Unsteady boundary layer of a Micropolar fluid flow past a moving wedge

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Abstract- We present an analytic and asymptotic solution of an unsteady laminar boundary layer of micropolar fluid flow past a moving wedge. Similarity transformations reduce the number of independent variables of partial differential equations in the governing system to a coupled system of ordinary differential equations. An exact solution obtained for particular values of parameters are extended to obtain an analytical solution for more general values of the parameters involved. Analytical results are consistent with the numerical results obtained by employing implicit finite difference method. The variations with position, material parameter and time, shown by Velocity, shear stress and gyration profiles obtained from both the solutions are analyzed.

Index Terms- Unsteady, Laminar boundary layer, Similarity transformations, Exact solution, Asymptotic solution

I. Introduction

Most of the fluid flows in real life are unsteady. Flow of water from compressor pump is a perfect example for an unsteady flow as the velocity changes with respect to time. Also it is not always possible to maintain steady state conditions in fluid flows. Thereby, it becomes important to shift our focus from steady to unsteady flows. Unsteadiness is an inevitable feature in many engineering machinery. Some of the unsteady fluid flows of practical interest are the helicopter rotor, the cascades of turbo, machinery blade, the ship propeller and so forth.

In the past few decades, many authors have be successful in finding numerical solutions of unsteady micropolar fluid flows with certain special boundary conditions using different mathematical approaches. Govardhan and Kishan[1] have investigated the MHD effects on the early unsteady boundary layer flow over a stretching sheet and solved using Adams Predictor-Corrector method of fourth order. Saleh et al[2] worked on unsteady micropolar fluid over a permeable curved stretching and shrinking surface and have solved numerically using shooting method. Nazar et al[3] worked on analysis of unsteady boundary layer flow and heat transfer of micropolar fluid flow over a stretching sheet and solved the system numerically using Keller box method. Lok et al [] in their paper analyzed the growth of unsteady boundary layer flow of a Micropolar fluid which was started impulsively from rest near the forward[4] and also rear[5] stagnation point and solved numerically using Keller box method. Kumari and Nath[6] considered the flow , heat and mass transfer on the unsteady laminar layer in micropolar fluid flow at the stagnation point and have solved numerically using a quasilinear finite difference scheme. Many authors have employed a solution methodology based on the group theoretic method to reduce the number of independent variables of the partial differential equations of the governing equations and convert it into a system of ODE which is then solved by any of the DNS.

In this paper, we propose exact solution of an unsteady, laminar, incompressible, two dimensional boundary layer of micropolar fluid flow past a moving wedge. Exact solutions of the fluid flows are rare in fluid mechanics due to the complexity of the problem with an extra independent time variable even more so when immersed in micropolar fluid with microrotating microelements [23]. Also, nonlinear problems do not permit a superposition principle thereby ruling out the building up of complex solutions of simple ones. But exact solutions are important in their own right as solutions of particular problems but also more important in checking accuracy of numerical solutions. The exact method employed to obtain solution of two- dimensional, laminar, incompressible, unsteady boundary layer of micropolar fluid flow past a moving wedge is based on the derivation obtained by Kolomenskiy and Moffat [2012] which is similar to the derivation from the Lighthill's complex potential theory [8]

II. FORMULATION

An unsteady, two dimensional laminar boundary layer flow of an incompressible, viscous, micropolar fluid over a wall of a moving wedge with a constant velocity $U_w(x,t)$ is considered. x - axis is taken parallel to the wedge and y-axis is normal to it. Under usual boundary layer approximations [9] the governing equations for micropolar fluid flow past a wedge moving in the non-dimensional form with the absence of body forces and body couples, are

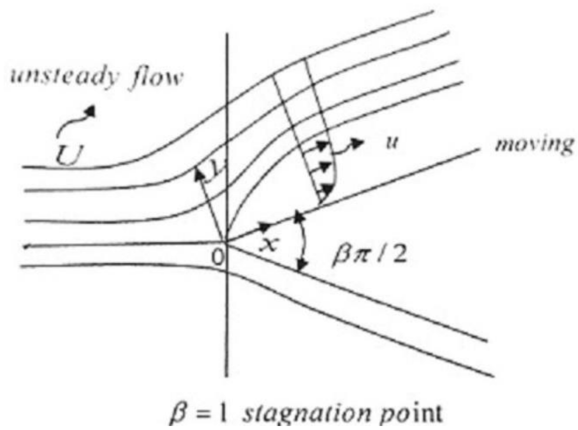


Figure1: Schematic diagram of boundary layer

Conservation of mass: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Conservation of momentum: $$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=U\frac{dU}{dx}+(\mu+\chi)\frac{\partial^2 u}{\partial y^2}+\chi\frac{\partial \omega}{\partial y} \tag{2}$$

Conservation of angular momentum: $$\rho j\left(\frac{\partial \omega}{\partial t}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}\right)=-\frac{\partial}{\partial y}\left(v\frac{\partial \omega}{\partial y}\right)-k\left(2\omega+\frac{\partial u}{\partial y}\right) \tag{3}$$

Conservation of micro-inertia: $$\frac{\partial s}{\partial t}+u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}=0 \tag{4}$$

where u and v are the velocity components in the x and y directions , ν is the kinematic viscosity of the fluid , $U(x,t)$ is the free stream velocity given by the power law $U(x,t)=U_\infty A(t)x^m$ [7] where x is the distance measured from the onset of the boundary layer, $A(t)>0$,U_∞, m are constants and $U_w(x,t)$ is the stretching surface velocity which obeys the power-law relation $U_w(x,t)=U_{0w}A(t)x^m$. Boundary conditions on velocity and microrotation are

$$\text{at } y=0, \quad u=U_w(x,0), v=0, w=-\frac{1}{2}\frac{\partial u}{\partial y} \tag{5}$$

$$\text{as } \frac{y}{\delta}\to\infty, \quad u=U, v=0, w=0 \tag{6}$$

Introducing the stream function $\psi(x,y,t)$ with $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$ and adopting the co-ordinate transformations from the variables (x,y) to the new dimensionless similarity variables([10] [11] [12])

$$y=\left(\frac{2\gamma x}{(m+1)U}\right)^{\frac{1}{2}}\eta,\quad \psi=\left(\frac{2\gamma xU}{(m+1)}\right)^{\frac{1}{2}} f(\eta),\quad \omega=U\left(\frac{(m+1)U}{2\gamma x}\right)^{\frac{1}{2}} h(\eta),\quad s=\left(\frac{2\gamma x}{(m+1)U}\right)^{\frac{1}{2}} i,$$

$$\rho=\left(\mu+\frac{\chi}{2}\right)\left(\frac{2\gamma x}{(m+1)U}\right)^{\frac{1}{2}} i, \tag{7}$$

The boundary layer equations transform to the following non-linear ordinary differential equations

$$(1+k)f'''+ff''+\frac{2m}{m+1}\left(1-f'^2\right)+kh'=D\left(\frac{\eta}{2}f''+f'-1\right) \tag{8}$$

$$(1+\frac{k}{2})(ih')'+i\left(fh'-\frac{(3m-1)}{(m+1)}hf'\right)-k(2h+f'')+Dhi=0 \tag{9}$$

$$Di-(1-m)if'+\frac{(m+1)}{2}fi'=0 \tag{10}$$

where $D=\left(\dfrac{2U_\infty^{m-2}A'(t)}{(m+1)\gamma^{m-1}A^2(t)}\right)^{\frac{1}{2}}$.

The boundary conditions are

$$f(0)=0,\ \ f'(0)=-\lambda,\ \ i(0)=0,\ \ h(0)=-\frac{1}{2}f''(0) \tag{11}$$

$$f'(\infty)\to 1,\quad h(\infty)\to 0$$

where η is a new similarity variable, $f(\eta)$ is the non-dimensional stream- function and $\lambda(=-\dfrac{U_{0w}}{U_\infty})$ is the ratio of free stream velocity and boundary value $\lambda<0$ corresponds to wedge moving in the direction of stream velocity whereas and $\lambda>0$ corresponds to that of opposite direction. $k=\dfrac{\chi}{\mu}$ where $\mu=\rho\gamma$ is the dimensionless viscosity ratio[22]. the stream wise pressure gradient is favorable pressure gradient when m > 0 and adverse pressure gradient when m < 0 whereas Blasius flow over a flat plate when m = 0[12]. The flow corresponding to stagnation point when m = 1[8] .

III ANALYTICAL SOLUTION

When D = 0, the solution of micro inertia density equation (10) satisfying boundary conditions (11) is

$$i=Af^{\frac{2(m-1)}{(m+1)}} \tag{12}$$

where $A=C^{\frac{2}{(m+1)}}$ is a non- dimensional constant of integration. Using the boundary condition i(0) = 0 leads to i(η) = 0 which is a trivial solution, in which case (9) reduces to

$$h = -\frac{1}{2} f'' \tag{13}$$

substituting (12) and (13) in (8) we get

$$(1+\frac{k}{2}) f''' + ff'' + \frac{2m}{m+1}\left(1 - f'^2\right) = 0 \tag{14}$$

In this paper, we obtain the solution analytically, asymptotically and numerically of this equation with the boundary conditions

$$f(0) = 0, \qquad f'(0) = -\lambda, \qquad f'(\infty) \to 1 \tag{15}$$

where primes denote differentiation with respect to η .We seek exact solution[16] of (14) with (15) .Exact Solution of (13) for m = -1/3[17] is obtained by integrating (14)twice and applying the boundary conditions (15)which results in a Riccati type equation and leads to the solution of (14)as

$$f = \eta + \delta - \frac{\delta\, e^{-(1+\frac{k}{2})^{-1}(\frac{\eta^2}{2} + \delta\eta)}}{1 - \frac{\delta(1+\frac{k}{2})^{\frac{-1}{2}}}{2} e^{\frac{\delta^2}{2(1+\frac{k}{2})}} \sqrt{\frac{\pi}{2}} \left[erf\left(\frac{(1+\frac{k}{2})^{\frac{-1}{2}} (\eta+\delta)}{\sqrt{2}} \right) - erf\left(\frac{(1+\frac{k}{2})^{\frac{-1}{2}} (\delta)}{\sqrt{2}} \right) \right]} \tag{16}$$

Provided $\delta^2 = -\frac{2(1+\lambda)}{\left(1+\frac{k}{2}\right)}$ .To obtain an exact analytical solution of the system (13) with (14) for different values of m and k, we rewrite the solution (15) as

$$f(\eta) = \eta + \delta - \frac{\delta}{G(\eta)} \tag{17}$$

where

$$G(\eta) = e^{\frac{\delta^2 (1+\frac{k}{2})}{2}} - \frac{\delta(1+\frac{k}{2})^{\frac{-1}{2}}}{2} e^{\frac{\delta^2}{2(1+\frac{k}{2})}} \sqrt{\frac{\pi}{2}} \left[erf\left(\frac{(1+\frac{k}{2})^{\frac{-1}{2}} (\eta+\delta)}{\sqrt{2}} \right) - erf\left(\frac{(1+\frac{k}{2})^{\frac{-1}{2}} (\delta)}{\sqrt{2}} \right) \right] e^{\left(1+\frac{k}{2}\right)^{=1}\left(\frac{\eta+\delta}{2}\right)^2} \tag{18}$$

For any m and D eqn (7) When i(0)=0 and $h = -\frac{1}{2} f''$ is

$$(1+\frac{k}{2}) f''' + ff'' + \frac{2m}{m+1}\left(1 - f'^2\right) = D\left(\frac{\eta}{2} f'' + f' - 1\right) \tag{19}$$

$$f(0) = 0, \qquad f'(0) = -\lambda, \qquad f'(\infty) \to 1 \tag{20}$$

Substituting (17) into (19) and (20), we get

$$\left(1+\frac{k}{2}\right)\left(G^2G''' - 6GG'G'' + 6G'^3\right) + (\eta + \delta - \frac{D\eta}{2})G^2G'' - 2(\eta + \delta - D)GG'^2 - \left(\frac{4m}{m+1} + D\right)G^2G' + \delta\left(2 - \frac{2m}{m+1}\right)G'^2 - \delta GG'' = 0 \tag{21}$$

with the boundary conditions

$$G(0) = 1,\ G'(0) = \frac{\delta}{2},\ G'(\infty) = 0. \tag{22}$$

The solution of (21) for $m = \frac{-1}{3}$, subject to (22) is given by (16). The error and exponential functions in equation (18) are entire functions with infinite radius of convergence about $\eta = 0$ and therefore can be expanded using Taylor series.

Further the solution (17) which is in series representation [20] for $m = \frac{-1}{3}$, plays an important role in further analysis for general values of m. Thus we let

$$G(\eta) = \sum_{n=0}^{\infty} a_n \eta^n \tag{23}$$

for general m and k. Substituting (23) into (21) and equating the coefficients of η^n to zero we get the coefficients a_n and in general

$$\begin{aligned} a_{n+3} = \frac{-1}{\left(1+\frac{k}{2}\right)(n+1)(n+2)(n+3)} & \left(-\left(1+\frac{k}{2}\right)\delta \sum_{i=1}^{n} (i+1)(i+2) a_{n-i} a_{i+2} + \left(\frac{-2}{m+1}\right)(n-i+1) a_{n-i+1} a_{i+1} \right) \\ & + \sum_{j=0}^{n-1} \Bigg(\sum_{i=0}^{n-j} \left(1+\frac{k}{2}\right)(j+3)((j+2)(j+1) a_{n-j-i} a_i a_{j+3} + \sum_{i=0}^{n-1-j} \left(1-\frac{D}{2}\right)(j+1)(j+2) a_i a_{j+2} - 2(i+1) a_{i+1} a_{j+1}) a_{n-j-i-1} \Bigg) \\ & + \sum_{j=0}^{n} \sum_{i=0}^{n-j} (j+1)(-6\left(1+\frac{k}{2}\right)(j+2)(i+1) a_{n-j-i} a_{i+1} a_{j+2} + 6\left(1+\frac{k}{2}\right)(i+1)(n-j-i+1) a_{n-j-i+1} a_{i+1} a_{j+1} \\ & + \left(1+\frac{k}{2}\right)\delta(j+2) a_{n-j-i} a_i a_{j+2} - \left(1+\frac{k}{2}\right)(2\delta + D)(i+1) a_{n-j-i} a_{i+1} a_{j+1} - \left(\frac{4m}{m+1} + D\right) a_{n-j-i} a_i a_{j+1})) \end{aligned} \tag{24}$$

where n = 1, 2, 3... and the coefficients a_n have been expressed in terms of a_2, δ, k, m. The value of coefficient of skin friction a_2 that satisfies the derivative boundary condition at far away from the wall has to be determined. This is same as determining the value of either a_2 of series (20) or $f''(0)$ of the system (14) and (15) as they are intrinsically related to each other by the following

$$a_2 = \frac{2f''(0) + \left(1+\frac{k}{2}\right)\delta^2}{4\left(1+\frac{k}{2}\right)\delta} \tag{25}$$

The coefficients a_n consists of two arbitrary constants, namely $f''(0)$ and δ.For $m = \frac{-1}{3}$, we match the series (23) with the exact solution (16) which gives $\delta = \frac{-4(1+\lambda)}{2+k}$, This constant δ plays an important role in this analysis. The solution of (14) exists only when the expression under the square root in δ,is positive. The other constant or a_2 needs to be determined. Thus, (14 - 15) have infinite solutions in the form of (23). The constant $f''(0)$ is determined in the following manner. We integrate (14) from $\eta = 0 \; or \, \eta = \infty$ and use (15) to get

$$\int_0^\infty \left(\frac{\left(1-\frac{D}{2}\right) f' - f'^2 + \frac{2m}{m+1}(1-f'^2)}{1+\frac{k}{2}} \right) d\eta + \frac{D}{2}\eta_\infty = f''(0) \tag{26}$$

Since skin friction $f''(0)$ appears on both sides of (25) and (26) it has to be determined iteratively using an appropriate initial approximations for it, taken from the known exact solution(16), (20) and (26) for all values of k, m and λ. $f''(0)$ converges when the derivative condition at far distance in (14) is satisfied (Kudenatti et al 2013). It is known that the series behaves well for small values of η enabling its integration. So Pade's approximants are used for the summation of the series. With an initial approximation of $f''(0)$ and a fewer iterations, $f''(0)$ can be obtained up to desired accuracy without any difficulty by numerically integrating the integral relation. Thus we obtain an exact solution of the equation for all the values of m, D and k. To prove the robustness of the method the values of skin friction $f''(0)$ obtained analytically are compared with that of direct numerical solution of the equation (14) with boundary condition (15) obtained using Keller Box method(Cebeci[14]) ,based on finite difference. It is observed that results agree well with the Numerical solution for all the values of parameters

IV ASYMPTOTIC SOLUTION

We analyze the far-field behavior of (19) with boundary condition (20) asymptotically for which we study large η behavior i.e $f'(\eta) = 1$ as $\eta \to \infty$ because the derivative boundary condition $f'(\eta)$ becomes linear as η increases away from zero. This helps us to define a new function

$$f(\eta) : \; \eta + E(\eta) \tag{27}$$

where $E(\eta)$ and their derivatives are assumed to be small. Substituting (23) with $f'(\eta) = 1 + E'(\eta) = 1 + F(\eta)$, $f''(\eta) = E''(\eta) = 1 + F'(\eta)$ and $f'''(\eta) = E'''(\eta) = 1 + F''(\eta)$

into (19) with the boundary conditions (20) and linearizing the resulting ordinary differential equation, we get

$$\left(1+\frac{k}{2}\right)F''(\eta)+\left(1-\frac{D}{2}\right)F'(\eta)-\frac{4m+D}{m+1}F(\eta)=0 \tag{28}$$

and boundary conditions take the form

$$F(0)=-(1+\lambda),\ \ F(\infty)=0 \tag{29}$$

whose solution eventually results in, Kummer's equation [27] with solution involving confluent hypergeometric series[26].Thus the solution to (28) is given by

$$F(\eta)=(1+\lambda)\left(-M\left(\frac{\frac{4m}{m+1}+D}{D-2},\frac{1}{2},\frac{-(2-D)}{4\left(1+\frac{k}{2}\right)}\eta^2\right)+\sqrt{\frac{2-D}{\left(1+\frac{k}{2}\right)}}\frac{\Gamma\left(1-\frac{\frac{4m}{m+1}+D}{D-2}\right)}{\Gamma\left(\frac{1}{2}-\frac{\frac{4m}{m+1}+D}{D-2}\right)}\eta M\left(\frac{1}{2}+\frac{\frac{4m}{m+1}+D}{D-2},\frac{3}{2},\frac{2-D}{4\left(1+\frac{k}{2}\right)}\eta^2\right)\right) \tag{30}$$

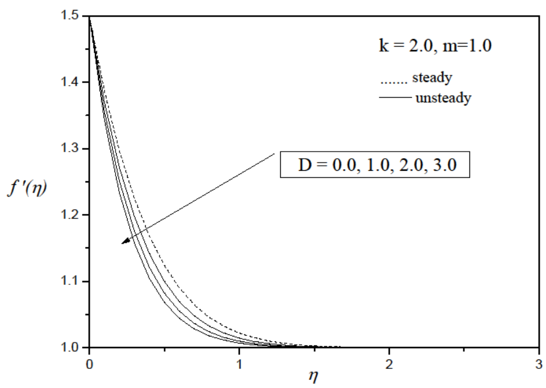
The solution in terms of $f(\eta)$ is given by $f'(\eta)=1+E'(\eta)=1+F(\eta)$

V RESULTS AND DISCUSSION

Exact solution of unsteady boundary layer of a micropolar fluid flow past a moving wedge is obtained by using error and exponential function for particular values of pressure gradient m and unsteady parameter D and is then extended using series solution to more general values of m, D. To substantiate the method of exact solution, the analytical results of skin friction are compared with those obtained by DNS (Keller box method) and presented in table 1. Analysis of velocity profiles helps to know the significance of the method employed and the physical nature of the unsteady micropolar flow in the boundary layer. Also, interesting physical dynamics of the model over the range of parameters is shown in the profiles.

Table 1: Comparison of the skin friction $f''(0)$ obtained by analytical method and numerical method.

	K = 0.0		K = 1.0		K = 2.0	
λ	Exact	Numerical	Exact	Numerical	Exact	Numerical
-1.1	-0.2000956	-0.201269	-0.161143	-0.162525	-0.137365	-0.139835
-1.2	-0.405645	-0.406175	-0.326743	-0.328015	-0.281375	-0.282235
-1.3	-0.613243	-0.614666	-0.494653	-0.4964291	-0.425126	-0.427165
-1.4	-0.824321	-0.826697	-0.665643	-0.667726	-0.5727634	-0.574589
-1.5	-1.040943	-1.042217	-0.840876	-0.841868	-0.722734	-0.724476



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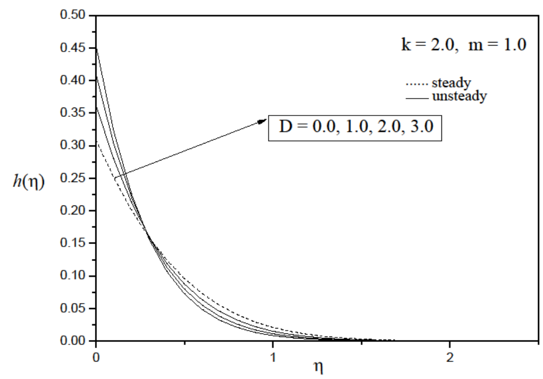
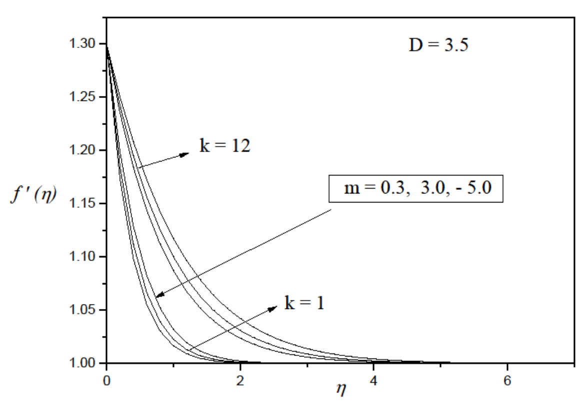


(3)

Figure : Variation of velocity profiles $f'(\eta)$ and $h(\eta)$ with η for different values of unsteady parameter D.

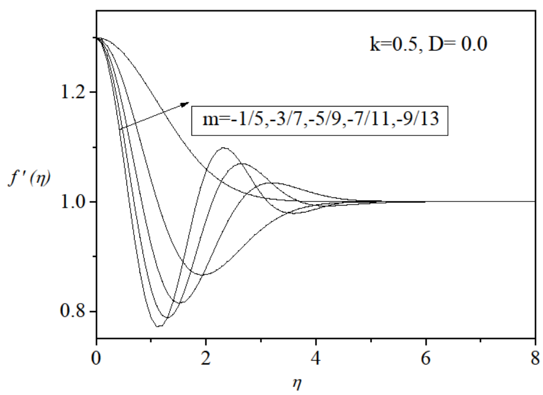
In figure 2, the velocity curves decrease monotonically to achieve the derivative boundary condition at infinity. When *D* = 0 velocity curve represents the steady flow model which is shown in dashed line in the figure. It is also observed that as unsteady parameter *D* increases the velocity of the fluid is found to be increasing which results into an increase in the Reynolds number, and thus, the boundary layer thickness is thinner. Since, for positive *D*, flow is considered to be accelerated, it is expected that the velocity is essentially increases. It is anticipated that, though the large *D* asymptotics is not performed here, for increasing D the flow turns out to be steady for which wall shear stress is almost constant. In figure 3, we also see the gyration profile $h(\eta)$ plotted against non-dimensional co-ordinate in which angular velocity decreases satisfying the boundary condition far away from the wedge wall. As D increases the angular velocity increases when closer to the wall but decreases with increase of D at a certain distance from the wall.



(4)

Figure : Variation of velocity profiles $f'(\eta)$ with η for different values of pressure gradient m for k=1 and k=12 when unsteady parameter is 3.5 is taken

In figure 4, as the pressure gradient increases the increases and the boundary layer thickness decreases.for a value of material parameter k =1.The same pattern is observed for k=12. However the velocity decreases with increase of the material parameter showing increase in the boundary layer thickness.



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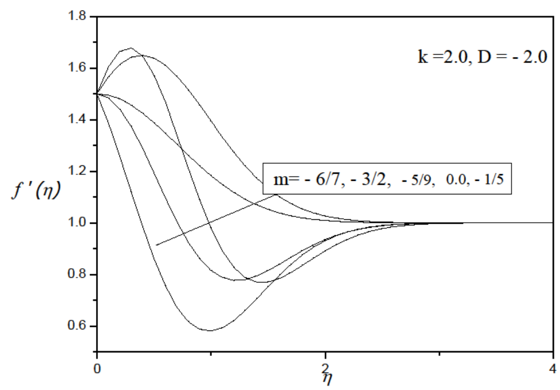


(6)

Figure : Asymptotic velocity profiles $f'(\eta)$ with η for different values of *m* when D=0 and D=-2.0

Figure 5 presents the asymptotic results obtained from (30) when values of pressure gradient *m* are held negative this corresponds to the adverse pressure gradient. As interesting velocity profiles are noticed in the unsteady boundary layer which are rather new. There are finite number of oscillations in the boundary layer for example *m* =-1/5 there are four modes in the velocity curve before decaying onto the mainstream, This corresponds to undershoots (i.e., $f'(\eta)$ <1for some η) in the boundary layer . Oskam and Veldman (1982) have also noticed the similar oscillatory-type boundary layer profiles for negative pressure gradient. The same typical trend is observed for all negative values of m.

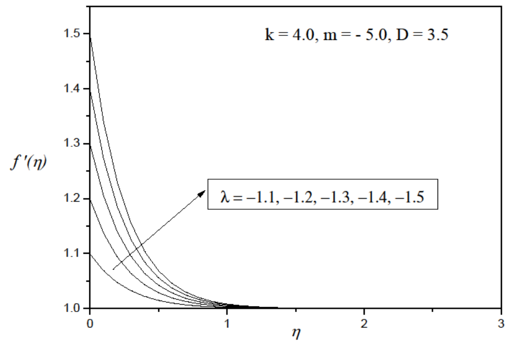
We intentionally plotted the velocity profiles for other set of *m* and *D* in figure 6. It is noticed that the same typical nature occurs quite often. Since *k* and *D* are different, we observe that there are less number of oscillations compared to the resulta of figure 5.

Table 2: Comparison of the skin friction $f''(0)$ obtained by asymptotic method with Numerical method.

	K = 0.0		K = 1.0		K = 2.0	
λ	Asymptotic	numerical	Asymptotic	numerical	Asymptotic	numerical
-1.1	-0.364038	-0.277144	-0.210177	-0.157831	-0.171609	-0.128443
-1.2	-0.728076	-0.561549	-0.420355	-0.319899	-0.343218	-0.260352
-1.3	-1.09211	-0.853064	-0.630532	-0.486115	-0.514827	-0.395656
-1.4	-1.45615	-1.151545	-0.840709	-0.656395	-0.686436	-0.534284
-1.5	-1.82019	-1.456854	-1.05089	-0.830662	-0.858045	-0.676173

Table 2 shows the comparison of the asymptotic values of skin friction with the numerical values. We see that the values agree closely with each other though asymptotic results are obtained at far distance. Hence, there is a slight variation in the skin-friction but however the corresponding velocity profiles satisfy the derivative conditions.

In figure 7, it is observed that wedge velocity for increasing λ is greater than the main stream velocity .Therefore different velocity nature is observed for different λ .For λ = -1.0, there is no boundary layer formation and hence coincides with wedge wall.



(7)

Figure: Asymptotic velocity profiles $f'(\eta)$ with η for different values of

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