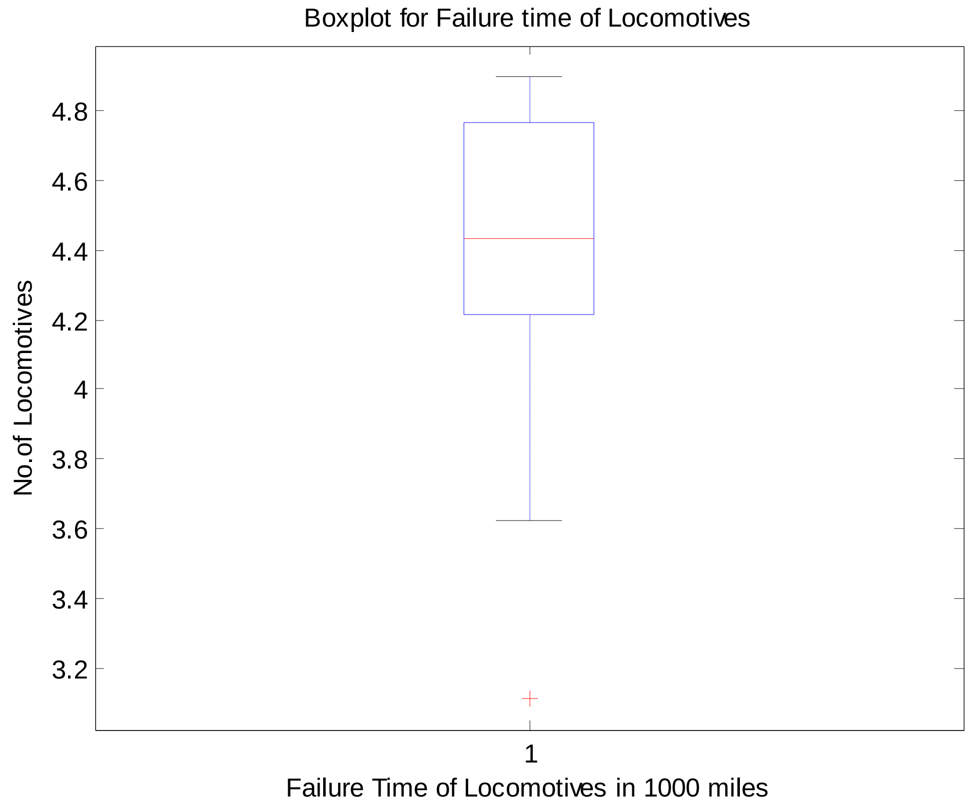


**Figure 2: Boxplot for Failure time of Locomotives**