An Inventory Model for Weibull deteriorating items with price-dependent and stock dependent demand taking shortages under inflation and time discounting in fuzzy and bi-fuzzy Environment

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Abstract - We have studied an inventory model for weibull deteriorating items with price-dependent and stock-dependent demand taking shortages under inflation and time discounting in fuzzy and bi-fuzzy environment. We have derived an inventory model for deteriorating items with stock and price dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon. This is the model with stock dependent consumption rate simultaneously considered the inflation and time value of money when shortages are allowed over a finite planning horizon. In addition, the effects of inflation and time value of money on replenishment policy under instantaneous replenishment with zero lead-time are also considered. Next we have extended this model in both fuzzy and bi-fuzzy environment. Numerical examples are being studied to illustrate the optimization procedure. The sensitivity analysis of the optimal solution with respect to parameters of the system is being carried out.

Index Terms - Inventory, Stock dependent and Price-dependent demand, Deterioration, Inflation, Shortages.

I. INTRODUCTION

The classical inventory models consider the demand rate to be either constant or time-dependent but independent of the stock status. However, for certain types of inventory, particularly consumer goods, the consumption rate may be influenced by the stock levels, that is, the consumption rate may go up or down with the on-hand stock level. The selling rate is assumed to be a function of current inventory level. Sarker et al.[16] have been done for order-level lot size inventory model with inventory-level dependent demand and deterioration. However, Sarker et al.[16] presented some assumptions that differ from this paper. That is, they assumed that there is the nature of decreasing demand, the replenishment rate is regarded to be finite and the planning horizon is infinite. Datta and Paul[4] analyzed an inventory system where the demand rate is influenced by both displayed stock level and selling price. More recently, Balkhi and Benkherouf[1] presented an inventory model for deteriorating items with stock-dependent and time-varying demand rates over a finite planning horizon.

The assumption that the goods in inventory always preserve their physical characteristics is not true in general because there are some items which are subject to risks of breakage, evaporation, obsolescence etc. Decay, change or spoilage that prevent the items from being used for its original purpose are usually termed as deterioration. Food items, pharmaceuticals, photographic film, chemicals and radioactive substances, to name only a few items in which appreciable deterioration can take place during the normal storage of the units. Incorporating the effects of deteriorating items and stock-dependent demand to develop the order-level lot size inventory models were discussed by Mandal and Phaujdar[11], Datta and Pal[3], Sarker et al.[16]. Roy et al. [15] developed an inventory model for a deteriorating item over a random planning horizon. Yao et al. [21], Liu[10] and Mendel[12] discussed inventory problems in the fuzzy sense. Prof. Zadeh[22] first applied a new concept Fuzzy Set Theory to accommodate the uncertainty in non-stochastic sense. Bi-fuzzy sets were originally presented by Zadeh [23]. In addition, numerous inventory models for deteriorating items under a permissible delay in payment have been studied by Jamal et al.[7], Sarker et al.[17], and Liao et al.[9]. Other issues relating to deterioration were also addressed by Chung et al.[2], Padmanabhan and Skouri[13], Goyal and Giri[5] and Teng et al.[18]. Jana et al.[8] discussed a bi-fuzzy approach to a production-recycling-disposal inventory problem with environment pollution cost via genetic algorithm.

One of the assumptions in most derivations of the inventory model has been a negligible level of inflation. But in recent times many countries have been confronted with fluctuating inflation rates that often have been far from negligible. Moreover, the effects of inflation and time value of money are vital in practical environment, especially in the developing countries with large scale inflation. Therefore, the effect of inflation and time value of money cannot be ignored in real situations. Datta and Pal[3] developed a model with linear time-dependent demand rate and shortages to investigate the effects of inflation and time value of money on ordering policy over a finite time horizon. Ray and Chaudhuri[14], Sarker et al.[17] and Wee and Law[19] all have investigated the effects of inflation, time value of money and deterioration on inventory models. Hou [6] formulated an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Here we have extended Hou's model by taking price dependent demand and formulated the corresponding fuzzy model in bi-fuzzy environment.
Models with stock-dependent consumption rate simultaneously considered the inflation and time value of money when shortages are allowed over a finite planning horizon. In this paper, a finite planning horizon inventory model for deterioration items with stock-dependent consumption rate and shortages is developed. In addition, the effects of inflation and time value of money on replenishment policy under instantaneous replenishment with zero lead-time are also considered. By incorporating the effects of deterioration rate, the inflation and time value of money on replenishment policy over a finite planning horizon and where the consumption rate is assumed to be dependent on current stock level. An optimization framework is presented to derive optimal replenishment policy when the present value of total cost is minimized. A numerical example is provided to illustrate the optimization procedure. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

II. NECESSARY KNOWLEDGE ABOUT FUZZY AND BI-FUZZY SETS

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let X be a collection of objects and x be an element of X, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$ which maps X to the membership space G which is considered as the closed interval $[0,1]$.

Fuzzy Number: A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on real line $\Re$ such that (i) it exists exactly one $x_0 \in \Re$ with $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{G}$),
(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Example 1: In particular if $\tilde{A} = (a_1, a_2, a_3)$ be a Triangular Fuzzy Number (TFN)(Fig. 1), then $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 < x \le a_3 \\ 0 & otherwise \end{cases}$$

where a_1, a_2 and a_3 are real numbers.

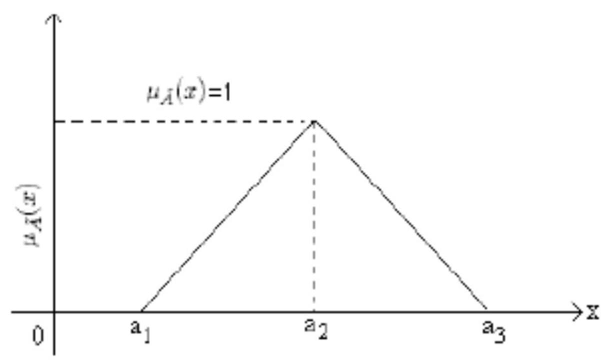

Fig. 1. Triangular Fuzzy Number(TFN)

Lemma-1: Let $\tilde{a} = (a_1, a_2, a_3)$ be a triangular fuzzy number and r is a crisp number. The expected value of $\tilde{a}$ is

$$E[\tilde{a}] = \frac{1}{2}[(1-\rho)a_1 + a_2 + \rho a_3], \quad 0 < \rho < 1.$$
$$= \frac{a_1 + 2a_2 + a_3}{4}, \quad \rho = 0.5. \qquad \ldots.(1)$$

Proof: The Proof of the lemma-1 is in reference in Liu and Liu[10].

II.1 Bi-fuzzy set

Generally speaking, a level-2 fuzzy set is a fuzzy set in which the elements are also fuzzy sets, and the bi-fuzzy variable is a fuzzy variable with fuzzy parameters. Level-2 fuzzy sets were originally presented by Zadeh[22]. Such sets are fuzzy sets whose elements themselves are ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine some elements for a fuzzy set.

Definition 1: In Mendel[10] , a type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $(x \in X$ and $u \in J)\ x \subseteq [0,1]$, i.e.,

$$\tilde{V} = \left\{\left(\tilde{V}, \mu_{\tilde{V}}(\tilde{V})\right) / \forall x \in \tilde{\Gamma}(U) : \mu_{\tilde{V}} > 0\right\} \qquad \ldots .(2)$$

where each ordinary fuzzy set $\tilde{V}$ is defined by

$$\tilde{V} = \{(x, \mu_{\tilde{V}}(x)) / \forall x \in U : \mu_{\tilde{V}} > 0\} \qquad \ldots .(3)$$

For convenience, the membership grades $\mu_{\tilde{V}}(\tilde{V})$ of the fuzzy sets $.\tilde{V} \in \tilde{\Gamma}(U)$are called 'outer-layer' membership grades, whereas the membership grades $\mu_{\tilde{V}}(x)$ of the elements $x \in U$ are called inner-layermembership grades. Since level-2 fuzzy sets are still fuzzy sets, their mathematical behavior is defined by the fuzzy set operators. Type-2 fuzzy sets were introduced by Zadeh[24] as another extension of the concept of an ordinary fuzzy set, and it was elaborated by Mendel[12]. Such sets are fuzzy sets whose membership grades them as ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Normally speaking, a Fu-Fu variable is a fuzzy variable under fuzzy environment.

Example 2: $\tilde{\tilde{\xi}} = (s_L, \tilde{\xi}, s_R)$ with $\rho = (\rho_L, \rho_M, \rho_R)$ is called Fu-Fu variable,(Fig. 2), if the outer-layer and inner-layer membership functions are as follows

$$\mu_{\tilde{\xi}}(x) = \begin{cases} \left(\dfrac{x - s_L}{\tilde{\rho} - s_L}\right) & for\ s_L \le x < \tilde{\rho} \\ 0 & otherwise \\ \left(\dfrac{s_R - x}{s_R - \tilde{\rho}}\right) & for\ \tilde{\rho} < x \le s_R \end{cases}$$

and

$$\mu_{\tilde{\rho}}(x) = \begin{cases} \left(\dfrac{x' - \rho_L}{\rho_M - \rho_L}\right) & for\ \rho_L \le x' < \rho_M \\ 0 & otherwise \\ \left(\dfrac{\rho_R - x}{\rho_R - \rho_M}\right) & for\ \rho_M < x' \le \rho_R \end{cases}$$

where $\tilde{\rho}$ is the center of $\tilde{\xi}$, which is a triangular fuzzy variable, $s_L \in R$ and $s_R \in R$ are the smallest possible value and the largest possible value of $\tilde{\tilde{\xi}}$, $s_L \in R$, $s_M \in R$ and $s_R \in R$ are the smallest possible value, the most promising value and the largest possible value of $\tilde{\rho}$, respectively.

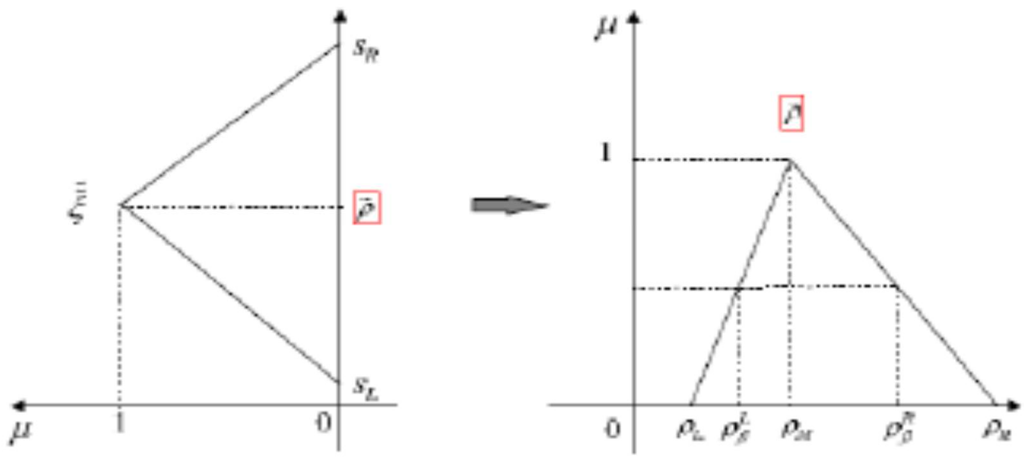

Fig. 2. Triangular Bi-fuzzy variable

Lemma-2: The expected value for the bi-fuzzy variable $\tilde{\tilde{c}} = (\tilde{c} - l_1, \tilde{c}, \tilde{c} + r_1)$ with $\tilde{c} = (c - l_2, c, c + r_2)$ we obtain that

$$E[\tilde{\tilde{c}}] = c + \frac{(r_1+r_2)-(l_1+l_2)}{4} \qquad \ldots. (4)$$

Proof: Let $\tilde{\tilde{c}} = (\tilde{c} - l_1, \tilde{c}, \tilde{c} + r_1)$ where $\tilde{c} = (c - l_2, c, c + r_2)$.Therefore

$$E[\tilde{\tilde{c}}] = \frac{E(\tilde{c}-l_1)+2E(\tilde{c})+E(\tilde{c}+r_1)}{4} \qquad \text{(Using Lemma-1)}$$

$$= \frac{E(\tilde{c})-l_1+2E(\tilde{c})+E(\tilde{c})+r_1}{4}$$

$$= \frac{4E(\tilde{c})-l_1+r_1}{4}$$

$$= E(\tilde{c}) + \frac{r_1-l_1}{4}$$

$$= c + \frac{r_2-l_2}{4} + \frac{r_1-l_1}{4}$$

$$= c + \frac{(r_1+r_2)-(l_1+l_2)}{4} .$$

Particular case: When $l_2 = 0 = r_2 \Longrightarrow \tilde{\tilde{c}} = \tilde{c} \Rightarrow E[\tilde{\tilde{c}}] = c + \frac{r_1-l_1}{4}$.

Lemma-3: Assume that ξ and η are fuzzy/ bi-fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta] \qquad \ldots. (5)$$

Proof: The proof of the Lemma-3 is in reference Xu and Zhou [20].

III. ASSUMPTIONS AND NOTATION

III.1 Assumptions

(i) Demand D(t) at time t is assumed to be $a(p) + bI(t), 0 \le t \le t_1$ and $a(p)$, $t_1 \le t \le T$;
where $a(p) = a_1 p^{-a_2}$, a_1 , a_2 and b are positive constant, p is the stock-dependent consumption rate parameter and I(t) is the inventory level at time t.
(ii) The replenishment rate is infinite and lead time is zero.
(iii) The system operates for a prescribed period of a planning horizon.
(iv) Shortages are completely backordered.
(v) The constant rate of deterioration is known and only applied to on-hand inventory.
(vi) Product transactions are followed by instantaneous cash flow.

III.2 Notations

r	discount rate, representing the time value of money
i	inflation rate
R	$r-i$, representing the net discount rate of inflation is constant
H	planning horizon
T	replenishment cycle
m	the number of replenishment during the planning horizon, $m = \frac{H}{T}$
T_j	the total time that is elapsed up to and including the jth replenishment cycle ($j = 1,2,\ldots,m$) where $T_0 = 0, T_1 = T$ and $T_m = H$.
t_j	the time at which the inventory level in the jth replenishment cycle drops to zero ($j = 1,2,\ldots,m$)
$T_j - t_j$	time period when shortage occurs ($j = 1,2,\ldots,m$)
Q	the 2nd, 3rd, . . .,mth replenishment lot size
I_m	maximum inventory level
$\theta(t)$	Deterioration rate , $\alpha\beta t^{\beta-1}$, $0<\alpha<1$, $\beta>0$, $t>0$.
A	cost per replenishment, \$/order
c	per unit cost of the item, \$/unit
$G(t)$	$c_3 t^\gamma$, $\gamma \geq 1$, per unit Inventory holding cost per unit time,\$/unit/unit time,
c_2	per unit shortage cost per unit time, \$/unit/unit time

IV. MODEL FORMULATION

Suppose that the planning horizon H is divided into m equal parts of length $T = \frac{H}{m}$. Hence, the reorder times over the planning horizon H are $T_j = jT$ ($j = 0,1,2,\ldots,m$). When the inventory is positive, consumption rate is dependent on stock levels, whereas for negative inventory, the demand (backlogging) rate is constant. Therefore, the period for which there is no-shortage in each interval $[jT,(j+1)T]$ is a fraction of the scheduling period T and is equal to kT $(0<k<1)$. Shortages occur at time $t_j = (k+j-1)T$ ($j = 1,2,\ldots,m$) and are accumulated until time $t = jT$ ($j = 1,2,\ldots,m$) before they are backordered. This model is illustrated in Fig. 3.

The first replenishment lot size of I_m is replenished at $T_0 = 0$. During the time interval $[0,t_1]$, the inventory level decreases due to stock-dependent consumption rate and deterioration until it is zero at $t = t_1$. During the time interval $[t_1,T]$, shortages occur and are accumulated until $t = T_1$ before they are backordered. Therefore, the inventory system at any time t can be represented by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -[a(p) + bI(t)] , \qquad 0 \le t \le t_1 , \qquad \ldots\ldots\ldots (6)$$

$$\frac{dI(t)}{dt} = -a(p) , \qquad t_1 \le t \le T , \qquad \ldots\ldots\ldots (7)$$

The solutions of the above differential equations after apply the boundary condition $I(t_1) = 0$, are

$$I(t) = a(p)e^{-(\alpha t^\beta + bt)}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left({t_1}^{\beta+1} - t^{\beta+1}\right) + \frac{b}{2}(t_1^2 - t^2) + \frac{\alpha b}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right)\right], \quad 0 \le t \le t_1 , \qquad \ldots\ldots\ldots (8)$$

$$I(t) = -a(p)(t - t_1) , \qquad t_1 \le t \le T , \qquad \ldots\ldots\ldots (9)$$

Therefore, the maximum inventory level during the first replenishment cycle

$$I_m = a(p)\left[\frac{kH}{m} + \frac{\alpha}{\beta+1}\left(\frac{kH}{m}\right)^{\beta+1} + \frac{b}{2}\left(\frac{kH}{m}\right)^2 + \frac{\alpha b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right] , \qquad \ldots\ldots\ldots (10)$$

and maximum shortage quantity during the first replenishment cycle

$$I_b = a(p)(T - t_1) = a(p)(1-k)\frac{H}{m} , \qquad \ldots\ldots\ldots (11)$$

Since replenishment in each cycle is done at the start of each cycle, the present value of ordering cost during the first cycle is

$$C_r = A , \qquad \ldots\ldots\ldots (12)$$

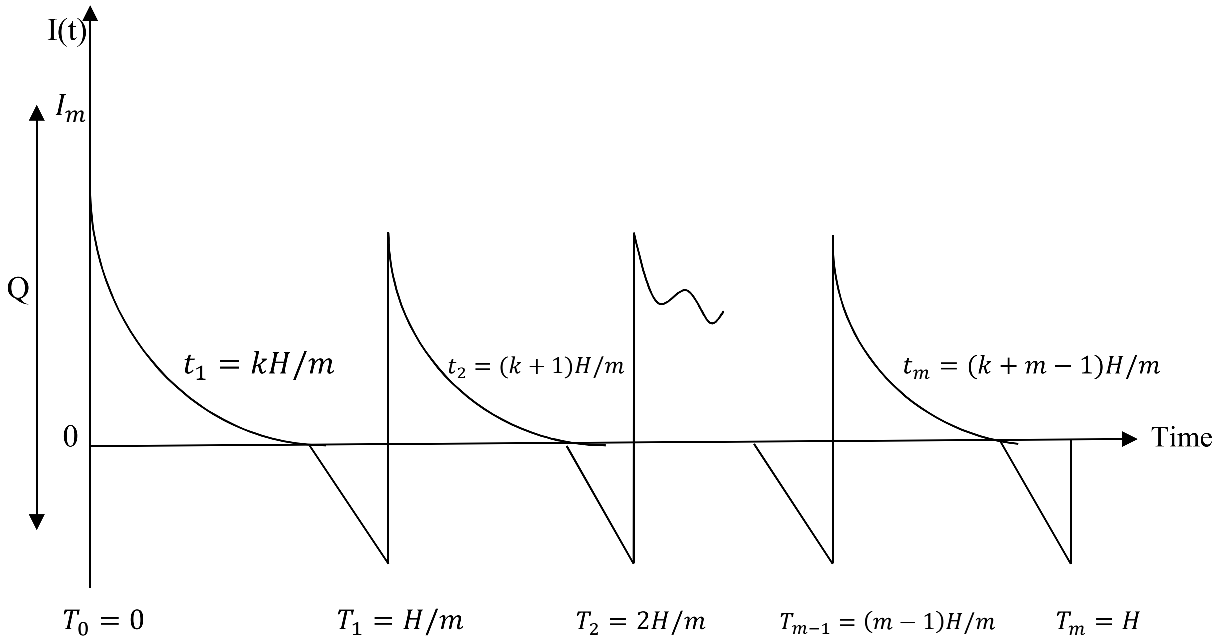

Fig.3. The graphical representation of the inventory cycles.

Inventory occurs during period t_1 therefore, the present value of holding cost during the first replenishment cycle is

$$C_h = \int_0^{t_1} c_3 t^\gamma I(t) e^{-Rt} dt$$

$$= c_3 a(p)\Bigg[\frac{t_1^{\gamma+2}}{(\gamma+1)(\gamma+2)} + \frac{\{b-R(\gamma+1)\}\, t_1^{\gamma+3}}{(\gamma+1)(\gamma+2)(\gamma+3)} + \frac{\{R^2(\gamma+2)-2bR\}t_1^{\gamma+4}}{2(\gamma+2)(\gamma+3)(\gamma+4)} + \frac{\alpha\beta\ {}_1^{\gamma+\beta+2}}{(\gamma+1)(\gamma+\beta+1)(\gamma+\beta+2)} + \frac{\alpha\beta\{b(2\gamma+\beta+3)-R(\gamma+1)(\gamma+\beta+1)\}t_1^{\gamma+\beta+3}}{(\gamma+1)(\gamma+2)(\gamma+\beta+1)(\gamma+\beta+2)(\gamma+\beta+3)} +$$
$$\frac{\alpha R\{2b(3\gamma^2+14\gamma+2\gamma\beta+5\beta+\quad)+R(\gamma+2)(\gamma+\beta+2)\}t_1^{\gamma+\beta+4}}{2(\gamma+2)(\gamma+3)(\gamma+\beta+2)(\gamma+\beta+4)} + \frac{\alpha b\ \ {}^2 t_1^{\gamma+\beta+5}}{2(\gamma+3)(\gamma+\beta+5)}\Bigg], \qquad \ldots\ldots\ldots(13)$$

The maximum shortage level is $I_b = a(p)(1-k)\frac{H}{m}$, all shortages during the interval $[t_1, T_1]$ will be completely backordered at T_1, the present value of shortage cost during the first replenishment cycle is

$$C_s = c_2 \int_{t_1}^{T_1} a(p)(t-t_1)e^{-Rt} dt$$

$$= \frac{c_2 a(p)}{R^2}\left[R\left(\frac{kH}{m} - \frac{H}{m}\right) + e^{-R\left(\frac{kH}{m}-\frac{H}{m}\right)} - 1\right]e^{-\frac{RH}{m}}\ , \qquad \ldots\ldots\ldots(14)$$

Replenishment is done at $t = 0$ and T, the replenishment items are consumed by demand as well as deterioration during t_1 . The present value of material cost during the first replenishment cycle is

$$C_p = cI_m + ce^{-RT}\int_0^{T-t_1} a(p)dt$$

$$= ca(p)\left\{\frac{kH}{m} + \frac{\alpha}{\beta+1}\left(\frac{k+1}{m}\right)^{\beta+1} + \frac{b}{2}\left(\frac{kH}{m}\right)^2 + \frac{\alpha b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right\} ca(p)e^{-\frac{RH}{m}}\left\{\frac{H}{m} - \frac{kH}{m}\right\}, \qquad \ldots\ldots\ldots(15)$$

Consequently, the present value of total cost of system during the first replenishment cycle can be formulated as
$$TRC = C_r + C_h + C_s + C_p\ , \qquad \ldots\ldots\ldots(16)$$

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There are m cycles during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at $T_m = H$ is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, there are $m+1$ replenishments in the entire planning horizon H. The first replenishment lot size is I_m , the $2nd$, $3rd$, . . .,mth replenishment lot size is

$$Q = I_m + \int_0^{T-T_1} a(p)dt \,, \qquad \ldots\ldots\ldots (17)$$

and the last or $(m+1)$th replenishment cost size is

$$I_b = a(p)\left(\frac{H}{m} - \frac{kH}{m}\right) \,, \qquad \ldots\ldots\ldots (18)$$

So, the present value of total cost of system over a finite planning horizon H is

$$TC(m,k) = \sum_{j=0}^{m-1} TRC e^{-RjT} - Ae^{-RH}$$

$$= TRC\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}\right) - Ae^{-RH} \quad , \qquad \ldots\ldots\ldots (19)$$

where $T = H/m$ and TRC derived by substituting Eqs. (12)–(15) into Eq. (16).
On simplification and summation, we get

$$TC(m,k) = A\left[\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1} - e^{-R}\right] + c_3 a(p)\left[\frac{\left(\frac{kH}{m}\right)^{\gamma+2}}{(\gamma+1)(\gamma+2)} + \frac{\{b-R(\gamma+1)\}\left(\frac{kH}{m}\right)^{\gamma+3}}{(\gamma+1)(\gamma+2)(\gamma+3)} + \frac{\{R^2(\gamma+2)-2bR\}\left(\frac{kH}{m}\right)^{\gamma+4}}{2(\gamma+2)(\gamma+3)(\gamma+4)} + \frac{\alpha\beta\left(\frac{kH}{m}\right)^{\gamma+\beta+2}}{(\gamma+1)(\gamma+\beta+1)(\gamma+\beta+2)} + \frac{\alpha\beta\{b(2\gamma+\beta+3)-R(\gamma+1)(\gamma+\beta+1)\}\left(\frac{kH}{m}\right)^{\gamma+\beta+3}}{(\gamma+1)(\gamma+2)(\gamma+\beta+1)(\gamma+\beta+2)(\gamma+\beta+3)} + \frac{\alpha R\{2b(3\gamma^2+14\gamma+2\gamma\beta+5\beta+\quad)+R(\gamma+2)(\gamma+\beta+2)\}\left(\frac{kH}{m}\right)^{\gamma+\beta+4}}{2(\gamma+2)(\gamma+3)(\gamma+\beta+2)(\gamma+\beta+4)} + \frac{\alpha b R^2\left(\frac{kH}{m}\right)^{\gamma+\beta+5}}{2(\gamma+3)(\gamma+\beta+5)}\right]\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}\right) + \frac{c_2 a(p)}{R^2}\left\{R(k-1)\frac{H}{m} + e^{-\frac{R(k-1)}{m}} - 1\right\}\left(\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1}\right) + ca(p)\left\{\frac{kH}{m} + \frac{\alpha}{\beta+1}\left(\frac{kH}{m}\right)^{\beta+1} + \frac{b}{R}\left(\frac{kH}{m}\right)^2 + \frac{\alpha b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right\}\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}\right) + ca(p)\frac{H}{m}(1-k)\left(\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1}\right), \qquad \ldots\ldots\ldots (20)$$

IV.1 Formulation of corresponding fuzzy inventory model

$\tilde{A}$ fuzzy cost per replenishment, \$/order
$\tilde{c}$ per unit fuzzy cost of the item, \$/unit
$\tilde{c_2}$ per unit fuzzy shortage cost per unit time, \$/unit/unit time
$\tilde{G}(t)$ $\tilde{c_3}t^{\gamma}, \gamma \geq 1$, per unit fuzzy inventory holding cost per unit time,\$/unit/unit time,
$\tilde{\tilde{\theta}}(t)$ bifuzzy deterioration rate , $\tilde{\tilde{\alpha}}\beta t^{\beta-1}$
$\tilde{A} = (A_1, A_2, A_3)$, $\tilde{c} = (c_{11}, c_{12}, c_{13})$ $\tilde{c}_2 = (c_{21}, c_{22}, c_{23})$, $\tilde{c}_3 = (c_{31}, c_{32}, c_{33})$,
$\tilde{\tilde{\alpha}} = (\tilde{\alpha} - \alpha_1, \tilde{\alpha}, \tilde{\alpha} + \alpha_3)$, $\tilde{\alpha} = (\alpha - \alpha_2, \alpha, \alpha + \alpha_4)$.
Then,$E(\tilde{A}) = \frac{A_1+2A_2+A_3}{4}$, $E(\tilde{c}) = \frac{c_{11}+2c_{12}+c_{13}}{4}$, $E(\tilde{c}_2) = \frac{c_{21}+2c_{22}+c_{23}}{4}$,
$E(\tilde{\tilde{\alpha}}) = \alpha + \frac{(\alpha_3+\alpha_4)-(\alpha_1+\alpha_2)}{4}$, $E(\tilde{c}_3) = \frac{c_{31}+2c_{32}+c_{33}}{4}$.

Therefore, , the defuzzified present value of total cost of system

$$E(TC) = E(\tilde{A})\left[\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1} - e^{-RH}\right] + E(\tilde{c}_3)a(p)\left[\frac{\left(\frac{kH}{m}\right)^{\gamma+2}}{(\gamma+1)(\gamma+2)} + \frac{\{b-R(\gamma+1)\}\left(\frac{kH}{m}\right)^{\gamma+3}}{(\gamma+1)(\gamma+2)(\gamma+3)} + \frac{\{R^2(\gamma+2)-2bR\}\left(\frac{kH}{m}\right)^{\gamma+4}}{2(\gamma+2)(\gamma+3)(\gamma+4)} + \frac{E(\tilde{\tilde{\alpha}})\beta\left(\frac{kH}{m}\right)^{\gamma+\beta+2}}{(\gamma+1)(\gamma+\beta+1)(\gamma+\beta+2)} + \frac{E(\tilde{\tilde{\alpha}})\beta\{b(2\gamma+\beta+3)-R(\gamma+1)(\gamma+\beta+1)\}\left(\frac{kH}{m}\right)^{\gamma+\beta+3}}{(\gamma+1)(\gamma+2)(\gamma+\beta+1)(\gamma+\beta+2)(\gamma+\beta+3)} + \frac{E(\tilde{\tilde{\alpha}})R\{2b(3\gamma^2+14\gamma+2\gamma\beta+5\beta+16)+R(\gamma+2)(\gamma+\beta+2)\}\left(\frac{kH}{m}\right)^{\gamma+\beta+4}}{2(\gamma+2)(\gamma+3)(\gamma+\beta+2)(\gamma+\beta+4)} + \frac{E(\tilde{\tilde{\alpha}})bR^2\left(\frac{kH}{m}\right)^{\gamma+\beta+5}}{2(\gamma+3)(\gamma+\beta+5)}\right]\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}\right) + \frac{E(\tilde{c}_2)a(p)}{R^2}\left\{R(k-1)\frac{H}{m} + e^{-\frac{R(k-1)}{m}} - 1\right\}\left(\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1}\right) + E(\tilde{c})a(p)\left\{\frac{kH}{m} + \frac{E(\tilde{\tilde{\alpha}})}{\beta+1}\left(\frac{kH}{m}\right)^{\beta+1} + \frac{b}{R}\left(\frac{kH}{m}\right)^2 + \frac{E(\tilde{\tilde{\alpha}})b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right\}\left(\frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}\right) + E(\tilde{c})a(p)\frac{H}{m}(1-k)\left(\frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1}\right). \qquad \ldots\ldots\ldots (21)$$

IV.2 Solution procedure

The present value of total cost $TC(m,k)$ is a function of two variables m and k where m is a discrete variable and k is a continuous variable. For a given value of m, the necessary condition for $TC(m,k)$ to be minimized is

$$\frac{dTC(m,k)}{dk} = 0. \qquad \ldots\ldots\ldots (22)$$

Furthermore, $\frac{d^2TC(m,k)}{dk^2}$ is positive. So, for a given positive integer m, the optimal value of k can be obtained from Eq. (22). Using the optimal solution procedure, we can find the maximum inventory level and optimal order quantity to be

$$I_m = a(p)\left[\frac{kH}{m} + \frac{E(\tilde{\tilde{\alpha}})}{\beta+1}\left(\frac{kH}{m}\right)^{\beta+1} + \frac{b}{2}\left(\frac{kH}{m}\right)^2 + \frac{E(\tilde{\tilde{\alpha}})b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right]$$

and

$$Q^* = a(p)\left[\frac{kH}{m} + \frac{E(\tilde{\tilde{\alpha}})}{\beta+1}\left(\frac{kH}{m}\right)^{\beta+1} + \frac{b}{2}\left(\frac{kH}{m}\right)^2 + \frac{E(\tilde{\tilde{\alpha}})b}{\beta+2}\left(\frac{kH}{m}\right)^{\beta+2}\right] + E(\tilde{\tilde{\alpha}})\left((H/m) - (k*H/m)\right)$$

respectively, by eqs(10) and (17).

V. NUMERICAL EXAMPLE

An example is devised here to illustrate the effects of the general model developed in this paper with the following data:
The inventory parametric values $a_1 = 600$ units/year, $a_2 = 0.02$, $b = 0.15$, $p = 20$, $A_1 = \$150$ /order, $A_2 = \$200$ /order, $A_3 = \$250$ /order, $\alpha = 2$, $\alpha_1 = 0.4$, $\alpha_2 = 0.9$, $\alpha_3 = 1.2$, $\alpha_4 = 1.7$, $\beta = 4$, $c_{11} = 3.1$ /unit , $c_{12} = 4.4$ /unit , $c_{13} = 5.2$ /unit , $c_{21} = 2.2$ /unit/year , $c_{22} = 2.9$ /unit/year , $c_{23} = 3.6$ /unit/year, $c_{31} = 0.8$ /unit/year , $c_{32} = 1.5$ /unit/year , $c_{33} = 2.1$ /unit/year , $R = 0.20$, $H = 10$ year.
Using the solution procedure described above, the results are presented in Table 1. From this table, we see that the number of replenishments $m = 5$, the total cost TC becomes minimum. Hence, the optimal values of m and k are $m^* = 5$ and $k^* = 0.638$, respectively and the minimum total cost $TC(m^*, k^*) = \$21050.05$.
We then have $T^* = \frac{H}{m^*} = 2$, $t_1^* = k^*T^* = 1.276$, and $Q^* = 3990.169$.

Table 1: Optimal solution with shortages

m	$k^a(m)$	T^a	Q^a	$TC(m^a, k^a)$
3	0.434	3.333	6985.245	36842.69
4	0.571	2.5	6560.961	34613.72
5	0.638^a	2^a	3990.169^a	21050.05^a
6	0.785	1.667	4446.360	23462.36
7	0.935	1.429	4869.634	25699.67

^{a} optimal solution .

Table 2: Effects of changing the stock-dependent consumption rate b on the optimal replenishment policy

b	m^a	$k^a(m)$	T^a	Q^a	$TC(m^a, k^a)$
0.15	5	0.638	2	3990.169	15062.15
0.20	5	0.586	2	2855.870	21050.05
0.25	5	0.640	2	4207.059	21165.83
0.30	5	0.631	2	4012.139	22194.81

^{a} optimal solution .

Table 3: Effects of changing the deterioration rate β on the optimal replenishment policy

β	m^a	$k^a(m)$	T^a	Q^a	$TC(m^a, k^a)$
2	5	0.8	2	6430.087	33931.49
4	5	0.571	2	6560.961	34613.72
6	5	0.694	2	9088.658	43231.76
8	5	0.679	2	8249.661	47960.17

^{a} optimal solution .

Table 4: Effects of changing the net discount rate of inflation R on the optimal replenishment policy

R	m^a	$k^a(m)$	T^a	Q^a	$TC(m^a, k^a)$
0.20	5	0.638	2	3990.169	21050.05
0.25	5	0.644	2	4172.944	22014.87
0.30	5	0.678	2	5185.060	26950.89
0.35	5	0.675	2	5108.044	27357.43

^{a} optimal solution .

In addition, the effects of changing the parameters b, β and R on the optimal replenishment policy are studied. The results are summarized in Tables 2–4. Based on Tables 2–4, the observations can be made as follows.
(1) When the stock-dependent consumption rate b is increasing, the optimal cost is increasing.
(2) When the deterioration rate h is increasing, the optimal cost is increasing.
(3) When the net discount rate of inflation R is increasing, the optimal cost is increasing.

VI. SENSITIVITY ANALYSIS

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. Using the numerical example given in the preceding section, the sensitivity analysis of various parameters has been done. The results of sensitivity analysis are summarized in Table 5. The following inferences can be made from the results obtained:

Table 5: Sensitivity analysis with respect to various parameters on order quantity and total cost for stock-dependent consumption rate model

Parameters	% Change	Q	TC
b_1	-50% -20% +20% +50%	3246.714 3823.693 5246.086 5984.372	18541.42 20699.21 27678.84 31575.01
b	-50% -20% +20% +50%	3284.852 3377.239 3825.054 4268.638	17326.91 17814.55 20178.41 22519.90
R	-50% -20% +20% +50%	2541.787 3006.980 3820.302 5185.060	13261.59 15859.93 20153.36 27357.43
β	-50% -20% +20% +50%	6430.087 4987.042 3507.348 1088.658	33931.49 26312.73 18501.08 17960.17
α	-50% -20% +20% +50%	3922.284 3744.776 3373.734 3089.318	20694.26 19756.42 17792.26 16289.76

(1) When consumption rate (b_1) decreases or increases, the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase. Similarly, the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase as stock-dependent consumption rate (b) decreases or increases. That is, the changes in b_1 and b will lead to the positive changes in Q and TC.
(2) When deterioration parameter $(\alpha \text{ and } \beta)$ decreases or increases, the ordering quantity (Q) and the present value of total cost (TC) will increase or decrease. That is, the change in deterioration rate leads to a negative change in Q and TC.
(3) The change in net discount rate of inflation (R) leads to a positive change in the present value of total cost (TC) and the ordering quantity (Q)i.e., when R increases, TC and Q will also increase.

VII. CONCLUSIONS

This model incorporates some realistic features that are likely to be associated with some kinds of inventory. Deterioration over time is a natural feature for goods. Occurrence of shortages in inventory is a natural phenomenon in real situations. It has been observed in supermarkets that the demand is usually influenced by the amount of stock displayed on the shelves, i.e., the demand rate may go up or down if the on-hand inventory level increases or decreases. Next, since the inventory systems always need to invest large capital to purchase inventories, which it is highly correlated to the return of investment. Hence, it is important to consider the effects of inflation and the time value of money in formulating inventory replenishment policy. In keeping with this reality, these factors are incorporated into the present model. The model is very useful in the retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products which are more likely with the characteristics above. We have given an analytic formulation of the problem on the framework described above and have presented an optimal solution procedure to find optimal replenishment policy. From our research results, we have also verified that the effects of inflation and the time value of money in formulating replenishment

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policy result in smaller discounted total cost than a policy which does ignore the effects of these factors. Finally, the sensitivity of the solution to changes in the values of different parameters has been discussed. It is seen that changes in the consumption rate leads to significant effects on the order quantity. The total cost is sensitive to changes in the consumption rate parameter and net discount rate of inflation .

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