Some Discussion of Gravitational constant and cosmological constant Using Bianchi Type-I cosmological Model

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Abstract- Bianchi Type-I cosmological model in General Relativity is investigated. To get an exact solution of Einstein's Field equation we have taken Gravitational constant and Cosmological constant. It is observed that Einstein's Field Equations are solvable. Some physical and geometrical properties also discussed. Two phenomenological decay of cosmological constant have been discussed.

Index Terms- Cosmological constant, Gravitational constant, Bianchi Type-I metric and Decays Law.

I. Introduction

Many Researchers stated that the expansion of the Universe is undergoing a late time acceleration(Perlmutter et al.1997,1998,1999;Ries et al.1998,2004;Efstathiou et al.2002;Spergel et al.2003;Allen et al.2004;Sahni and Starobinski 2000;Peebles and Ratra 2003;Padmanabhan 2003;Lima 2004). In General theory of Relativity some sort of Dark energy or any constant varies only slowly with time and space dominates the current cosmos. In fact all scientists have come to take a decision that the Universe is expanding. Because origin and Nature of the Universe acceleration is a question for Researchers. The homogeneous anisotropic models have spatial sections which are flat but the expansion or contraction rate are dependable on direction. From different point of view many researchers have investigated Bianchi Type-I models. Bianchi Type-I homogeneous models have flat spatial sections and the direction of expansion or contraction are dependable. Many Researchers have investigated Bianchi Type-I models from different point of view for observing the effects of anisotropy in the early Universe.

Cosmological constant is the energy density of the vacuum space. It was originally introduced as an addition to his General theory of Relativity to "hold back gravity" and achieve a static Universe. For dark energy existence we can say that energy density stored on the vacuum state of all existing fields is the Universe. In the early stage of the Universe value of Λ was large and decayed with evolution (Dolgov 1983). Einstein introduced the cosmological constant in 1917.According to him Cosmological constant act as a repulsive force to keep the Universe in static equilibrium. Another definition is cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. In Modern Cosmology, it is the leading parameter for Dark Energy, which is one of the reasons of acceleration of Universe. vacuum energy and the cosmological constant have identical behaviour in general relativity, as long as the vacuum energy density is identified with, $\rho^{vac} = \frac{\Lambda}{8\pi G}$.The term Λ arises naturally in general relativistic quantum field theory, where it is interpreted as the energy density of the vacuum[1-4].Some authors [5-9] argued for the dependence of Λ . Bianchi Type-I models are studied by using variable G and Λ [10-16]. Bianchi type-I string cosmological model with bulk viscosity investigated by Tiwari and Sonia[17].

The gravitational constant is an empirical physical constant which is obtained in Newton's law of Universal gravitation and in Einstein's general theory of Relativity. It plays a role in the relation connecting the gravitational force between two bodies with the product of their masses and the inverse square of their distance. In general theory of Relativity it identifies the relation between Spacetime geometry and energy-momentum. Again Gravitational constant plays the role of a coupling constant between matter and geometry in the Einstein's Field equation. Dirac[16-18] and Dicke[14] suggested a time varying gravitational constant for first time. The large number hypothesis (LNH) proposed by Dirac leads to a cosmology when G varies with time. In astrophysics variation of G has many interesting consequence. Canuto and Narlikar [15] have shown that G-varying cosmology is consistent.

II. The Metric, Field Equations and Solutions

Let us consider Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2$$

The energy momentum tensor of a perfect fluid is represented by

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}$$

where ρ is the energy density, p is the thermodynamical pressure and v_i is the four velocity vectors of the fluid satisfying the relation

$$v_i v^i = -1$$

The Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij}$$

For the metric (1) and the energy-momentum tensor (2) in commoving system of coordinates, the field equation (4) yields

$$8\pi G p - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} ,$$

$$8\pi G p - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} ,$$

$$8\pi G p - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} ,$$

$$8\pi G \rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} .$$

In view of vanishing divergence of Einstein tensor, we get

$$8\pi G \left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0$$

The usual energy conservation equation $T^{j}_{i;j} = 0$ yields

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0$$

Combining (9) and (10) we get,

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0$$

Let R denotes the average scale factor of Bianchi type-I cosmology, then

$$R^3 = \sqrt{-g} = ABC$$

Subtracting (6) from (5) we get,

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC} = 0$$

$$\Rightarrow ABC\left(\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC}\right) = ABC \times 0$$

$$\Rightarrow \ddot{A}BC + \dot{A}\dot{C}B - A\ddot{B}C - A\dot{B}\dot{C} = 0$$

$$\Rightarrow \ddot{A}BC + \dot{A}\dot{C}B + \dot{A}\dot{B}C - \dot{A}\dot{B}C - A\ddot{B}C - A\dot{B}\dot{C} = 0$$

$$\Rightarrow d(\dot{A}BC) - d(A\dot{B}C) = 0$$

Integrating we get,

$$\dot{A}BC - A\dot{B}C = k_1, \quad (Constant\ of\ integration)$$

$$\Rightarrow \frac{\dot{A}BC}{ABC} - \frac{A\dot{B}C}{ABC} = \frac{k_1}{ABC}$$

$$\Rightarrow \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3} \qquad [Using\ (12)] \tag{13}$$

Similarly, subtracting (7) from (6)

$$\Rightarrow \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3} \tag{14}$$

Integrating (13),

$$\ln A - \ln B = \int \frac{k_1}{R^3} dt$$

$$\Rightarrow \ln\frac{A}{B} = k_1 \int \frac{dt}{R^3}$$

$$\Rightarrow \frac{A}{B} = exp\left[k_1 \int \frac{dt}{R^3}\right]$$

$$\Rightarrow A = B exp\left[k_1 \int \frac{dt}{R^3}\right] \tag{15}$$

Similarly integrating (14) we get,

(4)

$$B = C exp\left[k_2 \int \frac{dt}{R^3}\right] \tag{16}$$

Multiplying (15) by (16) we get,

$$AB = BC\, exp\left[(k_1 + k_2) \int \frac{dt}{R^3}\right]$$

(5)

From (12)

(6)

$$ABC = R^3$$

$$\Rightarrow BC\, exp\left[(k_1 + k_2) \int \frac{dt}{R^3}\right] \cdot C = R^3$$

(7)

$$\Rightarrow BC^2\, exp\left[(k_1 + k_2) \int \frac{dt}{R^3}\right] = R^3$$

(8)

$$\Rightarrow C exp\left[k_2 \int \frac{dt}{R^3}\right] C^2\, exp\left[(k_1 + k_2) \int \frac{dt}{R^3}\right] = R^3$$

$$\Rightarrow C^3\, exp\left[(k_1 + 2k_2) \int \frac{dt}{R^3}\right] = R^3$$

(9)

$$\Rightarrow C^3 = R^3\ exp\left[-(k_1 + 2k_2) \int \frac{dt}{R^3}\right]$$

$$\Rightarrow C = m_3 R\, exp\left[-\frac{(k_1 + 2k_2)}{3} \int \frac{dt}{R^3}\right] \tag{17}$$

(10)

From (16) we get,

$$B = R \cdot exp\left[-\frac{(k_1 + 2k_2)}{3} \int \frac{dt}{R^3}\right] \cdot exp\left[k_2 \int \frac{dt}{R^3}\right]$$

(11)

$$\Rightarrow B = R \cdot exp\left[\frac{-k_1 - 2k_2 + 3k_2}{3} \int \frac{dt}{R^3}\right]$$

(12)

$$\Rightarrow B = m_2 R \cdot exp\left[\frac{(k_2 - k_1)}{3} \int \frac{dt}{R^3}\right]$$

From (15) we get,

$$A = R \cdot exp\left[\frac{(k_2 - k_1)}{3} \int \frac{dt}{R^3}\right] \cdot exp\left[k_1 \int \frac{dt}{R^3}\right]$$

$$\Rightarrow A = R \cdot exp\left[\frac{(k_2 - k_1 + 3k_1)}{3} \int \frac{dt}{R^3}\right]$$

$$\Rightarrow A = m_1 R \cdot exp\left[\frac{(2k_1 + k_2)}{3} \int \frac{dt}{R^3}\right]$$

where m_1, m_2 and m_3 are constants of integration and $m_1 \cdot m_2 \cdot m_3 = 1$.

Once the values of A, B and C are obtained, we now discuss the following two cases:

Case 1: Cosmological constant Λ is constant

∴ From (11),

$$\dot{G} = -\frac{\dot{\Lambda}}{8\pi\rho} \tag{21}$$

When $\Lambda = \text{constant}, \dot{\Lambda} = 0$

∴ From (21),

$$\dot{G} = 0$$

$$\Rightarrow G = constant$$

Thus we observed that when cosmological constant is constant then the gravitational constant is also behaves as constant.

From (20) we have,

$$\dot{A} = m_1 R \cdot exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]\cdot\frac{(2k_1+k_2)}{3}\cdot\frac{1}{R^3} + m_1\dot{R}\cdot exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]$$

$$\Rightarrow \frac{\dot{A}}{A} = \frac{2k_1+k_2}{3R^3} + \frac{\dot{R}}{R}$$

Similarly,

$$\frac{\dot{B}}{B} = \frac{k_2-k_1}{3R^3} + \frac{\dot{R}}{R}$$

$$\frac{\dot{C}}{C} = \frac{-(k_1+2k_2)}{3R^3} + \frac{\dot{R}}{R}$$

Again

$$\ddot{A} = m_1 R^{-2} exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]\cdot\frac{(2k_1+k_2)}{3}\cdot\frac{1}{R^3}\cdot\frac{(2k_1+k_2)}{3}$$

$$+m_1(-2)R^{-3} exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]\cdot\frac{(2k_1+k_2)}{3}$$

$$+m_1\dot{R} exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]\cdot\frac{(2k_1+k_2)}{3}\cdot\frac{1}{R^3} + m_1\ddot{R} exp\left[\frac{(2k_1+k_2)}{3}\int\frac{dt}{R^3}\right]$$

$$\Rightarrow \frac{\ddot{A}}{A} = \frac{(2k_1+k_2)^2}{9R^6} - \frac{2(2k_1+k_2)}{3R^4} + \frac{(2k_1+k_2)\dot{R}}{3R^4} + \frac{\ddot{R}}{R}$$

Similarly,

$$\frac{\ddot{B}}{B} = \frac{(k_2-k_1)^2}{9R^6} - \frac{2(k_2-k_1)}{3R^4} + \frac{(k_2-k_1)\dot{R}}{3R^4} + \frac{\ddot{R}}{R}$$

$$\frac{\ddot{C}}{C} = \frac{(k_1+2k_2)^2}{9R^6} + \frac{2(k_1+2k_2)}{3R^4} - \frac{(k_1+2k_2)\dot{R}}{3R^4} + \frac{\ddot{R}}{R}$$

We now find the behaviour of the density and pressure

From (6),

$$p = \frac{1}{8\pi G}\left[\Lambda - \left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC}\right)\right]$$

$$= \frac{1}{8\pi G}\left[\Lambda - \frac{3k_1^2+4k_1k_2+3k_2^2}{9R^6} + \frac{2k_1-2k_2}{3R^4} - \frac{(2k_1-2k_2)\dot{R}}{3R^4} - \frac{2\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2\right] \qquad (28)$$

Let us assume that $p = \omega\rho, \quad where\ 0 \le \omega \le 1$

$$\Rightarrow \rho = \frac{p}{\omega}$$

$$= \frac{1}{8\pi G\omega}\left[\Lambda - \frac{3k_1^2+4k_1k_2+3k_2^2}{9R^6} + \frac{2k_1-2k_2}{3R^4} - \frac{(2k_1-2k_2)\dot{R}}{3R^4} - \frac{2\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2\right] \qquad (29)$$

The relationship among physical quantities, *Hubble parameter H, volume expansion scalar θ, shear σ and deceleration paprametere q* are

$$H = \frac{\dot{R}}{R}$$

$$\theta = 3H = \frac{3\dot{R}}{R},$$

$$\sigma = \frac{k}{\sqrt{3}R^3}, \quad k\ is\ a\ positive\ constant.$$

$$i.e.\ \sigma \propto \frac{1}{R^3}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}$$

Case 2: Cosmological constant Λ is variable

We consider the decays laws

i. $\Lambda \propto H^2$

ii. $\Lambda \propto \frac{1}{R^2}$

[1]

[2] First we assume $\Lambda = 3\beta H^2$ (i.e. $\Lambda \propto H^2$)

$$\Rightarrow \dot{\Lambda} = 6\beta H\dot{H}$$

$\therefore$ from (21)

$$\dot{G} = -\frac{3\beta}{4\pi\rho}H\dot{H} \qquad (22)$$

$$\Rightarrow \frac{dG}{dt} = -\frac{3\beta}{4\pi\rho}H\frac{dH}{dt} \qquad (23)$$

$$\Rightarrow dG = -\frac{3\beta}{4\pi\rho}HdH \qquad (24)$$

$$Integrating, \quad G = -\frac{3\beta H^2}{8\pi\rho}$$

$$i.e.\ G \propto H^2$$

Hence when time increases, pressure decreases, Gravitational constant is directly proportional to the square of the Hubble parameter and assumed the density as a constant.

[3] Next we assume $\Lambda = hR^{-2}; h\ is\ constant. (i.e.\ \Lambda \propto \frac{1}{R^2})$

[4] $\therefore \dot{\Lambda} = -2hR^{-3}\dot{R}$

[5] From (21)

$$\dot{G} = \frac{hR^{-3}\dot{R}}{4\pi\rho} \qquad (25)$$

$$\Rightarrow \frac{dG}{dt} = \frac{hR^{-3}}{4\pi\rho}\frac{dR}{dt} \qquad (26)$$

$$Integrating, \quad G = -\frac{h}{4\pi\rho}\frac{1}{R^2} \qquad (27)$$

$$i.e. G \propto \frac{1}{R^2}$$

It should be noted that G and Λ appears as indirectly coupled fields, similar to the case of G in the original Brans-Dicke theory [24].

III. Physical Interpretations

Deceleration Parameter is

$$q = -\frac{R\ddot{R}}{\dot{R}^2}$$

(30)

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\Lambda}{\theta^2}$$

From this we get **(31)**

When

$$\Lambda \ge 0, \quad 0 < \frac{\sigma^2}{\theta^2} \le \frac{1}{3}$$

And

$$0 < \frac{\rho}{\theta^2} \le \frac{1}{3}$$

It is seen that the ratio between density and square of expansion scalar lies between 0 and 1/3 for cosmological constant 0 or positive. This indicates about upper limit of Anisotropy. From equation (a) we obtain

$$\frac{3\sigma^2}{3H^2} = 1 - \frac{\rho}{\rho_C} - \frac{\rho_\Lambda}{\rho_C}$$

(32)

$$\rho_C = 3H^2$$

Is the critical density and

$$\rho_\Lambda = \Lambda$$

Is the vacuum density. Hubble Parameter H can also be expressed as

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

It is observed that Hubble parameter gives a decrease value with the increase of time. Also Expansion scalar decreases with time increases.

In case ii, we consider decaying vacuum energy density. i,e

$$\Lambda = kH^2$$

(a)

Where k is a constant. From field equation (8) we get

$$3H^2 - \sigma^2 = \rho + \Lambda$$

(b)

When the Universe expanded energy density decreases and an infinitely large R corresponds to a ρ close to zero. For this

$$3H^2 - \Lambda \to 0$$

(i) Cosmological constant must be positive

(ii)Since Hubble parameter zero R becomes constant

(iii)For positive cosmological constant the evaluation of Universe never comes to halt. It is observed that the presence of dark energy i,e positive cosmological constant gives the accelerated universe.

From (a) and (b)

$$k = \frac{3\Omega}{1+\Omega}\left(1 - \frac{\sigma^2}{27\theta^2}\right)$$

Where

$$\Omega = \frac{\Lambda}{\rho}$$

(c)

Deceleration Parameter q can be obtained as

$$q = 2 - k$$

Recent observations by S.Perlmutter [25] gives that the present universe is accelerating phase and deceleration parameter lies somewhere in the range $0 < q \le 1$. It also follows that our model of the universe consistent with recent observations. The deceleration parameter q approaches the value zero (when a= -3) as in the case of De-sitter universe.

IV. Conclusions

We have investigated Bianchi type-I cosmology containing barotropic equation of state and cosmological term proportional to the square of Hubble parameter and inversely proportional to the square of the scale factor. It is observed that this model gives a Universe which is not start from early stage. As proper volume becomes infinitely large as t tends to infinity, the other physical quantities such as density, pressure, shear becomes insignificant. Also shear tends to zero faster than the expansion. From equation (28) and (29) it is seen that Pressure and density decreases with the expansion of the Universe. It is found that deceleration parameter value is 2 when k=0. And for k=2, deceleration parameter is 0. Thus it is observed that contraction or expansion of the Universe depends on deceleration parameter. Also it is seen that Gravitational constant behaves like cosmological constant behaviour. When cosmological constant is constant, Gravitational constant also constant. Equation (32) gives an empty Universe When time t→∞. Another important matter that during variation of cosmological constant density observerved as a constant.

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