On Revisiting the Name of the Chi-squared Test for Homogeneity of Proportions

Bernard L. Dillard
Professor of Mathematics
Fashion Institute of Technology

Department of Science and Mathematics
New York, NY 10001

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ABSTRACT

Approximately a century and a quarter ago, Karl Pearson placed his statistical stake in the ground, coining his famous chi-squared test for comparing differences between proportions. A staple methodology in the statistical community—both domestic and international—it is the go-to path taken in reference to exploiting truths involving basic contingency tables. Upon closer inspection, however, one discovers that this notion, in all of its glory and splendor, has endured an entire lifetime with a glaring misnomer. Its updated name should be obvious.

KEYWORDS Chi-squared test for homogeneity of proportions, proportion, ratio, test of independence

A BRIEF LOOK BACK

Elevated to almost-genius status, Karl Pearson occupies rarefied air with respect to his contributions to the field of statistics. While many progenitors laid much of the foundation within the discipline in their own right, Pearson surely claims the lion's share of respect for laying much of the groundwork for mathematical statistics (Hutcheson & Brown 2024). A number of topics find refuge under the umbrella of Pearson's extensive expertise and intellectual guidance. From regression to hypothesis testing to basic notions of standard deviation, Pearson's influence is vastly felt by budding and veteran statistical scholars alike. Perhaps the most known of his contributions is the chi-squared test for the equality of two proportions or, more generally, the chi-squared test for homogeneity of proportions.

WHAT WE THOUGHT WE KNOW

In any chi-squared test in which the equality of proportions is tested, the null hypothesis generally states that all population proportions are equal, whereas the alternative states that at least one pair of population proportions is significantly different. One remembers that it is typical to display both arguments as such:

$$H_0: \pi_1 = \pi_2 = \pi_3 = \ldots = \pi_k$$

H_1: not all π_k's are equal.

Decisions of rejection or acceptance of the hypotheses are buoyed by calculations from a standard contingency table, where observed values are analyzed alongside expected values. After the addition of several iterations of values in each cell, a chi-squared test statistic is compared to a cutoff value or a *p*-value to alpha. Finally, one concludes that population proportions are equal. Or some may not be. If not, one invokes the Marascuilo procedure to zero in on which pairs are significantly different. That is the end of the story. Maybe.

WHAT WE KNOW BUT OVERLOOKED

For ages, the mathematical community has defined a classical ratio as a way of comparing two or more quantities—a quantitative relation between two values, whereby one such value is "contained" within the other (Mooldijk et al. 2025).

In non-mathematical or -statistical environments, the term proportion has undergone a marked evolution. Having Latin etymological roots around the 14th century, it initially carried the basic idea meaning "share, part, or for the part" (Cohen 2014). Later morphing into its modern-day interpretation, the idea began to encompass two notions in relation to each other being measured against two other notions also in relation to each other (Wittkower 1960).

In statistical spheres, however, the concept of proportion has remained rather constant, avoiding any significant etymological change. It has consistently tended to encapsulate this idea of the comparison of two unique sets of relationships (or ratios). Born ages before the 14th century, Eudoxus, one of the most significant contributors to Euclid's *Elements*, summarizes the essence of proportions in Definition 5 of Book V, in which

> [m]agnitudes are said to be in the same ratio, the first to the second and the third to the fourth. [This is so] when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order. (Byrne 1847, p. 153).

Stated even more simply in Book VI, Euclid posits, "Numbers are proportional when the first is the same multiple, part, or parts of the second that the third is of the fourth" (Fine 1917, p. 71). In sum, over centuries, within mathematical and statistical contexts, a proportion has consistently been viewed as this equality of two ratios (Son 2013).

Especially within the field of geometry, the concept of proportion connotes this equality of pairs. Specifically, the notion of the Divine Proportion links two sets of ideas related to the Golden Ratio. Although reference to this Ratio and Proportion are sometimes used interchangeably, it is technically fallacious, perhaps dating back to Luca Pacioli's alleged plagiarism of geometric and arithmetic perspectives espoused by della Francesca (Magnaghi-Delfino & Norando 2018). Pacioli in his *Divina proportione* arguably lifted several of the painter's ideas without consideration for specific meanings undergirding relevant concepts. Hence, his coining of the Divine Proportion nomenclature as a substitute for the Golden Ratio is widely known and accepted but is inaccurate.

The Golden Ratio summarizes the relationship between two numbers such that their quotient is 1.618. Discrete mathematics highlights the convergence of the Fibonnaci sequence to this very value, or phi. Of course, its application to architecture, shapes, and even physical human beauty markers suggests that two quantities whose ratios possess this numerical quality retain a certain uniqueness within their own realm of existence. Or, given a and b, $\frac{a}{b} = \phi$.

With respect to the Divine Proportion, nonetheless, given a and b, the ratio of their sum to the larger of the two numbers is equal to the ratio of the same larger number to the smaller number. Or, $\frac{a+b}{a} = \frac{a}{b}$. One may recall that this specific property holds true conformity among sides of the Golden Rectangle. This delineation between ratio and proportion, though seemingly trivial, remains consistent, even within this mathematical realm.

WHAT WE HAVE TO KNOW OUT LOUD

Against the backdrop of this terse but distinct history exists the issue of primary focus in the stated statistical observation and arises a simple proposal. It must be the case that the historical chi-squared test that assesses the homogeneity of *proportions* should thus be rebranded as the chi-squared test that assesses the homogeneity of *ratios*. Although it has existed under the former nomenclature for ages on end, the test must be renamed to reflect the real essence of the term that has remained consistent in mathematical and statistical circles.

The previously known chi-squared test for the homogeneity of proportions shall henceforth and forever be referred to as the chi-squared test for the homogeneity of ratios. In the pervasive specific case of a *2 x 2* contingency table, we dub it simply as the chi-squared test for the difference between two ratios (not proportions).

HOW TO SHOW WHAT WE KNOW: A CALL TO ACTION

As the statistical community basks in the celebration of chi-squared's 125th birthday this year, let there be, henceforth and forever, a relabeling of the technique's name in lectures, in workshops, in seminars, in conferences, and in publications to adequately reflect the true essence of the term's denotation.

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